# Towards Perfect and Compact Symmetry Breaking: Computing Unavoidable Subgraphs Using SAT Solvers

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# Outline

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Symmetry-breaking methods have been highly crucial for solving **Boolean satisfiability problems (SAT)** with large numbers of symmetries.

However, existing state-of-the-art symmetry-breaking methods, such as **shatter**, are not powerful enough to help SAT solvers successfully solving hard graph-related problems.

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| #graphs | $2^1$ | $2^3$ | $2^6$ | $2^{10}$ | $2^{15}$ | $2^{21}$ | $2^{28}$ |
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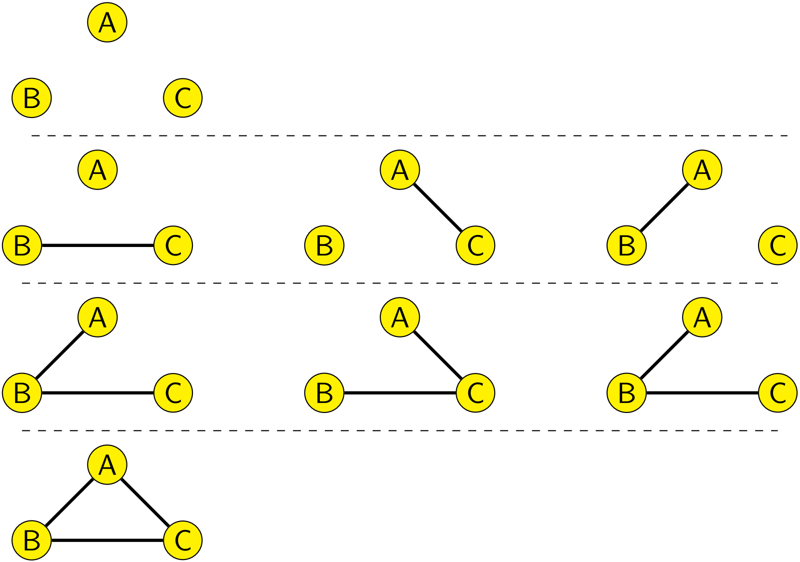

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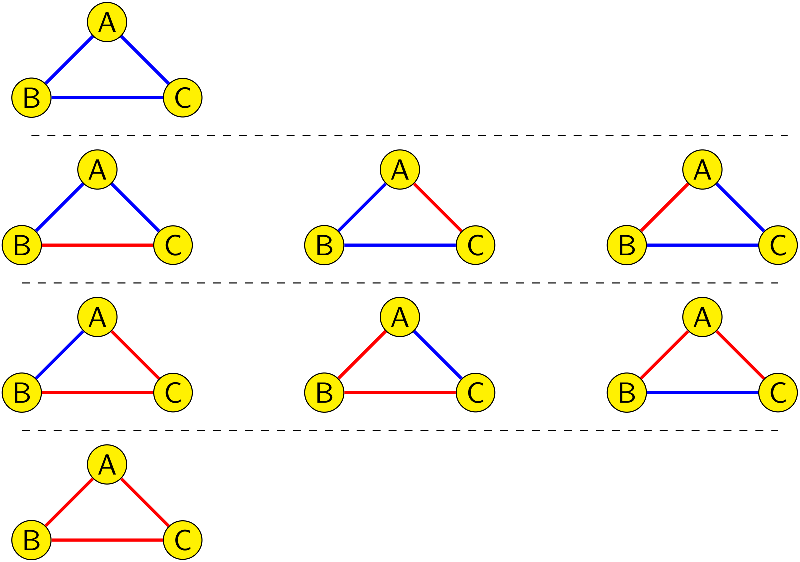

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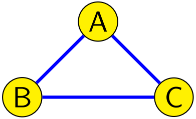
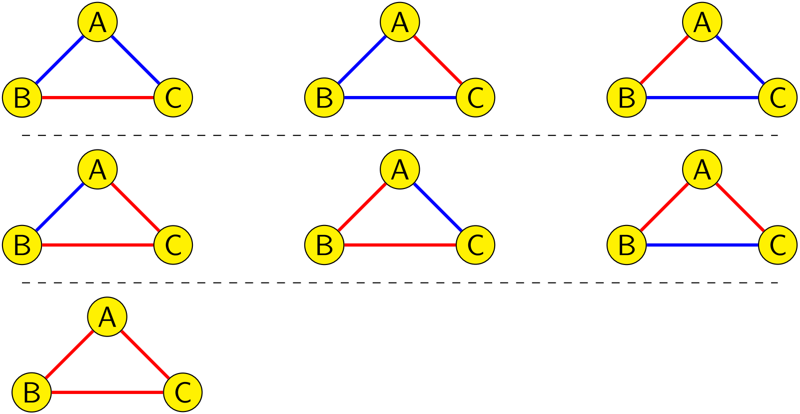



A path of two edges has the same color.

(Unavoidable subgraph)



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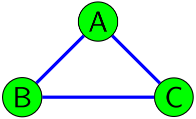
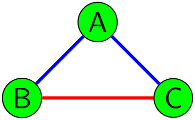
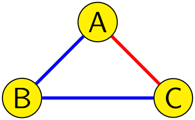


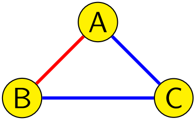

Convert an unavoidable subgraph to

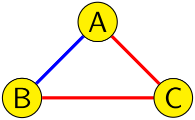
a constraint (an isolator):

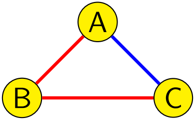
Path B-A-C has the same color.

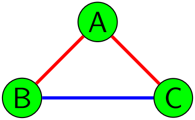


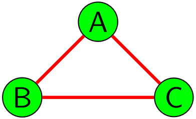



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The number of all possible two-color edge-labelings of $K_n$ is $2^{|E_{K_n}|}$.

Using a constraint derived from an unavoidable subgraph $G$ with $m$ connected components, the search space is reduced to $2^{|E_{K_n}|-(|E_G|-m)}$.

# Introduction

In this talk, I will focus on answering the following three questions:

1. What is an unavoidable subgraph?
2. How to efficiently compute unavoidable subgraphs of a given complete graph?
3. How to derive a constraint, say an isolator, from unavoidable subgraphs so that it can help to avoid examining isomorphic graphs?

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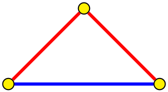

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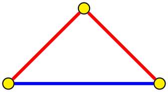

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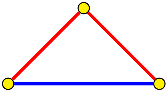

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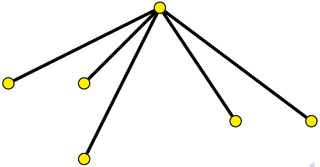

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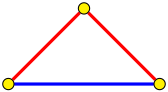

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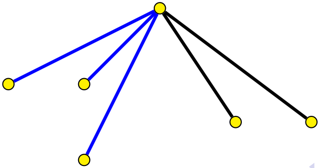

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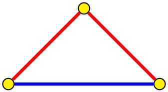

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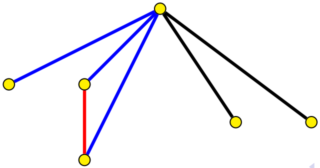

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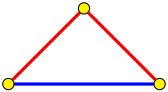

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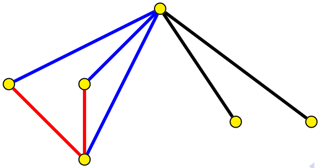

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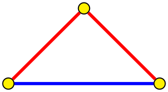

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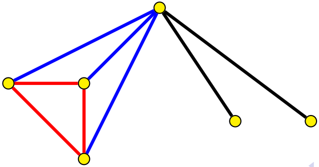
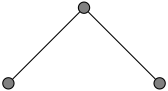
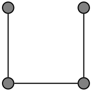


# Largest Connected Unavoidable Subgraph Examples

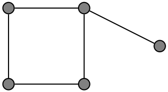
$K_3$ & $K_4$:



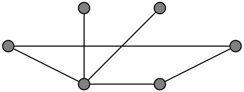
$K_5$:



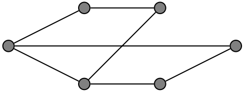
$K_6$:



$K_7$:



$K_8$:



# Unavoidable Subgraph

Many "nicely" structured unavoidable subgraphs have been studied. E.g., cliques (Ramsey numbers), cycles, paths, stars, trees, wheels, etc.

Our approach tries to compute a largest unavoidable subgraph for a given complete graph. The subgraph's structure doesn't have to be "nice".

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**Approach:** Start with computing unavoidable subgraphs for small graphs, which can be computed easily by SAT solvers. Then, these unavoidable subgraphs will be converted into an isolator for computing unavoidable subgraphs of bigger graphs. And so on.

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# SAT Encoding of Unavoidable Subgraphs

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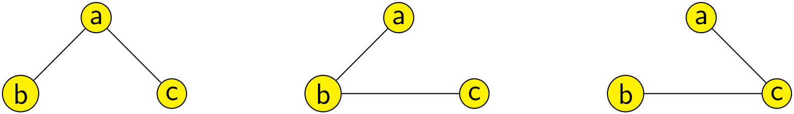

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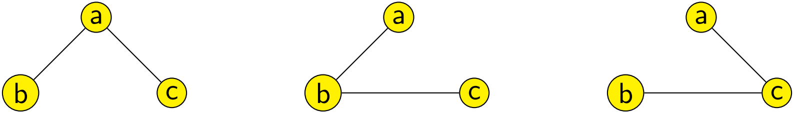

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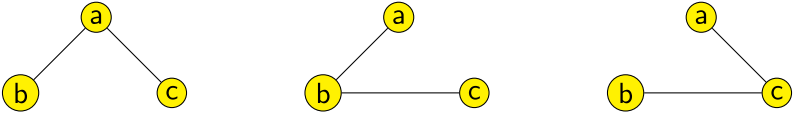


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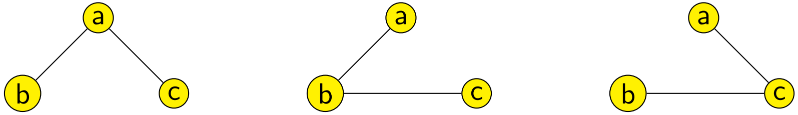


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$$(ab \wedge ac) \vee (\overline{ab} \wedge \overline{ac})$$

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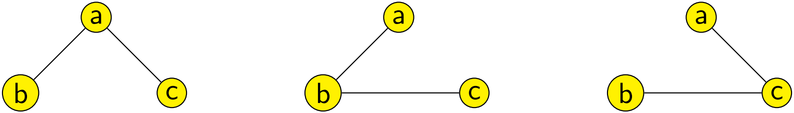


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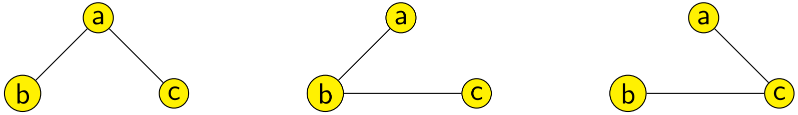


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$$(ab \land ac) \lor (\overline{ab} \land \overline{ac}) \quad (ab \land bc) \lor (\overline{ab} \land \overline{bc}) \quad (ac \land bc) \lor (\overline{ac} \land \overline{bc})$$

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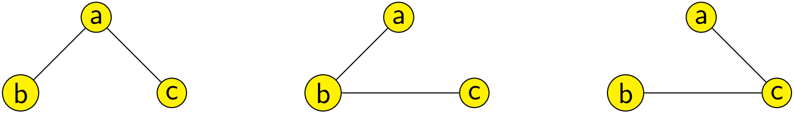


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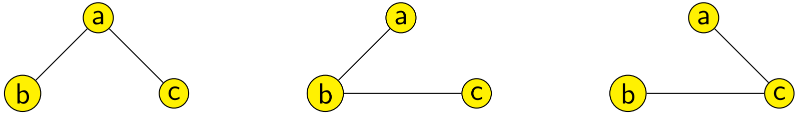


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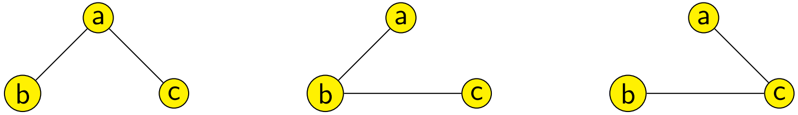


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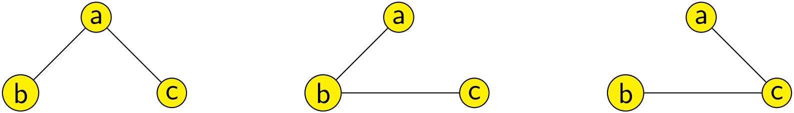


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$\overline{\mathcal{F}_G}$ is UNSATISFIABLE $\Leftrightarrow$ $\mathcal{F}_G$ is VALID $\Leftrightarrow$ A path of two edges is an unavoidable subgraph of $K_3$.

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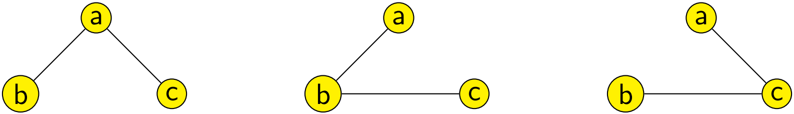


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$$\mathcal{F}_G = (ab \wedge ac) \vee (\overline{ab} \wedge \overline{ac}) \vee (ab \wedge bc) \vee (\overline{ab} \wedge \overline{bc}) \vee (ac \wedge bc) \vee (\overline{ac} \wedge \overline{bc})$$

$$\overline{\mathcal{F}_G} = (\overline{ab} \vee \overline{ac}) \wedge (ab \vee ac) \wedge (\overline{ab} \vee \overline{bc}) \wedge (ab \vee bc) \wedge (\overline{ac} \vee \overline{bc}) \wedge (ac \vee bc)$$

$\overline{\mathcal{F}_G}$ is UNSATISFIABLE $\Leftrightarrow$ $\mathcal{F}_G$ is VALID $\Leftrightarrow$ A path of two edges is an unavoidable subgraph of $K_3$.

Since $\overline{\mathcal{F}_G}$ is in CNF format, SAT solvers can solve it directly.

# SAT Encoding of Unavoidable Subgraphs

**Encoding algorithm:** Construct $\overline{\mathcal{F}_G}$.

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3. $\mathcal{F}_r$ = disjunction of all positive literals representing the red color of the edges in $H$.
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6. Return $\overline{\mathcal{F}_G}$.

# Computing Isomorphic Graphs

**Inputs:**

1. An adjacency matrix $A_G$ representing a graph $G$ of order $k$.
2. $n$ (number of nodes of $K_n$).

**Output:**

A set $\mathcal{G}$ of all subgraphs of $K_n$ that are isomorphic to $G$.

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**Algorithm:**

1. $\mathcal{G} = \varnothing$. For each combination of $k$ nodes taken from $n$ nodes:
2. $\quad ACC = \varnothing$.
3. $\quad$ Compute all permutations $P$ of $k$ nodes. For each $P_i \in P$:
4. $\quad\quad$ Construct a graph $H$ by applying $A_G$ to $P_i$. $H$ is isomorphic to $G$.
5. $\quad\quad$ If $H \notin ACC$, then add $H$ to $ACC$.
6. $\quad \mathcal{G} = \mathcal{G} \cup ACC$.
7. Return $\mathcal{G}$.

# Computing Unavoidable Subgraphs of $k$ Nodes for $K_n$

The tool **nauty** can generate all non-isomorphic graphs of $k$ ($\leq 10$) nodes in adjacency matrix format very quickly.

When $n$ is small ($\leq 20$), we are able to compute all unavoidable subgraphs of order $k$ ($\leq 6$) automatically with the help of symmetry-breaking predicates generated from **shatter** (We haven't tried with subgraphs from 7 to 10 nodes). These experiments were performed on TACC (Texas Advanced Computing Center).

# Computing Unavoidable Subgraphs of $k$ Nodes for $K_n$

**Inputs:**

1. $k$ (number of nodes of $G$).
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A set $\mathcal{H}$ of all non-isomorphic graphs of order $k$ that are unavoidable in $K_n$.

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A set $\mathcal{H}$ of all non-isomorphic graphs of order $k$ that are unavoidable in $K_n$.

**Algorithm:**

1. Generate symmetry-breaking predicates $SB$ for $K_n$ using **shatter**.

2. Generate all non-isomorphic graphs of order $k$ in adjacency matrix format using the **nauty** package.

3. $\mathcal{H} = \varnothing$. For each graph $G$ generated in Step 2:

4. Construct $\overline{\mathcal{F}_G}$ (as described).

5. Check the satisfiability of $(\overline{\mathcal{F}_G} \wedge SB)$ using a SAT solver.

6. If $(\overline{\mathcal{F}_G} \wedge SB)$ is UNSATISFIABLE, then add $G$ to $\mathcal{H}$.

7. Return $\mathcal{H}$.

# Outline

# Deriving An Isolator from Unavoidable Subgraphs

Converting an unavoidable subgraph $G$ into an isolator by forcing all edges in each connected component of $G$ to have the same color.

$$e_1 \leftrightarrow e_2 \leftrightarrow e_3 \leftrightarrow \cdots \leftrightarrow e_m \equiv e_1 \rightarrow e_2 \rightarrow e_3 \rightarrow \cdots \rightarrow e_m \;(e_m \rightarrow e_1)$$

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Each monochromatic connected component of $m$ edges can be encoded as a CNF formula consisting of $m$ binary clauses.

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**Example:** Convert a path of two edges into an isolator for $K_3$.

$\mathcal{I}_3 = ab \leftrightarrow ac = (\overline{ab} \vee ac) \wedge (\overline{ac} \vee ab)$

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**Example:** Convert a path of two edges into an isolator for $K_3$.

$\mathcal{I}_3 = ab \leftrightarrow ac = (\overline{ab} \vee ac) \wedge (\overline{ac} \vee ab)$

$\mathcal{I}_3$ has 4 satisfying assignments:

1. $ab := F, ac := F, bc := F$ (a graph with zero edges).
2. $ab := F, ac := F, bc := T$ (a graph with one edge).
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$\Rightarrow \mathcal{I}_3$ is also a perfect isolator for $K_3$, i.e., only one graph from each isomorphism class of all graphs of order 3 satisfies $\mathcal{I}_3$.

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# Outline

# Summary

We are in the first step towards constructing perfect isolators (i.e., isolators that break all symmetries) with reasonable sizes (polynomial in the size of the corresponding graph-related problem): computing initial isolators from unavoidable subgraphs.

We have presented a method for automatically computing all unavoidable subgraphs of order $k$ for a given complete graph of order $n$ using SAT solvers.

Deriving an isolator from unavoidable subgraphs is straightforward as presented.

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Isolators derived from unavoidable subgraphs of small graphs can be used to compute unavoidable subgraphs of bigger graphs.

Have some "crazy" moments with C programming.

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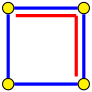

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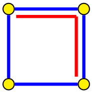

2. Study a technique for constructing a perfect isolator from an initial isolator, which is derived from (asymmetric) unavoidable subgraphs.

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Combine (1) symmetry-breaking predicates generated from **shatter** with (2) isolators derived from known unavoidable subgraphs in computing unknown unavoidable subgraphs.

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**Requirement:** Constraints in (2) have to be consistent with constraints in (1). Otherwise, they can remove completely certain isomorphism classes of the original problem.

Thank You!