Development of a Verified, Efficient Checker for SAT Proofs

Matt Kaufmann
(In collaboration with Marijn Heule and Warren Hunt, Jr.)

ACL2 Seminar
The University of Texas at Austin

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ABSTRACT

I'll present a case study, consisting of a sequence of verified checkers that validate SAT proofs. These culminate in an efficient checker that can be used in SAT competitions and in industry. No background in SAT is assumed.

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- Nathan Wetzler wrote and verified an ACL2 program that validates SAT proofs.
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Underlining denotes links to the ACL2+books online manual.

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- ... but not *that* simple, and *inspection is error-prone*.

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Background:

Variables, Literals, Clauses, Formulas

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A formula is *satisfiable* if it is true under some assignment; otherwise, it is *unsatisfiable*.

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- All addition steps *preserve satisfiability* (see next slide).

Proofs (2)

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Then Π *preserves satisfiability* when for each addition step p_i, if F_{i-1} is satisfiable then F_i is satisfiable.

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All checkers discussed today use a formalization like the one on the next slide, based on RAT.

Formalizing Soundness

Below, `proofp` is a recognizer for proofs, and `solutionp` checks that a formula is true under a given assignment,

```
(defun refutationp (proof formula)
  (declare (xargs :guard (formulap formula)))
  (and (proofp proof formula)
       (member *empty-clause* proof)))

(defun-sk exists-solution (formula)
  (exists assignment
          (solutionp assignment formula)))

(defthm main-theorem
  (implies (and (formulap formula)
                (refutationp clause-list formula))
           (not (exists-solution formula))))
```

Formalizing Soundness (2)

The following is easily proved by induction.

Lemma. Suppose that $\Pi = \langle p_1, p_2, ..., p_k \rangle$ is a proof and F_0 is satisfiable. Then each F_i is satisfiable.

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Soundness argument:

1. Deletion steps clearly preserve satisfiability.
2. Addition steps preserve satisfiability. [Must be proved!]
3. By the lemma, if F_0 is satisfiable then F_k is satisfiable.
4. Since p_k adds the empty clause, F_k is unsatisfiable.
5. It follows immediately that F_0 is unsatisfiable.

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Marijn's request: a formally verified checker for SAT competitions

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- Its efficiency benefits in part from some techniques not yet invented at the time of Nathan's work.

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2. *DRAT-trim* [2] takes inputs Π_0 and *F*; outputs alleged proof Π_1 for checker, in a format amenable to efficient checking.
3. Verified ACL2 checker validates that Π_1 is a proof for *F*.

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This table shows times (in seconds) for some checker runs, on examples provided by Marijn.

test	**[rat]** *(Wetzler)*	**[drat]** *(deletion)*	**[lrat-1]** *(fast-alist)*	**[lrat-2]** *(shrink)*	**[lrat-3]** *(clean up)*	**[lrat-4]** *(stobjs)*
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tph6[-dd]	-	-	6.18	0.56	0.54	0.46
R_4_4_18	~1 week	-	217.91	9.62	3.21	2.56
transform	-	-	47.80	9.59	8.82	8.77
schur	-	-	4674.18	1872.07	1884.23	246.94

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Times do not include parsing. Warren Hunt has sped up our original parser, and there are plans to speed it up further by using a *binary proof format* (not discussed further here).

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6. **[lrat-4]** Added **stobjs** for assignments

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Profiling (Marijn's suggestion) helped with discovering bottlenecks:

```
(include-book "centaur/memoize/old/profile"
              :dir :system)
(profile-all) ; or just profile specific functions
<evaluate forms>
(memsum)
```

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- Optimize the program for efficiency.
- Deal with proving correctness for the optimizations.

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But the **[drat]** checker is still slow. **Why?**

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The next slide breaks this line apart.

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For the RAT check on clause 163, no UP is performed.
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End of proof step:

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The next checker implements these efficiencies.

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  - This list is a *fast-alist*: ACL2 uses a hash-table to find c from i in essentially constant time.

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- A formula represents a list of pairs `(i . c)` where i is a natural number, the *index* of clause c.
  - This list is a *fast-alist*: ACL2 uses a hash-table to find c from i in essentially constant time.
  - A formula is a pair `(max . fal)`, where `fal` is its fast-alist and `max` is an upper bound on its indices.

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- Deletion of clause *i* simply extends the fast-alist with a pair `(`*i* `. *deleted-clause*)`.
  - The value of `*deleted-clause*` is a non-`nil` atom, hence not a clause.

[lrat-1] (3)

Soundness Proof Problem:
How to manage the substantial change from **[drat]** to **[lrat-1]**.

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Soundness Proof Problem:
How to manage the substantial change from **[drat]** to **[lrat-1]**.

- Painful to rework another's proof
- Decision: Sketch hand proof and manage a fresh proof
- Used top-down approach (see my 1999 ACL2 Workshop paper)

satisfiable-add-proof-clause.lisp

```
<hand proof in comment>
(in-package "ACL2")
(include-book "lrat-checker")

(local (encapsulate ()
   (local (include-book "satisfiable-add-proof-clause-rup"))
   (local (include-book "satisfiable-add-proof-clause-drat"))
   (set-enforce-redundancy t)
   (defthm satisfiable-add-proof-clause-rup
     ...)
   (defthm satisfiable-add-proof-clause-drat
     ...)))

(defthm satisfiable-add-proof-clause
  ...
  :hints
  (("Goal" :use (satisfiable-add-proof-clause-rup
                 satisfiable-add-proof-clause-drat)
    :in-theory (union-theories '(verify-clause)
                               (theory 'minimal-theory)))))
```

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The **[lrat-2]** checker improves on **[lrat-1]** in two ways:

- Shrink the formula's fast-alist when heuristics say to do so.
- RAT check recurs through the fast-alist instead of recurring down from the `max` index.

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Heuristically shrink the fast-alist at an addition proof step, based on experimentation:

- whenever $ndel > 10 * ncls$;
- when RAT check is necessary, shrink first if $ndel > 1/3 * ncls$.

To shrink a fast-alist (will discuss only if time):

```
(defun remove-deleted-clauses (fal acc)
  (declare (xargs :guard (alistp fal)))
  (cond ((endp fal) (make-fast-alist acc))
        (t (remove-deleted-clauses
            (cdr fal)
            (if (deleted-clause-p (cdar fal))
                acc
              (cons (car fal) acc))))))

(defund shrink-formula-fal (fal)
  (declare (xargs :guard (formula-fal-p fal)))
  (let ((fal2 (fast-alist-clean fal)))
    (fast-alist-free-on-exit
     fal2
     (remove-deleted-clauses fal2 nil))))
```

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Proved soundness by tweaking the **[lrat-1]** proof:

- Disabled the top-level “maybe shrink” function
- Re-ran the **[lrat-1]** proof on **[lrat-2]**
- Looked at key checkpoints on failure to determine lemmas to prove (about shrinking).

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- `Max` was only used for RAT check recursion, but **[lrat-2]** recurs through `fal`.
- This simplification seemed useful before starting the next checker, and it saves consing.
- Soundness proof for **[lrat-2]** was easy to modify for **[lrat-3]**.

[lrat-4]

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[lrat-4] solution: use **single-threaded objects** (**stobjs**) to model assignments.

- Lookup is a constant-time array reference.
- Avoids memory allocation (consing) when pushing new literals onto assignment.

[lrat-4]: ASSIGNMENTS

```
(defstobj a$
  (a$ptr :type (integer 0 *) ; stack pointer
         :initially 0)
  (a$stk :type (array t (1)) ; stack of a$arr indices
         :resizable t)
  (a$arr :type (array t (1)) ; array of 0, t, nil
         :initially 0
         :resizable t)
  :renaming ((a$arrp a$arrp-weak)
             (a$p a$p-weak)))
```

[lrat-4]: ASSIGNMENTS (2)

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KEY OBSERVATION: These operations generate calls to `nth` and `update-nth`, but for **[lrat-3]**, they are implemented with `cons` and `cdr`.

Tweaking the **[lrat-3]** proof seemed difficult! Instead....

[lrat-4]: PROOF

- I proved *correspondence theorems* relating **[lrat-3]** functions to **[lrat-4]** functions.

[lrat-4]: Proof

- I proved *correspondence theorems* relating **[lrat-3]** functions to **[lrat-4]** functions.

- Then I derived the soundness of **[lrat-4]** directly from those correspondence theorems and the soundness of **[lrat-3]**.

```
(defthm main-theorem-list-based
  (implies (and (formula-p formula)
                (refutation-p proof formula))
           (not (satisfiable formula)))
  :hints ...)

(defthm refutation-p-equiv
  (implies (and (formula-p formula)
                (refutation-p$ proof formula))
           (refutation-p proof formula)))

(defthm main-theorem-stobj-based
  (implies (and (formula-p formula)
                (refutation-p$ proof formula))
           (not (satisfiable formula)))
  :hints (("Goal"
           :in-theory '(refutation-p-equiv)
           :use main-theorem-list-based)))
```

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1. Developed that invariant, `(a$p a$)`
2. Verified guards (perhaps easier than correspondence theorems), which required invariance proofs
3. Proved correspondence theorems

[lrat-4]: PROOF (4)

I'll very briefly discuss the invariant:

```
(defun a$p (a$)
  (declare (xargs :stobjs a$))
  (and (a$p-weak a$)
       (<= (a$ptr a$) (a$stk-length a$))
       (equal (a$arr-length a$)
              (1+ (a$stk-length a$)))
       (good-stk-p (a$ptr a$) a$)
       (a$arrp a$)
       (arr-matches-stk (a$arr-length a$) a$)))
```

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The **[lrat-2]** function (originally used in **[lrat-3]**):

```
(defun negate-clause-or-assignment (clause)
  (declare (xargs :guard (clause-or-assignment-p clause)))
  (if (atom clause)
      nil
    (cons (negate (car clause))
          (negate-clause-or-assignment (cdr clause)))))
```

[lrat-4]: PROOF (6)

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What to do? Status when problem was discovered:

- Soundness for **[lrat-3]** was already established
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- Some equivalence proofs were complete.

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Then completed correspondence theorems, which yielded soundness for **[lrat-4]**.

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Checkers **[lrat-3]** and **[lrat-4]** are in the community books in these directories, respectively.

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Other checkers are available via links from the seminar page.

OUTLINE

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The next slide has references for citations in this talk.

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