A Unifying Principle for Clause Elimination in First-Order Logic

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  - Improve the efficiency of provers by simplifying the input.

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- Many clause-elimination techniques are used in SAT solving but not in first-order logic yet.
- We lifted SAT techniques to first-order logic without equality.
  - We proved correctness in a uniform way by introducing the principle of implication modulo resolution.

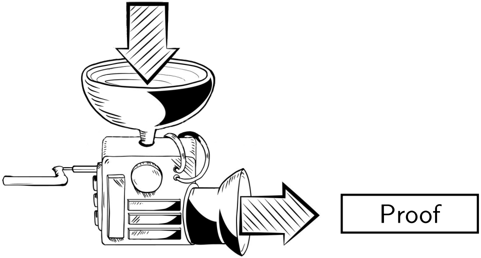
Outline

- First-order theorem proving and preprocessing in a nutshell.
- Details on one successful approach for preprocessing:
  - Clause-elimination techniques.
- Overview of techniques we lifted.
- The unifying principle of implication modulo resolution.
- Confluence results.
- Future work.

First-Order Theorem Proving

- Input: Formula in first-order logic.
- Output: Proof

$$Q(a, b) \land ((\forall x \forall y P(x, y) \leftrightarrow P(y, x)) \rightarrow (\neg P(a, b) \lor P(b, a)))$$



First-Order Theorem Proving

- Input: Formula in first-order logic.
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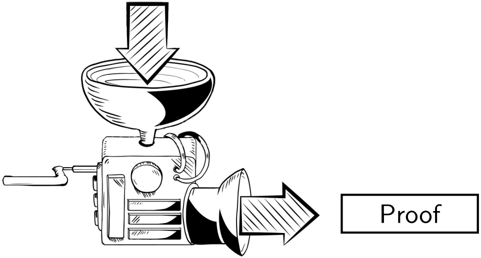
- Applications: Mathematics, verification of software and hardware, reasoning over knowledge bases, etc.

$$Q(a,b) \wedge ((\forall x \forall y P(x,y) \leftrightarrow P(y,x)) \rightarrow (\neg P(a,b) \vee P(b,a)))$$



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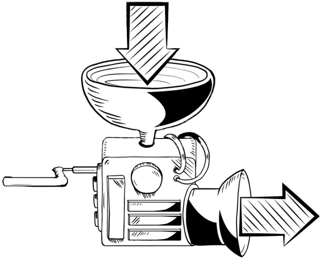

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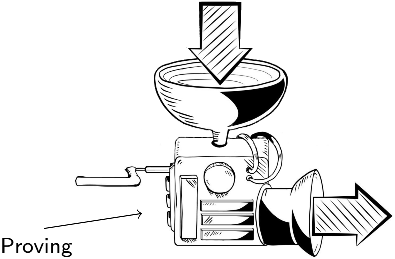

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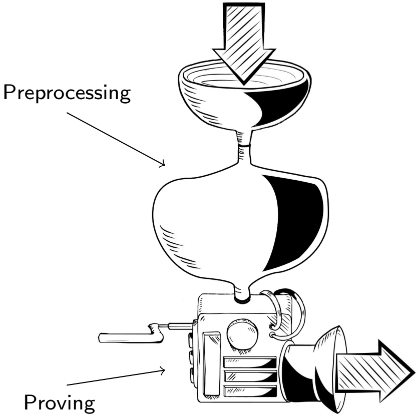

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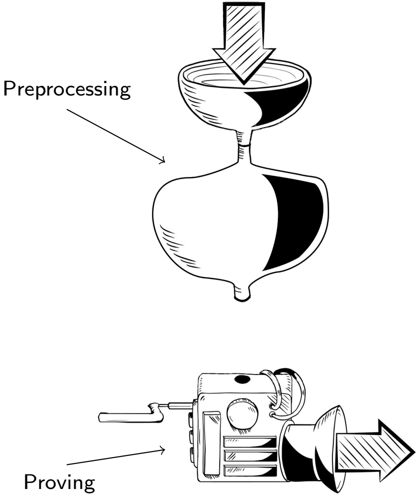

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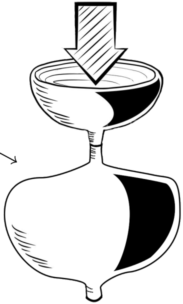
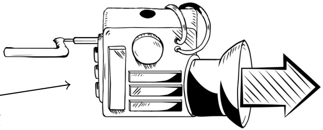
Preprocessing



Simplified Formula
in CNF (negated)

$$(\neg P(x,y) \vee P(y,x)) \wedge P(a,b) \wedge \neg P(b,a)$$



Proving

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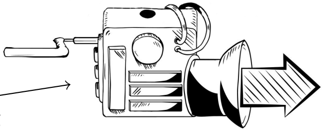

Proving

Refutation

Resolution Refutations (Propositional Logic)

- Resolution Rule: Derive $C \vee D$ from $C \vee L$ and $\neg L \vee D$:

$$\frac{C \vee L \qquad \neg L \vee D}{C \vee D}$$

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- Every unsatisfiable formula can be refuted by resolution.
- Example: $F = (\neg P \vee Q) \wedge (P) \wedge (\neg Q)$

$$\frac{\dfrac{\neg P \vee Q \qquad P}{Q} \qquad \neg Q}{\perp}$$

Resolution Refutations (First-Order Logic)

- Resolution Rule: Derive $(C \vee D)\sigma$ from $C \vee L(t_1, \ldots, t_n)$ and $\neg L(s_1, \ldots, s_n) \vee D$ if σ unifies $L(t_1, \ldots, t_n)$ and $L(s_1, \ldots, s_n)$:

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- Intuitively, a mapping σ unifies literals if it makes them equal:
  - $P(x, y)$ and $P(a, b)$ are unifiable $\rightarrow \sigma(x) = a$ and $\sigma(y) = b$.

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- Intuitively, a mapping σ unifies literals if it makes them equal:
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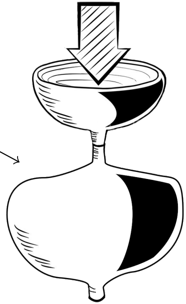
  - $P(b, a)$ and $P(b, a)$ are unifiable $\rightarrow$ no mapping necessary.
- Example Refutation:
  $F = (\neg P(x, y) \vee P(y, x)) \wedge P(a, b) \wedge \neg P(b, a)$

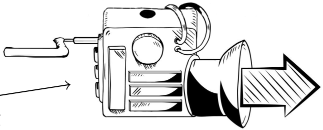
$$\dfrac{\dfrac{\neg P(x,y) \vee P(y,x) \qquad P(a,b)}{P(b,a)} \qquad \neg P(b,a)}{\bot}$$

Automatic First-Order Theorem Proving

$$Q(a,b) \land ((\forall x \forall y P(x,y) \leftrightarrow P(y,x)) \to (\neg P(a,b) \lor P(b,a)))$$



$$(\neg P(x,y) \lor P(y,x)) \land P(a,b) \land \neg P(b,a)$$



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What's going on here?

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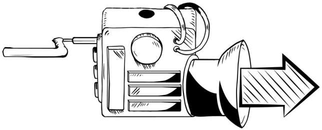
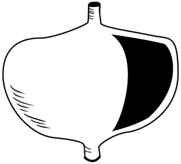


Resolution Refutation

Automatic First-Order Theorem Proving



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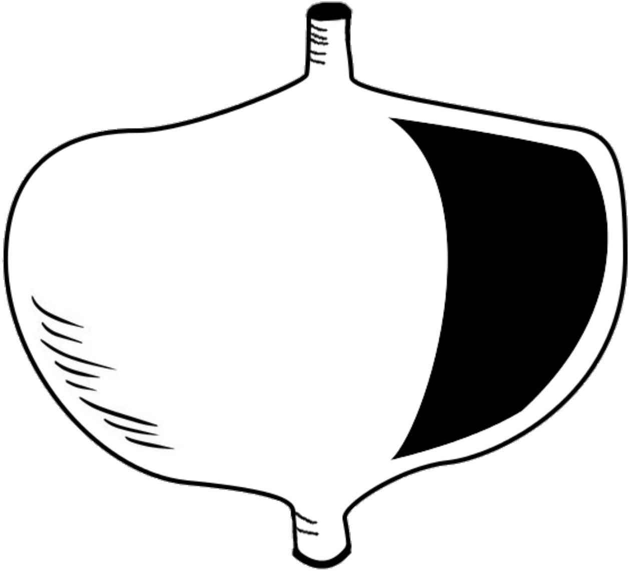
Preprocessing Pipeline



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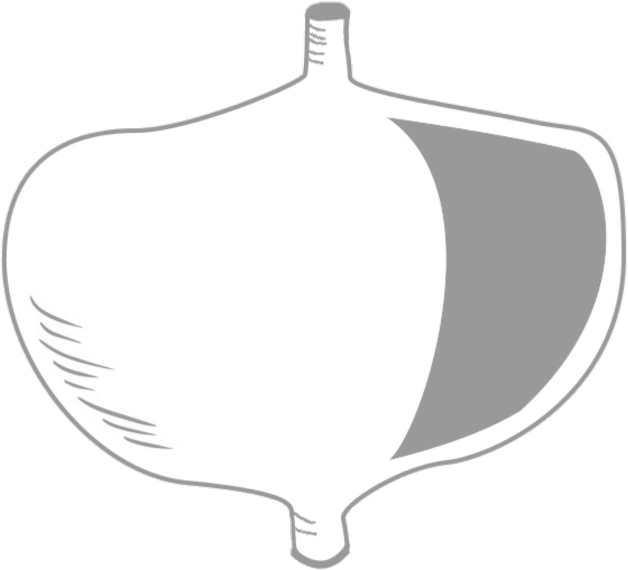
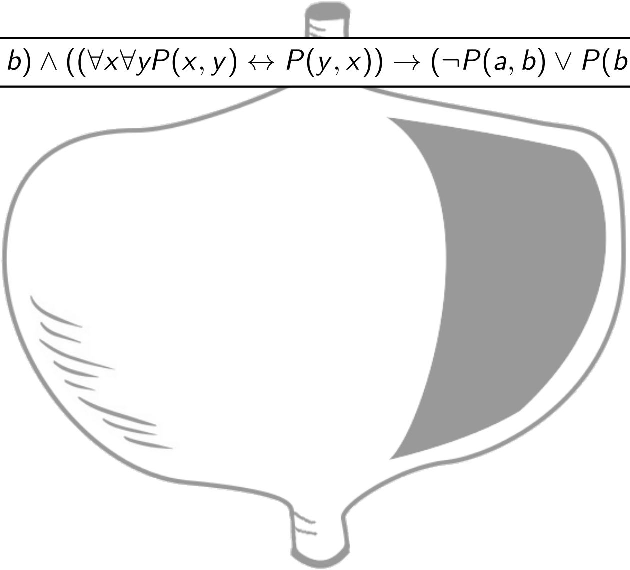


Preprocessing Pipeline

$$\boxed{Q(a,b) \wedge ((\forall x \forall y P(x,y) \leftrightarrow P(y,x)) \rightarrow (\neg P(a,b) \vee P(b,a)))}$$

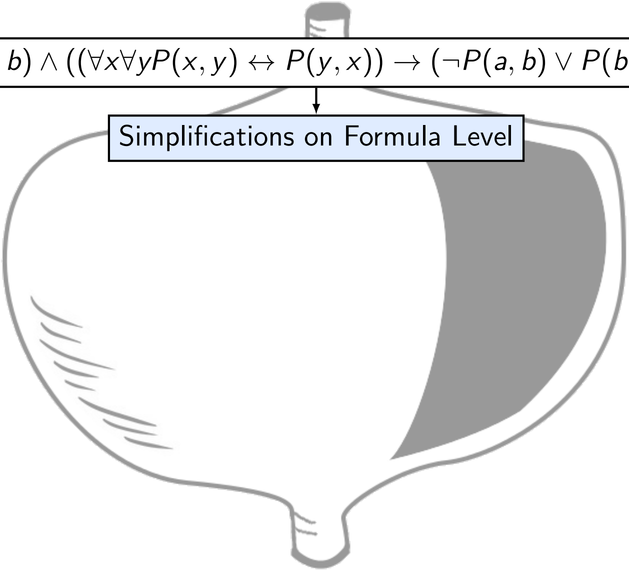


Preprocessing Pipeline

$$Q(a,b) \land ((\forall x \forall y P(x,y) \leftrightarrow P(y,x)) \rightarrow (\neg P(a,b) \lor P(b,a)))$$

$\downarrow$

Simplifications on Formula Level



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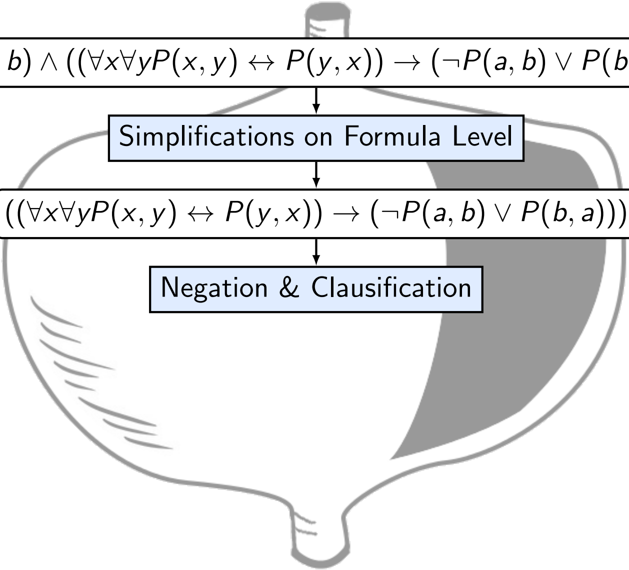

$Q(a,b) \land ((\forall x \forall y P(x,y) \leftrightarrow P(y,x)) \rightarrow (\neg P(a,b) \lor P(b,a)))$

↓

Simplifications on Formula Level

↓

$((\forall x \forall y P(x,y) \leftrightarrow P(y,x)) \rightarrow (\neg P(a,b) \lor P(b,a)))$

↓

Negation & Clausification

Preprocessing Pipeline

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Simplifications on Formula Level

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Negation & Clausification

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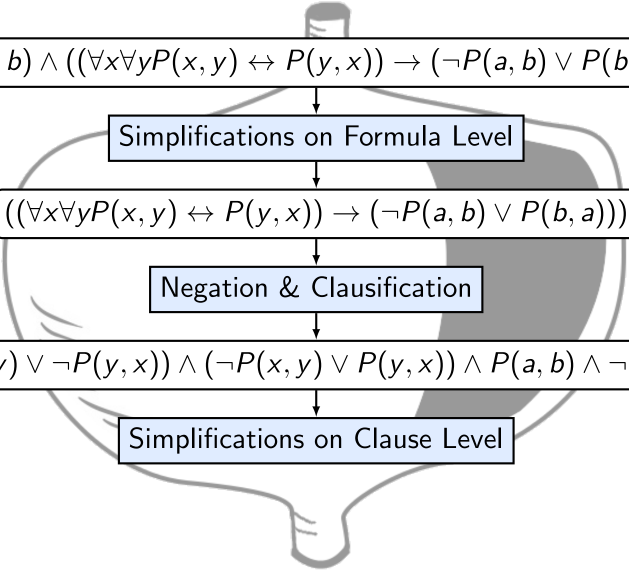
Preprocessing Pipeline



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Simplifications on Formula Level

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Negation & Clausification

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Simplifications on Clause Level

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↓

Simplifications on Clause Level

↓

$$(\neg P(x,y) \vee P(y,x)) \wedge P(a,b) \wedge \neg P(b,a)$$

Preprocessing Pipeline

- Topic of this talk: Simplifications on the clause level.

$$\boxed{(P(x,y) \vee \neg P(y,x)) \wedge (\neg P(x,y) \vee P(y,x)) \wedge P(a,b) \wedge \neg P(b,a)}$$

$$\downarrow$$

$$\boxed{\text{Simplifications on Clause Level}}$$

$$\downarrow$$

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- Remark: Redundant clauses need not be implied!

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- Problem: Checking if a clause is redundant is undecidable.

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Clause-Elimination Techniques: Success Stories

- Clause-elimination is successfully used in SAT and QSAT solving:
  - *Effective Preprocessing in SAT Through Variable and Clause Elimination* (Eén and Biere, SAT, 2005)
  - *Clause Elimination for SAT and QSAT* (Heule et al., JAIR, 2010)
  - *Covered Clause Elimination* (Heule et al., LPAR, 2010)
  - *Blocked Clause Elimination* (Järvisalo et al., TACAS, 2010)
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  - *Enhancing Search-Based QBF solving by Dynamic Blocked Clause Elimination* (Lonsing et al., LPAR, 2015)
  - ...
- Blocked-clause elimination can speed up first-order provers:
  - *Blocked Clauses in First-Order Logic* (Kiesl, Suda, Seidl, Tompits, and Biere, LPAR, 2017)

(Some) Types of Redundant Clauses in SAT Solving

Asymmetric Tautologies

Covered Clauses

Resolution Asymmetric Tautologies

Resolution Subsumed Clauses

Blocked Clauses

Tautologies

Asymmetric Blocked Clauses

Subsumed Clauses

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- Not available in first-order logic before!

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Asymmetric Blocked Clauses

Asymmetric Covered Clauses

- Not available in first-order logic before!
  - ➥ We lifted them.

Example: Blocked Clauses in Propositional Logic

- A clause C is blocked in a formula F if all resolvents upon one of its literals are tautologies.

$$\neg S \vee P \vee Q$$

$$P \vee Q \vee R \qquad \neg R \vee \neg Q$$

$$\neg R \vee \neg P$$

$$\neg T \vee S \vee Q$$

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$$\neg R \lor \neg Q$$

$$\neg R \lor \neg P$$

$$\neg T \lor S \lor Q$$

$$P \lor Q \lor \neg Q \qquad P \lor Q \lor \neg P$$

➥ $P \lor Q \lor R$ is a blocked clause.

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- Blocked clauses for first-order logic can be defined in a similar way as in propositional logic.
- Proving redundancy of blocked clauses in propositional logic is (relatively) simple.
- Proving redundancy of blocked clauses in first-order logic requires heavy machinery.
  - Herbrand's theorem,
  - factorization,
  - non-trivial properties of (most general) unification, etc.
- Required: A general theorem that helps us prove redundancy of several types of clauses in a unified way.

The Principle of Implication Modulo Resolution

- To prove correctness of the new techniques, we introduced the principle of implication modulo resolution.
  - A first-order variant of quantified implied outer resolvents (Heule, Seidl, and Biere, JAR, 2017).

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Definition

A clause C is implied modulo resolution by a formula F if all resolvents of C upon one of its literals are implied by $F \setminus \{C\}$.

The Principle of Implication Modulo Resolution

- To prove correctness of the new techniques, we introduced the principle of implication modulo resolution.
  - A first-order variant of quantified implied outer resolvents (Heule, Seidl, and Biere, JAR, 2017).

Definition

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➥ $F \setminus \{C\}$ might not imply C, but it implies all conclusions derived from C via resolution upon one of its literals.

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Theorem (Main Result)

If a formula F implies a clause C modulo resolution, then C is redundant with respect to F.

Implication Modulo Resolution: Examples

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  - There are no resolvents upon a pure literal $\Rightarrow$ every resolvent is implied.
- Resolution asymmetric tautologies (RATs), resolution-subsumed clauses, etc.

Confluent Clause-Elimination Techniques

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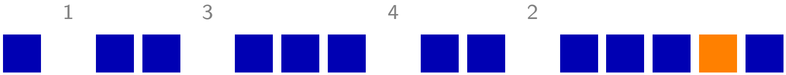

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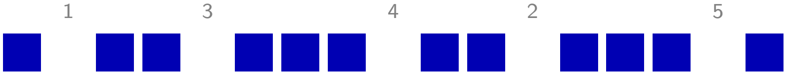

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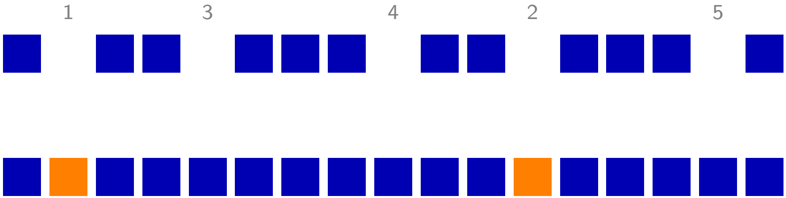

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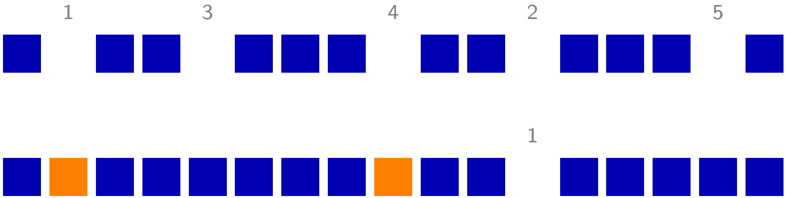

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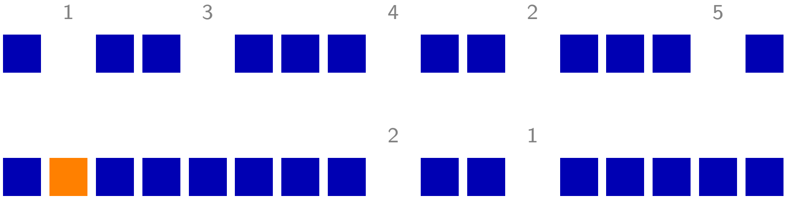

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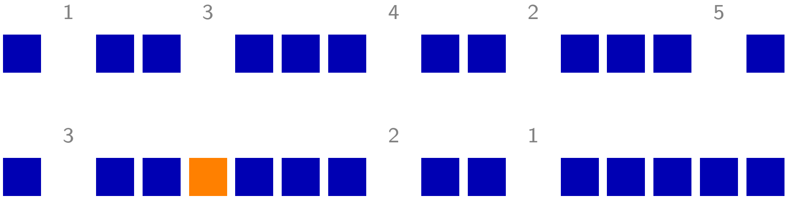

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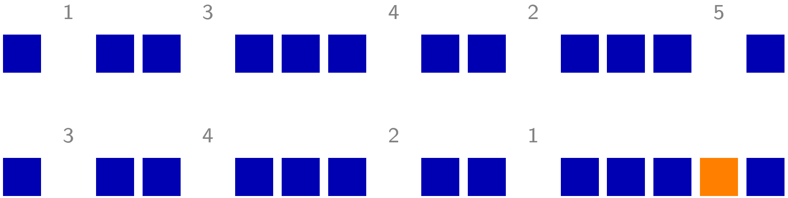

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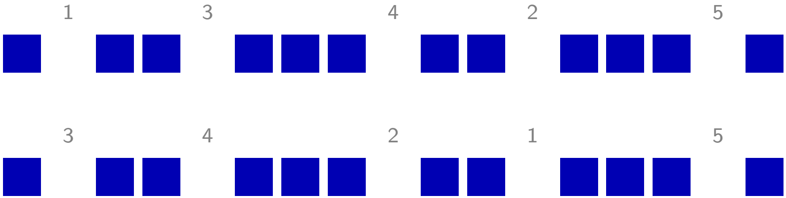

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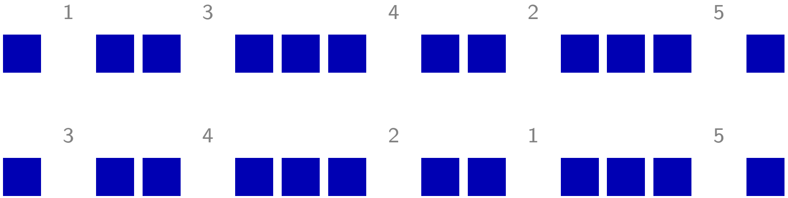

➥ We don't need to bother about the elimination order.

Confluence Results

Technique	Confluent
Blocked-Clause Elimination	✓
Covered-Clause Elimination	✓
Asymmetric-Tautology Elimination	✗
Resolution-Asymmetric-Tautology Elimination	✗
Resolution-Subsumed-Clause Elimination	✗

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Covered-Literal Addition	✓
Asymmetric-Literal Addition	✓

Future Work

- Implication modulo resolution for first-order logic with equality.
  - ➥ Lift all preprocessing techniques to first-order logic with equality.
- Implement and evaluate a preprocessor with our techniques.
  - Blocked-clause elimination is already implemented.
  - Preprocessor is based on Vampire.

Summary

- Lifted clause-elimination techniques from SAT to first-order logic.
- Correctness proofs via principle of implication modulo resolution.
- Confluence analysis.
- Not in this talk but in the paper:
  - Short correctness proof for predicate elimination (Khasidashvili and Korovin, SAT, 2016) via implication modulo resolution.