ACL2:
Implementation of a Computational Logic

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Joint work with Bob Boyer, J Moore,
and the ACL2 community

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Presented at JAF 2019

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Please feel free to ask questions (in person or via email; I'll put contact info and a link to the slides on the last slide).

OUTLINE

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Overview and Context

ACL2 Introduction

Logical Foundations for ACL2

Conclusion

Overview and Context

The ACL2 home page has many useful links, and begins with the following summary.

> *ACL2 is a logic and programming language in which you can model computer systems, together with a tool to help you prove properties of those models. "ACL2" denotes "**A** **C**omputational **L**ogic for **A**pplicative **C**ommon **L**isp".*

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But before we talk about ACL2, let's put it in context.

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- AMD, ARM, ArterisIP, Battelle, Centaur, GE, IBM, Intel, NXP, Kestrel, Oracle, Rockwell Collins

FORMAL VERIFICATION (2)

Two recent examples of ACL2 formalizations at UT Austin:

- **x86 interpreter**: models state transitions for x86 instructions
  - Testing validates faithfulness of this model to actual Intel x86 chips when running x86 machine code (approximately 3.3 million instructions per second).
  - Some x86 machine code programs have been proved correct.
  - It is under continued development at Centaur and Kestrel.
- **an *efficient* checker** for Boolean satisfiability (SAT) proofs
  - Used in recent international SAT competitions

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- Proof automation
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- Fast execution of programs
- Documentation (about 120,000 lines for just the system; many more for the libraries)

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  - *Boyer-Moore Theorem Provers* go back to their collaboration starting in 1971.

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- Interfaces include Emacs, ACL2 Sedan (Eclipse-based), none.

USING ACL2 (2)

A longer talk on ACL2, oriented towards CS graduate students and with more focus on *using* ACL2, is here:

http://www.cs.utexas.edu/users/kaufmann/talks/acl2-intro-2015-04/acl2-intro.pdf

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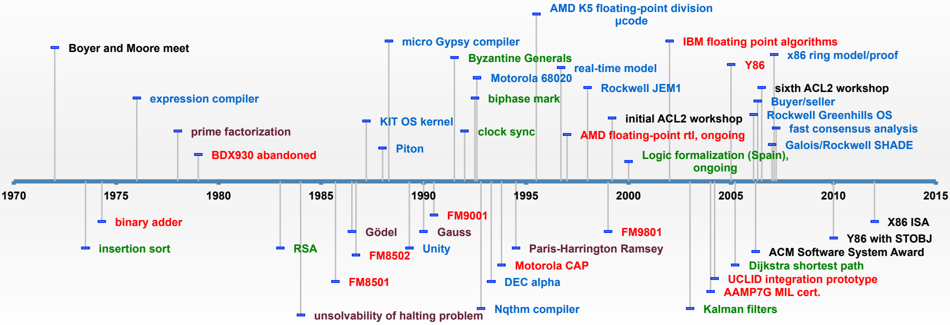
That talk mentions this link to several demos and their logs:

`http://www.cs.utexas.edu/users/kaufmann/talks/acl2-intro-2015-04/demos.tgz`

Partial timeline



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- . . . but it hasn't been a priority to work this out, let alone consider effects on the implementation.

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- closure under a pairing operation (`cons`).

This gives us lists, where the symbol `nil` represents the empty list. For example:

```
ACL2 !>(cons 1 (cons 2 (cons 3 nil)))
(1 2 3)
ACL2 !>
```

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- ... This holds even for recursive definitions, since "termination" must be provable.
- M. Kaufmann and J Moore, "Structured Theory Development for a Mechanized Logic." *Journal of Automated Reasoning* 26, no. 2 (2001) 161-203.
- So, one may introduce new concepts locally when carrying out proofs.

Extension principle: Definitions

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The definition may be recursive if some *measure* into ε_0 is proved to decrease on each recursive call.

EXTENSION PRINCIPLE: CHOICE (AND $\exists$)

Quantification is implemented using a choice operator. When asked to define
$P(\vec{x}) = \exists \vec{y} A(\vec{x}, \vec{y})$
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Conservatively introduce a Skolem (witness) function $w(\vec{x})$ and a predicate $P(\vec{x})$:

$w(\vec{x}) = \varepsilon \vec{y} A(\vec{x}, \vec{y})$

$P(\vec{x}) = A(\vec{x}, w(\vec{x}))$

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```
(defun-sk fermat-counterex (n)
  (exists (i j k)
    (and (posp i) (posp j) (posp k)
         (equal (+ (expt i n) (expt j n))
                (expt k n)))))
(defthm fermat
  (implies (and (integerp n) (< 2 n))
           (not (fermat-counterex n))))
```

EXTENSION PRINCIPLE: CHOICE (AND $\exists$) (2)

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Conservativity *with* induction follows from a model-theoretic forcing argument.

Extension principle: Constraints

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Example:

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A derived inference rule, *functional instantiation*, is often useful with constrained functions. The next slide shows an example.

```
(defun map2-fn (lst1 lst2)
  (if (consp lst1)
      (cons (fn (first lst1) (first lst2))
            (map2-fn (rest lst1) (rest lst2)))
    nil))
(defthm map2-fn-rev
  (implies (equal (len lst1) (len lst2))
           (equal (map2-fn (rev lst1) (rev lst2))
                  (rev (map2-fn lst1 lst2)))))
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(defun map2-* (lst1 lst2)
  (if (consp lst1)
      (cons (* (first lst1) (first lst2))
            (map2-* (rest lst1) (rest lst2)))
    nil))
(defthm map2-*-rev
  (implies (equal (len lst1) (len lst2))
           (equal (map2-* (rev lst1) (rev lst2))
                  (rev (map2-* lst1 lst2))))
  :hints (("Goal" :by (:functional-instance
                        map2-fn-rev
                        (fn *) (map2-fn map2-*)))))
```

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If internal predicate $P(n, x)$ holds for all standard natural numbers n, then $P(n, x)$ holds for some non-standard natural number n.

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- `overspill.lisp`: Clean formalization (which I'll flash on the next slide)
  25 lines
- `overspill-proof.lisp`: Ugly proof, but `LOCAL` to the main proof, by conservativity
  256 lines

Key parts of the book `overspill.lisp`:

```
(local (include-book "overspill-proof"))
(set-enforce-redundancy t)
(defstub overspill-p (n x) t)

(defun overspill-p* (n x)
  (if (zp n)
      (overspill-p 0 x)
    (and (overspill-p n x)
         (overspill-p* (1- n) x))))

(defchoose overspill-p-witness (n) (x)
  (or (and (natp n) (standardp n)
           (not (overspill-p n x)))
      (and (natp n) (i-large n)
           (overspill-p* n x))))

(defthm overspill-p-overspill
  (let ((n (overspill-p-witness x)))
    (or (and (natp n) (standardp n)
             (not (overspill-p n x)))
        (and (natp n) (i-large n)
             (implies (and (natp m)
                           (<= m n))
                      (overspill-p m x)))))
  :rule-classes nil)
```

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Efficient execution can be important for meta-theoretic reasoning!
We can return to this on an extra slide, if there is time and interest.

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- One can specify a *measure* in order to admit a recursive definition. But what if the measure is defined in terms of a function whose definition is `LOCAL`?
- *Congruence-based reasoning* allows replacing one subterm by another that is equivalent but not necessarily equal.

DEFATTACH (1)

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Example:

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Example:

- **Constraint for** a "specification" function, `spec`:
  $x \in \mathbb{Z} \implies \texttt{spec}(x) \in \mathbb{Z}$

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For details, including issues pertaining to evaluation, see the *Essay on Defattach* comment in the ACL2 sources.

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(include-book "projects/apply/top" :dir :system)
(defun$ norm^2 (x)
  (+ (* (car x) (car x)) (* (cdr x) (cdr x))))
(assert-event
 (equal (norm^2 (cons 3 4)) 25))
(thm (equal (norm^2 (cons 3 4)) 25))
(assert-event
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```

But this fails!

```
(thm (equal (apply$ 'norm^2
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However, the proof succeeds for the `thm` below, where the *warrant hypothesis*, `(warrant norm^2)`, asserts:

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Warrant hypotheses are not vacuous!
We show there is a natural *evaluation theory* where every warrant is attached to the constant "true" function.

ITERATION

A key application of `apply$`: *iteration*, which is useful for programming, and has reasoning support in ACL2. **Example**:

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where `sum$` is defined essentially as follows.

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(defun sum$ (fn lst)
  (if (endp lst)
      0
    (+ (apply$ fn (list (car lst)))
       (sum$ fn (cdr lst)))))
```

Outline

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- As an ITP system, it relies on user guidance for large problems but enjoys scalability.
- Mechanizing a logic, for efficient and flexible evaluation and proof, can present challenges.
- For more information, see the ACL2 home page, in particular links to The Tours and Publications, which links to introductory material.

THANK YOU!

THANK YOU!

Matt Kaufmann
matthew.j.kaufmann@gmail.com

Slides for this talk are available via links from my home page:
`http://www.cs.utexas.edu/users/kaufmann`

Extra Slides

We can go on, time permitting....

Some ACL2 features *not* discussed further today:

- Prover algorithms
  - Waterfall, linear arithmetic, Boolean reasoning, . . .
  - Rewriting: Conditional, congruence-based, rewrite cache, syntaxp, bind-free, . . .
- Using the prover effectively
  - The-method and introduction-to-the-theorem-prover
  - Theories, hints, rule-classes, . . .
  - Accumulated-persistence, brr, proof-checker, dmr, . . .
- Programming support, including (just a few):
  - Guards
  - Hash-cons and function memoization
  - Packages
  - Mutable State, stobjs, arrays, applicative hash tables, . . .
- System-level: Emacs support, books and certification, abbreviated printing, parallelism (ACL2(p)), . . .

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ACL2 supports a notion of “eval”, together with this sort of *meta* theorem, directing the use of `fn` to transform terms that are calls of `nth` or of `foo`.

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- ACL2 definitions are actually programs in the Common Lisp programming language.
- *Guards* specify intended domains of functions and support sound, efficient Common Lisp evaluation.
- Several features support efficient computation by reusing storage, yet with a first-order logic foundation.
  - *Single-threaded objects* including *state*
  - *Arrays*
  - *Function memoization* (reuse of saved results)
  - *Fast alists* (applicative hash tables)