Partial Clock Functions in ACL2

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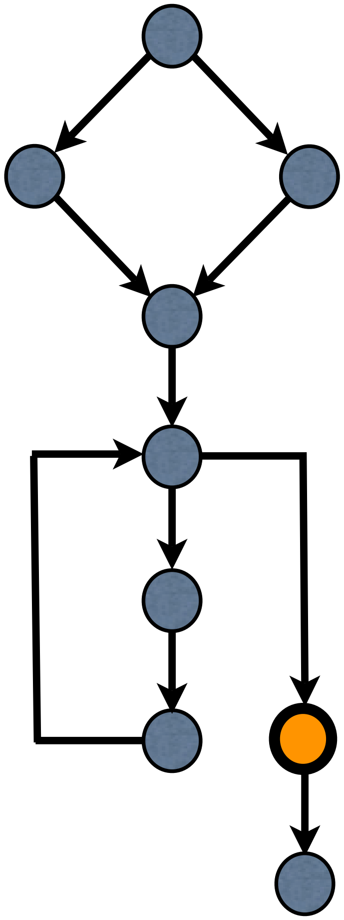
ACL2 Workshop 2004

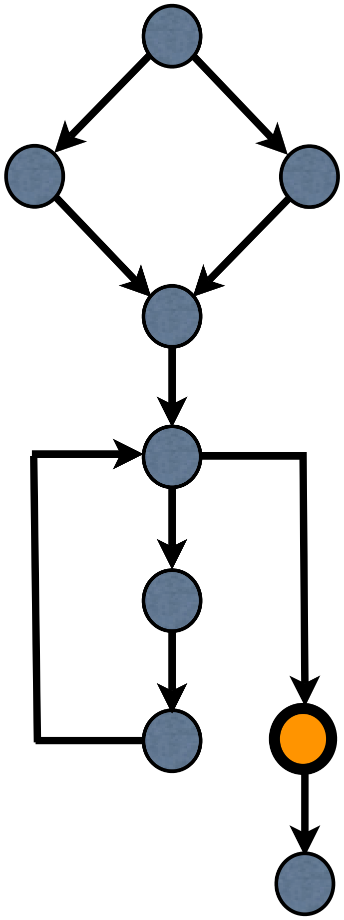
Goals

- **Given a state machine, we want:**
  - A termination proof: from a set of starting states, a desired goal state will always eventually be reached.
  - An efficient simulator: a function that steps machine until desired goal state is reached
  - Modularity: Be able to compose subroutine proofs and simulators



Goals

- **We don't want to**:
  - write a VCG (verification condition generator)
  - manually define a clock function
  - specify assertions or ordinal measures for every instruction in the subroutine
  - add a clock parameter to the simulator
- **Related work**:
  - First three conditions above met for partial correctness [Moore 2003]
  - First two conditions above met for total correctness [Ray & Moore 2004]



State machine model

- **State tuple:** represents current machine state
  - Defined as a stobj
  - Program, program counter are part of the state

```
(defstobj mstate
  (mem   :type (array (signed-byte 32) (1024))
  (progc :type integer)
  ...)
```

- **"next state" function:** executes one machine step

```
next : mstate => mstate
```

State machine model

- **Machine simulator (with clock parameter):** Executes machine for `n` steps
  - Returns current state if `n` is bogus

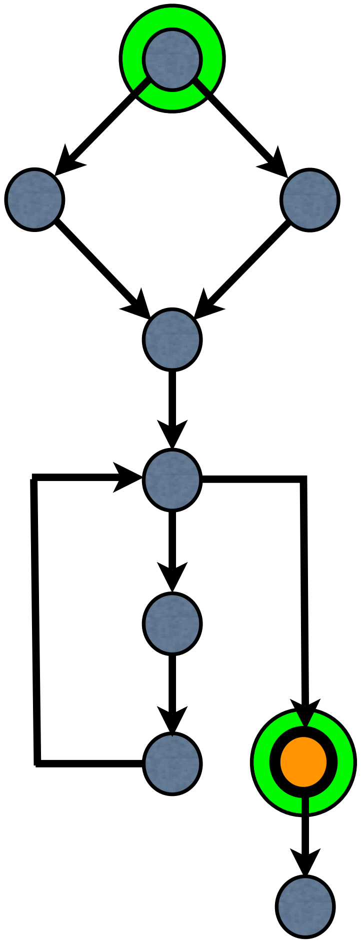
```
(defun run (n mstate)
  (declare (xargs :stobjs (mstate)
                  :guard (natp n)))
  (if (zp n)
      mstate
    (let ((mstate (next mstate)))
      (run (1- n) mstate))))
```

State machine model

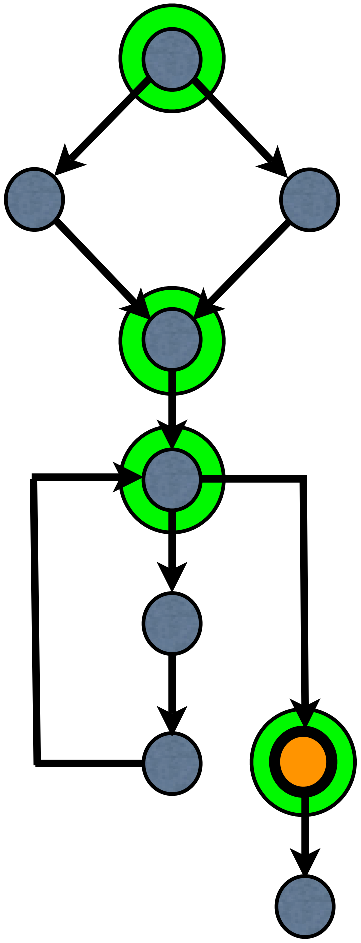
- **State assertion**: predicate about a machine state

```
(defun entering-fib-routine (n mstate)
  (and (program-loaded *fib-addr* mstate)
       (equal (progc mstate)
              *fib-addr*)
       (equal (top-of-stack mstate)
              n)))


(defun exiting-fib-routine (n mstate)
  (and (program-loaded *fib-addr* mstate)
       (equal (progc mstate)
              *fib-done-addr*)
       (equal (top-of-stack mstate)
              (fib n))))
```



State machine model

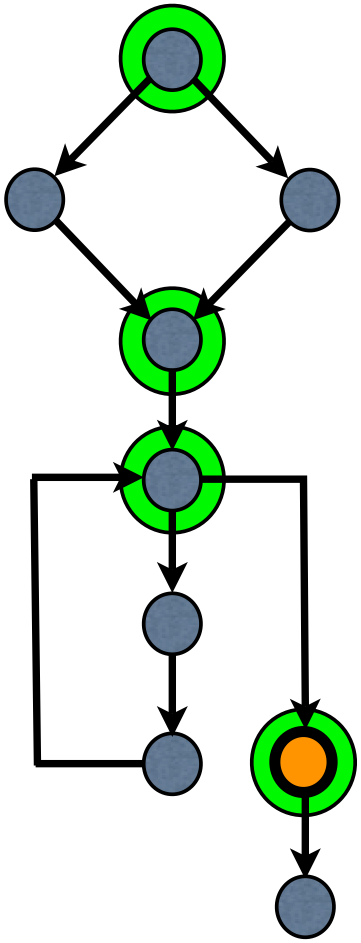


- **Cutpoints**: Finite collection of state assertions
  - Every program loop should be broken by at least one cutpoint
- **Exitpoint**: Desired end state assertion
  - Every exitpoint must be a cutpoint
  - Multiple exitpoints allowed
  - Exitpoints aren't necessarily halting
- **Internal cutpoint**: A cutpoint that is not an exitpoint

Termination proof

- **Total correctness**: Every cutpoint always leads to an exitpoint.
- Proof method:
  - Assign an ordinal measure to every cutpoint

    ```
    cutpoint-measure :
      mstate => ordinal
    ```
  - Symbolically simulate each control path from an internal cutpoint until another cutpoint is reached
  - Show that the newly-reached cutpoint is smaller according to **`cutpoint-measure`**



Symbolic simulation

- Symbolic simulation automated via a **partial clock function**
  - Has a generic, tail-recursive definition
  - Returns number of steps (- n) until next valid cutpoint state, if one is reachable
  - Undefined if no cutpoint state is reachable
  - Can be made "Executable"

```
(defpun steps-to-cutpoint-tail (n mstate)
  (if (at-cutpoint mstate)
      n
     (steps-to-cutpoint-tail (1+ n) (next mstate))))
```

Completed clock function

- Partial clock function is logically extended to a total function:
  - Tests whether value returned by steps-to-cutpoint-tail is a cutpoint:
    - If so, then return that value
    - If not, then return ω

```
(defun steps-to-cutpoint (mstate)
  (let ((steps (steps-to-cutpoint-tail 0 mstate)))
    (if (at-cutpoint (run steps mstate))
        steps
      (omega))))
```

Clock function rewrites

- Completed clock function has simpler rewrite rules
  - Rules use ordinal addition to handle unreachable cutpoints

```
(defthm steps-to-cutpoint-zero
  (implies (at-cutpoint mstate)
           (equal (steps-to-cutpoint mstate) 0)))

(defthm steps-to-cutpoint-nonzero-intro
  (implies (not (at-cutpoint mstate))
           (equal (steps-to-cutpoint mstate)
                  (o+ 1
                      (steps-to-cutpoint (next mstate))))))
```

Symbolic simulation

- Check termination by symbolically simulating machine, from each internal cutpoint to its next reachable cutpoint

```
(implies (and (at-cutpoint mstate)
              (not (at-exitpoint mstate)))
         (let* ((steps (steps-to-cutpoint (next mstate)))
                (cutpoint (run steps mstate)))
           (and (at-cutpoint cutpoint)
                (o< (cutpoint-measure cutpoint)
                    (cutpoint-measure mstate)))))
```

- But then machine gets simulated twice per internal cutpoint!
  - Once to compute number of steps to next cutpoint
  - Second time to compute next cutpoint's state tuple

Symbolic simulation

- Solution: use clock function to define a **next-cutpoint** function
  - Returns next cutpoint, if it is reachable
  - Returns a non-cutpoint value, otherwise

```
(defun next-cutpoint (mstate)
  (let ((steps (steps-to-cutpoint mstate)))
    (if (natp steps)
        (run steps mstate)
      nil)))
```

Symbolic simulation

- Next-cutpoint function agrees with machine simulator...

```
(thm
  (implies (at-cutpoint (next-cutpoint mstate))
           (equal (next-cutpoint mstate)
                  (run (steps-to-cutpoint mstate) mstate))))
```

...and still obeys good symbolic simulation rules

```
(defthm next-cutpoint-at-cutpoint
  (implies (at-cutpoint mstate)
           (equal (next-cutpoint mstate)
                  mstate)))

(defthm next-cutpoint-intro-next
  (implies (not (at-cutpoint mstate))
           (equal (next-cutpoint mstate)
                  (next-cutpoint (next mstate)))))
```

Symbolic simulation

- Now termination check symbolically simulates machine only once per internal cutpoint.

```
(implies (and (at-cutpoint mstate)
              (not (at-exitpoint mstate)))
         (let ((cutpoint (next-cutpoint (next mstate))))
           (and (at-cutpoint cutpoint)
                (o< (cutpoint-measure cutpoint)
                    (cutpoint-measure mstate)))))
```

Termination

- Can now define function to count steps from cutpoint to next exitpoint

```
(defun steps-to-exitpoint-from-cutpoint (mstate)
  (declare (xargs :measure (cutpoint-measure mstate)))
  (cond
   ((not (at-cutpoint mstate))
    0)
   ((at-exitpoint mstate)
    0)
   (t
    (+ 1 (steps-to-cutpoint (next mstate))
         (steps-to-exitpoint-from-cutpoint
           (next-cutpoint (next mstate)))))))
```

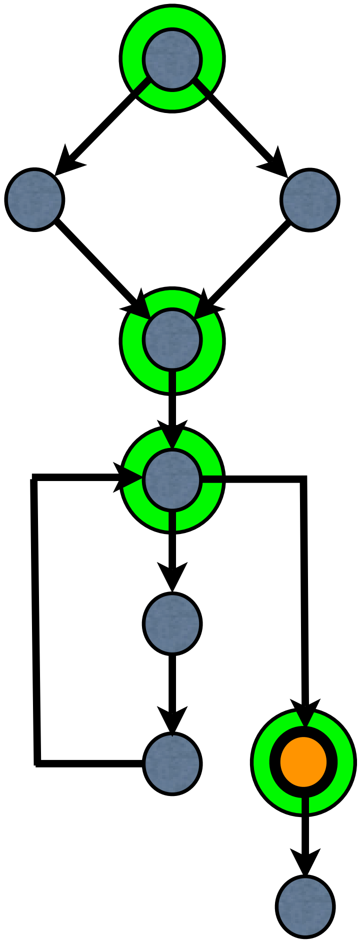
Termination

- Main termination theorem:

```
(defthm total-correctness-from-cutpoint
  (implies (at-cutpoint mstate)
           (at-exitpoint
              (run (steps-to-exitpoint-from-cutpoint mstate)
                   mstate))))
```

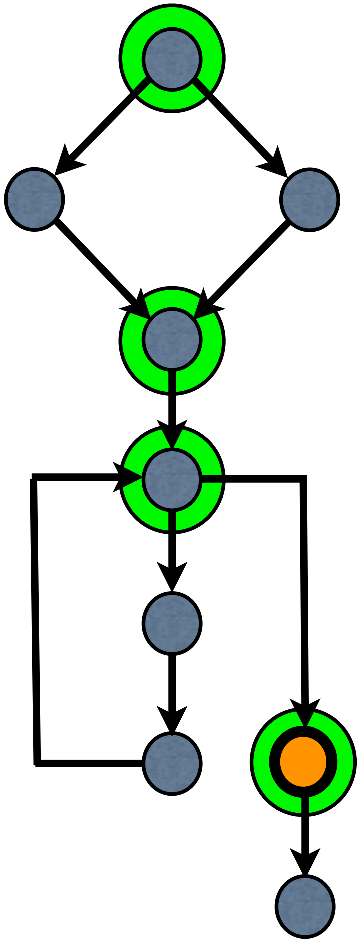
Efficient simulator

- Goal 2: Define an executable machine simulator function that doesn't use a step counter
  - Simulator returns the first reachable exitpoint state
  - Simulator guard: input state must be a cutpoint



Efficient simulator

- Defining the simulator:
  - First define a **cutpoint simulator**, that steps the machine from one cutpoint to the next cutpoint
  - Main simulator calls cutpoint simulator until exitpoint is reached
  - Use cutpoint measure to prove termination
- Main challenge: stobj syntactic restrictions



Stobj restrictions

- Want to use **`steps-to-cutpoint`** in guards, but not execute them

```
:guard (at-cutpoint
         (run (steps-to-cutpoint mstate) mstate))
```

- Problem: ACL2 requires guards to be executable
  - Difficult to make guards stobj-compliant
- This definition doesn't work, since **`defpun`** not stobj-compliant:

```
(defun steps-to-cutpoint (mstate)
  (declare (xargs :stobjs (mstate)))
  (let ((steps (steps-to-cutpoint-tail 0 mstate)))
    (if (at-cutpoint (run steps mstate))
         steps
       (omega))))
```

Stobj restrictions

- Need to write coercion functions between stobjs and ACL2 values

```
logical-mstatep   : * => bool
copy-from-mstate : mstate => *
copy-to-mstate   : (* mstate) => mstate

(defthm copy-from-mstate-correct
  (implies (mstatep mstate)
           (equal (copy-from-mstate mstate)
                  mstate)))

(defthm copy-to-mstate-correct
  (implies (and (mstatep mstate)
                (logical-mstatep copy))
           (equal (copy-to-mstate copy mstate)
                  copy)))
```

Stobj restrictions

- Next problem: guards are not allowed to modify stobjs

```
(defun steps-to-cutpoint (mstate)
  (declare (xargs :stobjs (mstate)))
  (let* ((mstate-copy (copy-from-mstate mstate))
         (steps
           (steps-to-cutpoint-tail 0 mstate-copy)))
    (if (at-cutpoint (run steps mstate))
        steps
      (omega))))
```

- "ACL2 value" version of `run` requires "ACL2 value" `next`
  - Basically need to redefine the entire machine semantics

Stobj restrictions

- Solution: create a `with-copy-of-stobj` macro
  - allocates a local copy of stobj object
  - Executes a stobj-compliant `mv-let` form on the local copy
    - Discards the `mv-let`'s final stobj
    - Returns the `mv-let`'s final value
- Modified `steps-to-cutpoint` function is now stobj-compliant
  - Can be used in guards
  - ACL2 runtime error if executed (but still sound)

Efficient simulator

- Clockless simulator, useful for cutpoint-induction proofs:
  - **`next-cutpoint-exec`** defined with stobj-compliant guard
  - called by cutpoint simulator **`cutpoint-to-cutpoint-exec`**
- Main simulator calls cutpoint simulator until exitpoint

```
(defun next-exitpoint-exec (mstate)
  (declare (xargs :stobjs (mstate)
                  :measure (cutpoint-measure mstate)
                  :guard (at-cutpoint mstate)))
  (if (mbt (at-cutpoint mstate))
      (if (at-exitpoint mstate)
          mstate
        (let ((mstate (cutpoint-to-cutpoint-exec mstate)))
          (next-exitpoint-exec mstate)))
    (dummy-mstate mstate)))
```

Efficient simulator

- Clockless simulator, useful for efficient execution (not in supporting materials):

```
(defun next-exitpoint-exec (mstate)
  (declare (xargs :stobjs (mstate)
                  :guard (cutpoint-reachable mstate)
                  :measure (steps-to-exitpoint
                            mstate)))
  (if (mbt (and (mstatep mstate)
                (cutpoint-reachable mstate)))
      (if (at-exitpoint mstate)
          mstate
        (let ((mstate (next mstate)))
          (next-exitpoint-exec mstate)))
    mstate))
```

Conclusions

- Partial clock functions and cutpoint symbolic simulation increase automation and robustness of termination proofs
- Termination proofs are modular, because exitpoints need not halt
- Possible to define efficient, clockless machine simulators
- Clockless stobj-compliant simulators will be easier to write when ACL2
  - allows nonexecutable guards
  - removes stobj syntax restrictions in logical portions of guards, **`mbt`**, and **`mbe`** macros
- In the meantime, a **`defstobj+`** ACL2 book has been written:
  - Automatically creates stobj coercion functions & theorems
  - Includes **`with-copy-of-stobj`** macro