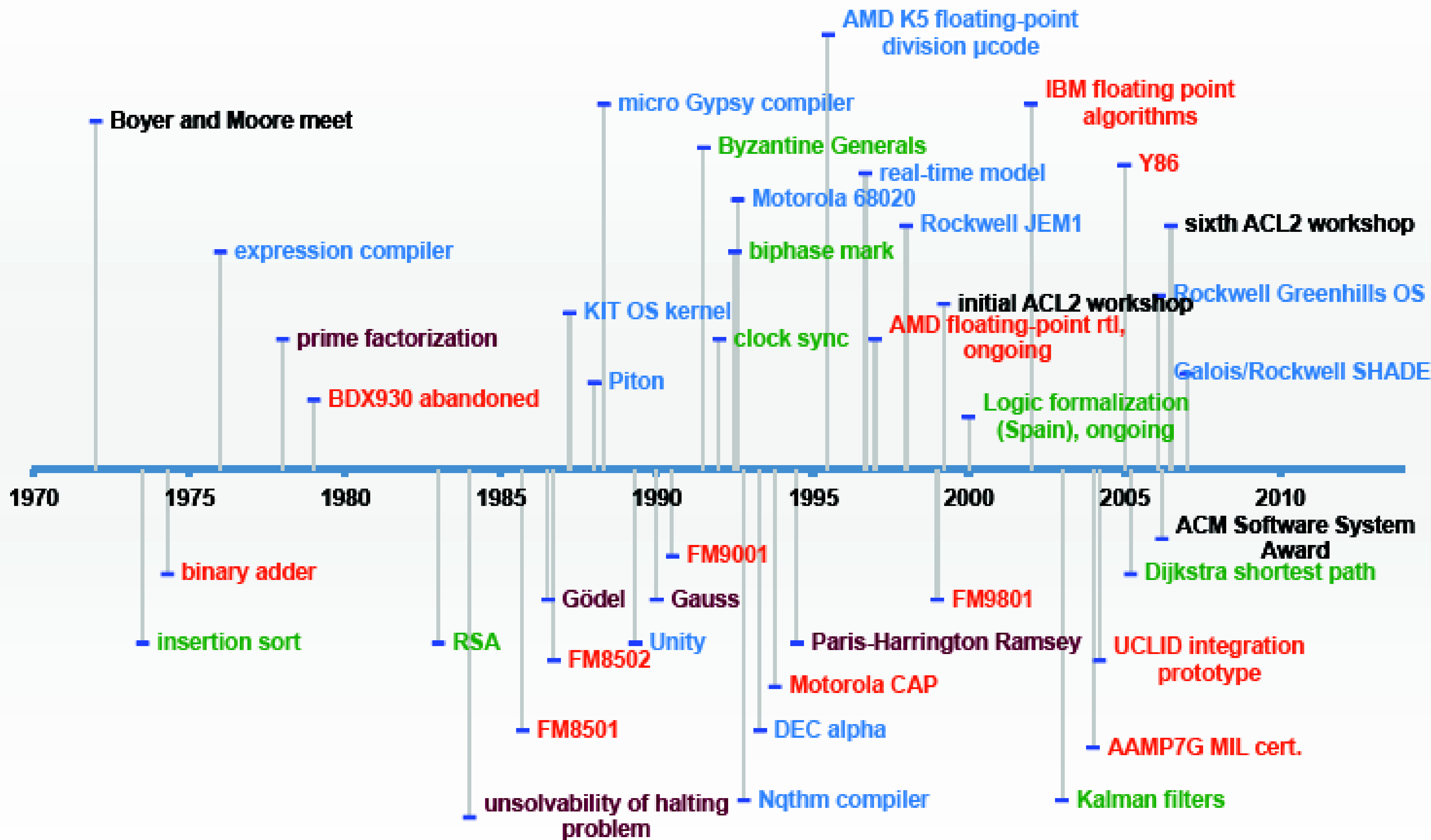
Inductive Assertion Style Proofs via Operational Semantics

J Strother Moore

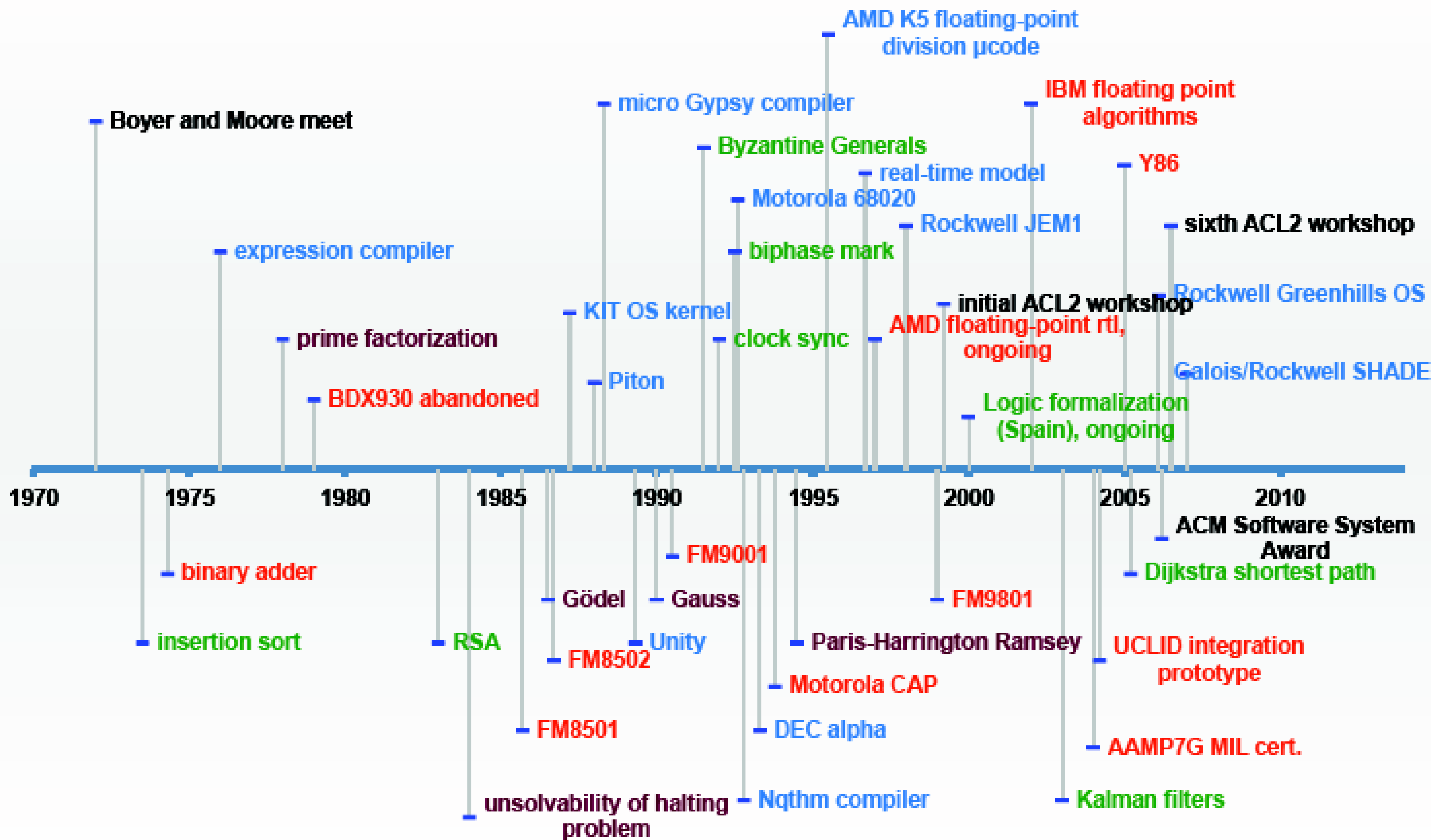
Department of Computer Sciences
The University of Texas at Austin

September, 2006



AMD K5 floating-point division μcode
IBM floating point algorithms
micro Gypsy compiler
Boyer and Moore meet
Byzantine Generals
real-time model
Y86
Motorola 68020
Rockwell JEM1
sixth ACL2 workshop
expression compiler
biphase mark
Rockwell Greenhills OS
initial ACL2 workshop
KIT OS kernel
AMD floating-point rtl, ongoing
prime factorization
clock sync
Galois/Rockwell SHADE
Piton
BDX930 abandoned
Logic formalization (Spain), ongoing
1970
1975
1980
1985
1990
1995
2000
2005
2010
ACM Software System Award
FM9001
binary adder
Dijkstra shortest path
Gödel
Gauss
FM9801
insertion sort
RSA
Unity
Paris-Harrington Ramsey
UCLID integration prototype
FM8502
Motorola CAP
FM8501
DEC alpha
AAMP7G MIL cert.
unsolvability of halting problem
Nqthm compiler
Kalman filters

Demo 1


AMD K5 floating-point division µcode
IBM floating point algorithms
micro Gypsy compiler
Boyer and Moore meet
Byzantine Generals
Y86
real-time model
Motorola 68020
Rockwell JEM1
sixth ACL2 workshop
expression compiler
biphase mark
Rockwell Greenhills OS
initial ACL2 workshop
KIT OS kernel
AMD floating-point rtl, ongoing
prime factorization
clock sync
Galois/Rockwell SHADE
Piton
BDX930 abandoned
Logic formalization (Spain), ongoing
1970
1975
1980
1985
1990
1995
2000
2005
2010
ACM Software System Award
FM9001
binary adder
Dijkstra shortest path
Gödel
Gauss
FM9801
insertion sort
RSA
Unity
Paris-Harrington Ramsey
UCLID integration prototype
FM8502
Motorola CAP
FM8501
DEC alpha
AAMP7G MIL cert.
unsolvability of halting problem
Nqthm compiler
Kalman filters

Our Secret

Pick a traditional mathematical logic, put your effort into building a powerful symbolic manipulation engine for it, and get on with the task of describing and analyzing computing systems.

So Much for History...

This talk is about how to do inductive assertion style proofs with an operational semantic model.

No Floyd-Hoare semantics need be expressed.

No verification condition generator (VCG) need be defined.

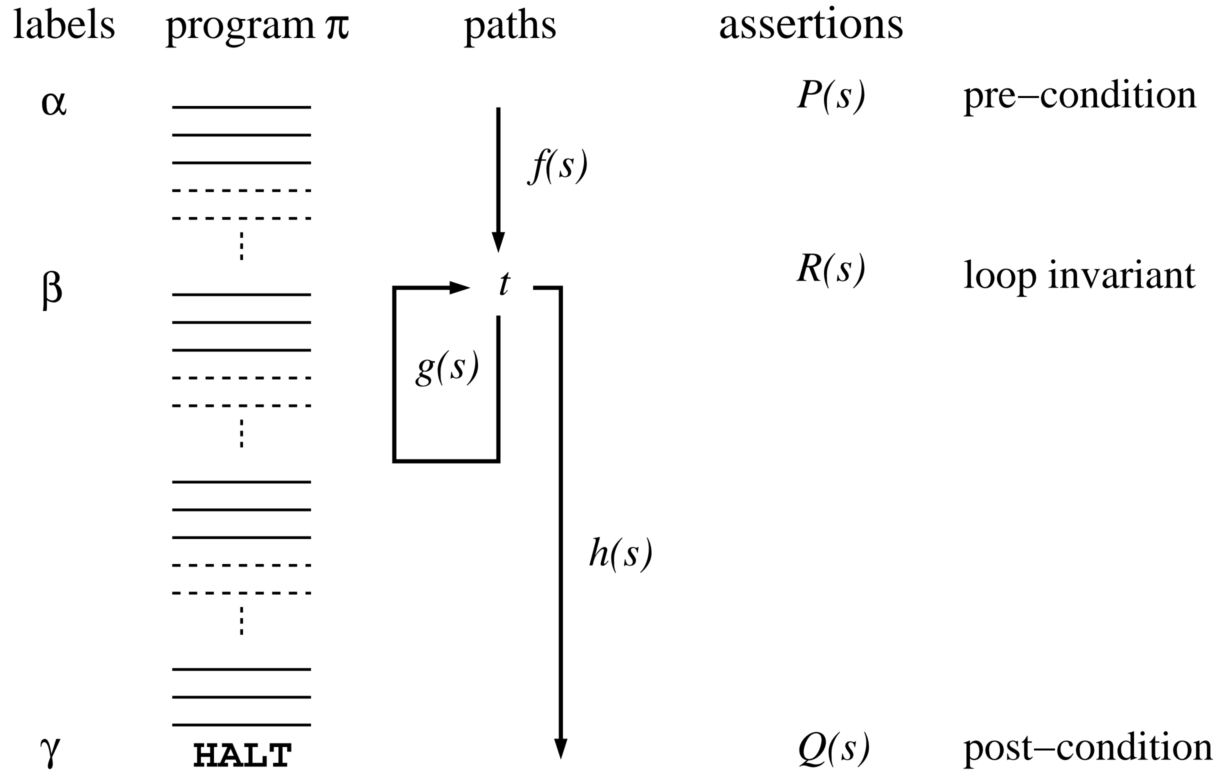
Operational Semantics

The *semantics* of the programming language may be given by a function run which "interprets" a program against some state and determines the "final" state.

$$run\,(k, s) = \begin{cases} s & \text{if } k = 0 \\ run\,(k-1, step\,(s)) & \text{otherwise} \end{cases}$$

Here, $step$ is the single step state transition function.

Conventions



VC1. $P(s) \rightarrow R(f(s))$,

VC2. $R(s) \wedge t \rightarrow R(g(s))$, and

VC3. $R(s) \wedge \neg t \rightarrow Q(h(s))$.

We assume the program in s, π, does not change during execution.

Let s_0 be the initial state of program π.

$pc\,(s_0) = \alpha$

Let s_k denote $run\,(k, s_0)$.

Formally Stated Correctness Theorems

Total:

$$\exists k : P\,(s_0) \rightarrow (Q(s_k) \wedge pc(s_k) = \gamma).$$

This is sometimes stated without the quantifier as

$$P\,(s_0) \rightarrow (Q(run\,(clock\,(s_0), s_0)) \wedge \ldots).$$

Partial:

$$P\,(s_0)\ \wedge\ pc(s_k) = \gamma \rightarrow Q(s_k).$$

Advantages of Operational Semantics

They

- are entirely within a logical framework and so permit logical analysis of programs by traditional formal proofs, without introduction of meta-logical transformers (VCGs),
- are generally *executable*,
- are easily related to implementations,
- allow derivation of language properties,

- may allow derivation of intensional properties (e.g., how many steps a program takes to terminate), and
- they allow verification of system hierarchies (e.g., the CLI stack).

A Disadvantage

Proofs of program properties can be complicated (or at least appear so) because of the presence of the interpreter, the program counter, the entire machine state, and the clock.

The inductive assertion method produces such nice proof obligations!

Conundrum

Can you prove

$$P\,(s_0) \;\wedge\; pc(s_k) = \gamma \rightarrow Q(s_k).$$

directly – where the only heavy-duty proof work is proving the verification conditions?

Do you need a trusted VCG?

Can you make the automatic proof attempt *generate* the standard verification conditions from the operational semantics?

Caveat

The observations I make below are not deep, but I think they have important practical implications:

Theorem: $P(s_0) \wedge pc(s_k) = \gamma \rightarrow Q(s_k)$

Proof: Define

$$Inv(s) \equiv \begin{cases} P(s) & \text{if } pc(s) = \alpha \\ R(s) & \text{if } pc(s) = \beta \\ Q(s) & \text{if } pc(s) = \gamma \\ Inv(step(s)) & \text{otherwise} \end{cases}$$

(Actually, we assert "$prog(s) = \pi$" at α, β and γ, but we omit that here by our convention that the program is always π.)

Theorem: $P(s_0) \wedge pc(s_k) = \gamma \rightarrow Q(s_k)$

Proof: Define

$$Inv(s) \equiv \begin{cases} P(s) & \text{if } pc(s) = \alpha \\ R(s) & \text{if } pc(s) = \beta \\ Q(s) & \text{if } pc(s) = \gamma \\ Inv(step(s)) & \text{otherwise} \end{cases}$$

Objection: Is this definition consistent? Yes: Every tail-recursive definition is witnessed by a total function. (Manolios and Moore, 2000)

Theorem: $P(s_0) \wedge pc(s_k) = \gamma \rightarrow Q(s_k)$

Proof: Define

$$Inv(s) \equiv \begin{cases} P(s) & \text{if } pc(s) = \alpha \\ R(s) & \text{if } pc(s) = \beta \\ Q(s) & \text{if } pc(s) = \gamma \\ Inv(step(s)) & \text{otherwise} \end{cases}$$

Assume

$$Inv(s) \rightarrow Inv(step(s)).$$

We'll see the proof in a moment.

Theorem: $P(s_0) \wedge pc(s_k) = \gamma \rightarrow Q(s_k)$

Proof: Define

$$Inv(s) \equiv \begin{cases} P(s) & \text{if } pc(s) = \alpha \\ R(s) & \text{if } pc(s) = \beta \\ Q(s) & \text{if } pc(s) = \gamma \\ Inv(step(s)) & \text{otherwise} \end{cases}$$

Thus $Inv(s_0) \rightarrow Inv(s_k)$.

But $pc(s_0) = \alpha$ and $pc(s_k) = \gamma$.

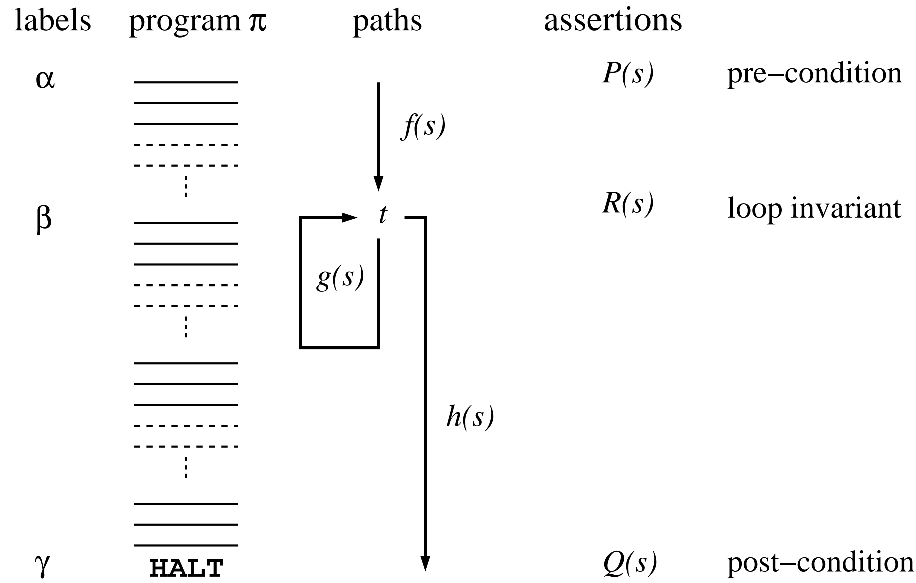
Thus $Inv(s_0) = P(s_0)$ and $Inv(s_k) = Q(s_k)$. $\square$

Lemma: $Inv(s) \rightarrow Inv(step(s))$

Proof: Consider cases on $pc(s)$ as suggested by def Inv. $\square$

$$Inv(s) \equiv \begin{cases} P(s) & \text{if } pc(s) = \alpha \\ R(s) & \text{if } pc(s) = \beta \\ Q(s) & \text{if } pc(s) = \gamma \\ Inv(step(s)) & \text{otherwise} \end{cases}$$

$$Inv(s) \rightarrow Inv(step(s))$$



$$Inv(s) \equiv \begin{cases} P(s) & \text{if } pc(s) = \alpha \\ R(s) & \text{if } pc(s) = \beta \\ Q(s) & \text{if } pc(s) = \gamma \\ Inv(step(s)) & \text{otherwise} \end{cases}$$

$Inv\,(s) \rightarrow Inv\,(step\,(s))$

case	*subgoal generated by expanding* Inv
$pc\,(s) = \alpha$:	$P\,(s) \rightarrow R\,(f\,(s))$
$pc\,(s) = \beta$:	$R\,(s) \wedge t \rightarrow R\,(g\,(s))$, and $R\,(s) \wedge \neg t \rightarrow Q\,(h(s))$
$pc\,(s) = \gamma$:	$step\,(s) = s$
otherwise:	$Inv\,(s) = Inv\,(step\,(s))$

Recap: Given the definition of Inv, the "natural" proof of

$$Inv\,(s) \rightarrow Inv\,(step\,(s))$$

generates the standard verification conditions

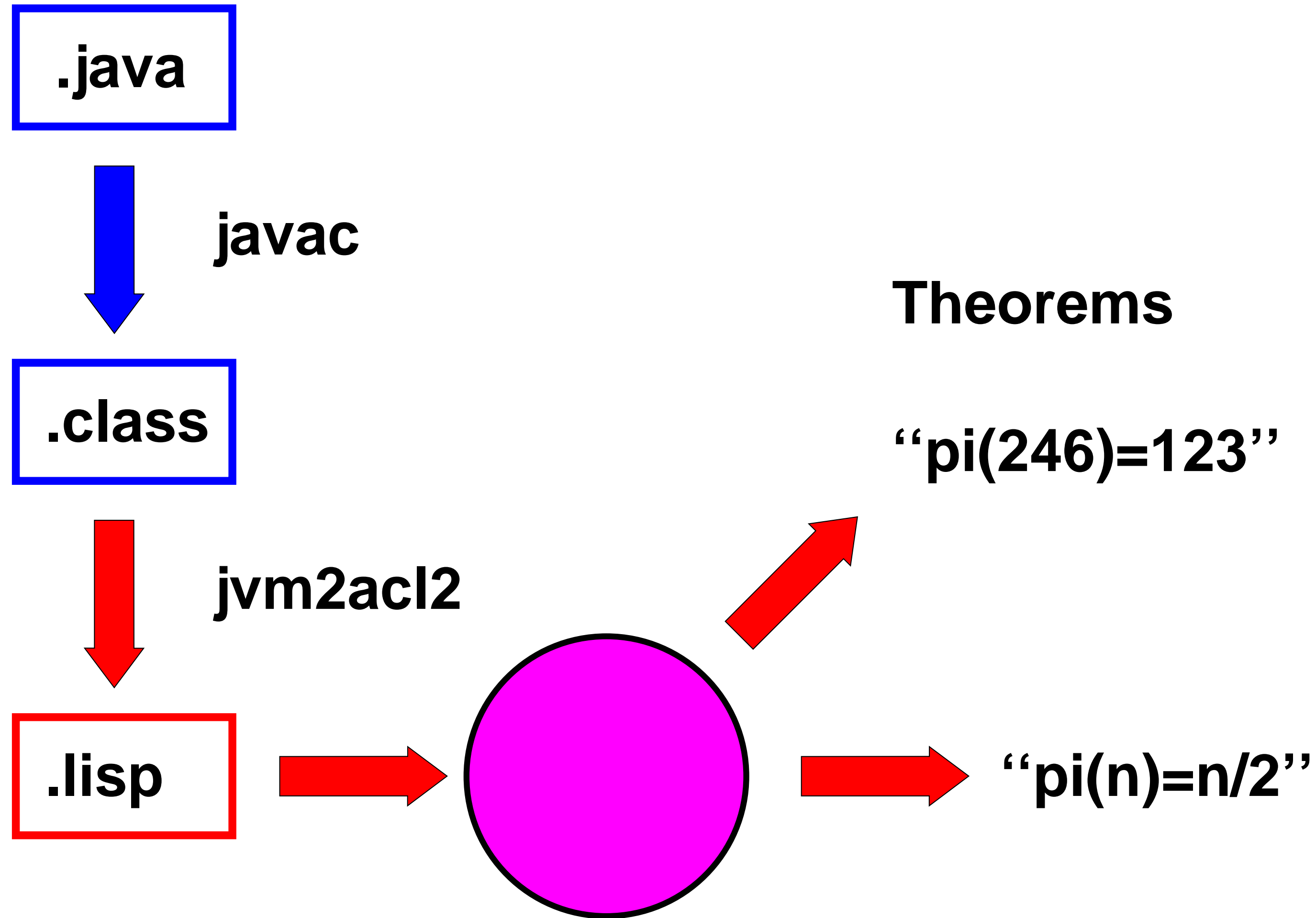
VC1. $P\,(s) \rightarrow R\,(f\,(s))$,
VC2. $R\,(s) \wedge t \rightarrow R\,(g\,(s))$, and
VC3. $R\,(s) \wedge \neg t \rightarrow Q\,(h(s))$
as subgoals from the operational semantics!

No VCG is necessary. The VCs are simplified as they are generated.

A Demonstration

- Java and the JVM – a program π to compute $n/2$
- a pre-existing operational semantics for the JVM
- execution of the operational semantics
- an inductive invariant proof of partial correctness

Java and the JVM



JVM Operational Semantics

Our “M6” model is based on an implementation of the J2ME KVM. It executes most J2ME Java programs (except those with significant I/O or floating-point).

M6 supports all CLDC data types, multi-threading, dynamic class loading, class initialization and synchronization via monitors.

We have translated the entire Sun CLDC API library implementation into our representation with 672 methods in 87 classes. We provide implementations for 21 out of 41 native APIs that appear in Sun's CLDC API library.

We prove theorems about bytecoded methods with the ACL2 theorem prover.

This work was supported by a gift from Sun Microsystems.

Disclaimers about Our JVM Model

Our thread model assumes

- sequential consistency and
- atomicity at the bytecode level.

This inductive assertion work does not exercise the thread model.

Demo 2

Discussion

We did not write a VCG for the JVM.

The VCs were generated directly from the operational semantics by the theorem prover.

Since VCs are generated by proof, the paths explored and the VCs generated are sensitive to the pre-condition specified.

The VCs are simplified (and possibly proved) by the same process.

We did not count instructions or define a "clock

function.”

We did not constrain the inputs so that the program terminated.

We have also handled total correctness via the VCG approach; a decreasing ordinal measure is provided at each cut point. “Clock functions” can be automatically generated and admitted from such proofs.

Primary Citation

J S. Moore, "Inductive Assertions and Operational Semantics," *CHARME 2003*, D. Geist (Ed.), Springer Verlag LNCS 2860, pp. 289–303, 2003.

Other Examples

Nested loops are handled exactly as by standard VCG methods.

```
public static int tfact(int n){  /* Factorial by repeated addition.     */
    int i = 1;                   /* Verified using inductive assertions */
    int b = 1;                   /* by Alan Turing, 1949.               */
    while (i<=n){
        int j = 1;
        int a = b;
        while (j < i) {
            b = a+b;
            j++;
        };
        i++;
    };
    return b;
}
```

Recursive methods can be handled.

```
public static int fact(int n){
  if (n>0)
    {return n*fact(n-1);}
  else return 1;
}
```

To handle recursive methods we

- modify run to terminate upon top-level `return`, and

- add a standard invariant about the shape of the JVM call stack.

Conclusion

If you have

- a theorem prover and
- a formal operational semantics,

you can prove formally stated *partial program correctness* theorems using *inductive assertions* without building or verifying a VCG.

Related Work

P. Y. Gloess, “Imperative Program Verification in PVS,” École Nationale Supérieure Électronique, Informatique et Radiocommunications de Bordeaux, 1999.

P. Homeier and D. Martin, “A Mechanically Verified Verification Condition Generator,” *The Computer Journal*, **38**(2), pp. 131–141, July 1995.

P. Manolios and J Moore, “Partial Functions in ACL2,” *JAR* 2003.

J. Matthews, J S. Moore, S. Ray, and D. Vroon: "Verification Condition Generation via Theorem Proving," to appear in M. Hermann and A. Voronkov, editors, *Proceedings of the 13th International Conference on Logic for Programming, Artifical Intelligence, and Reasoning* (LPAR 2006), Phnom Penh, Cambodia, November 2006, Springer-Verlag.