CS344M
Autonomous Multiagent Systems

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Good Afternoon, Colleagues

Are there any questions?

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- How can we apply game theory to RoboCup?
- Examples of game theory that aren't modeled as a matrix?
- What about irrational agents?
- Pure vs mixed strategy?

Logistics

- Progress reports due in 2 weeks

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Game Theory Premises

- Simultaneous actions: (mutual exclusivity?)

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- No communication
- Outcome depends on **combination** of actions
- Utility (payoff) encapsulates **everything** about preferences over outcomes

Solution Concepts

- Dominant strategy
- Nash equilibrium
- Pareto optimality
- Maximum social welfare
- Maximin strategy

Prisoner's Dilemma

		Column	
		C(1)	D(2)
Row	C(1)	3,3	0,5
	D(2)	5,0	1,1

Chicken

		Column	
		C(1)	D(2)
Row	C(1)	3,3	1,5
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Bach/Stravinsky

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- Propose a payoff matrix

Bach/Stravinsky

		Wife	
		S	B
Me	S	2,1	0,0
	B	0,0	1,2

Nash Equilibrium

- Does every game have a pure strategy Nash equilibrium?

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- We each put a penny down covered
- If they match, I win, if they don't, you win

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  - Nobel prize **and** academy award!
- Not known if complexity of finding one is NP-complete or in P

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- Is a Pareto optimal outcome necessarily the result of Nash equilibrium strategies?
- Is the maximum social welfare outcome necessarily Pareto optimal?
- If both players play maximin, is it necessarily a Nash equilibrium?

Mixed strategy equilibrium

		Player 2	
		Action 1	Action 2
Player 1	Action 1	4,8	2,0
	Action 2	6,2	0,8

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Rock/Paper/Scissors

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- Why is anything else **not** an equilibrium?

Correlated Equilibria

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Want only S,S or B,B - 50% each