# CS344M <br> Autonomous Multiagent Systems 

Patrick MacAlpine

Department of Computer Science The University of Texas at Austin

## Good Afternoon, Colleagues

Are there any questions?

## Logistics

- Progress reports due in 1 week


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- Progress reports due in 1 week
- Topic change for next week: multiagent learning
- Talks in the department:
- FAI Talk - Sergey Levine, Frdiay 11 am GDC 6.302


## Mixed strategy equilibrium

## Player 2 <br> Action 1 Action 2

Action 1
3,7
2,2

Player 1
Action 2
6,5
1,7

## Mixed strategy equilibrium

$$
\begin{array}{lll}
\hline & \text { Player } & 2 \\
\text { Action } 1 & \text { Action } 2
\end{array}
$$

$$
\begin{array}{lll}
\text { Action } 1 & 3,7 & 2,2
\end{array}
$$

Player 1
Action 2
6,5
1,7

- What if player 2 picks action $13 / 4$ of the time?
- What if player 2 picks action 1 1/4 of the time?
- Player 1 must be indifferent between actions 1 and 2
- Player 2 must be indifferent between actions 1 and 2


## Correlated Equilibria

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Sometimes mixing isn't enough: Bach/Stravinsky
Wife

|  | $S$ | $B$ |
| :---: | :---: | :---: |
| $S$ | 2,1 | 0,0 |

Me
B
0,0
1,2

Want only S,S or B,B-50\% each

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- We have no way of getting in touch.
- When and where?
- What are the Nash equilibria?


## Incomplete Information Games

- We each get one of 3 cards: 1,2,3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum


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Card ?

$$
\begin{array}{ccc} 
& \mathrm{R} & \mathrm{~F} \\
\mathrm{R} & 5,-5 & 1,-1
\end{array}
$$

Card 3

$$
\begin{array}{ll}
F & -1,1
\end{array}
$$

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| :---: | :---: | :---: |
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Card 3

| $F$ | $-1,1$ | 0,0 |
| :--- | :--- | :--- |

Card ?
R F
$\begin{array}{lll}\mathrm{R} & -5,5 & 1,-1\end{array}$
Card 1
F
$-1,1$
0,0

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With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium

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- What if one player isn't rational?
- What can'† game theory simulate?


## Repeated games

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- Doran's ICML slides

