Type checking

Slides adapted from CS 412 (Cornell) and CS 164 (Berkeley)

Types

- What is a type?
  - The notion varies from language to language
- Consensus
  - A set of values
  - A set of operations on those values
- Classes are one instantiation of the modern notion of type

Why Do We Need Type Systems?

Consider the assembly language fragment

addi $r1, $r2, $r3

What are the types of $r1, $r2, $r3?

Types and Operations

- Most operations are legal only for values of some types
  - It doesn't make sense to add a function pointer and an integer in C
  - It does make sense to add two integers
  - But both have the same assembly language implementation!

Type Systems

- A language's type system specifies which operations are valid for which types
- The goal of type checking is to ensure that operations are used with the correct types
  - Enforces intended interpretation of values, because nothing else will!
- Type systems provide a concise formalization of the semantic checking rules

What Can Types do For Us?

- Can detect certain kinds of errors
- Memory errors:
  - Reading from an invalid pointer, etc.
- Violation of abstraction boundaries:

```
class FileSystem {
  open(x : String) : File {
     …
  }
…
}
```

```
class Client {
  f(fs : FileSystem) {
     File fdesc <- fs.open("foo")
     …
  } -- f cannot see inside fdesc !
}
```

Dynamic And Static Types

- A *dynamic type* attaches to an object reference or other value
  - A run-time notion
  - Applicable to any language
- The *static type* of an expression or variable is a notion that captures all possible dynamic types the value of the expression could take or the variable could contain
  - A compile-time notion

Dynamic and Static Types. (Cont.)

- In early type systems the set of static types correspond directly with the dynamic types:
  - for all expressions E,

    dynamic_type(E) = static_type(E)

    (in **all** executions, E evaluates to values of the type inferred by the compiler)
- This gets more complicated in advanced type systems

Subtyping

- Define a relation X ≤ Y on classes to say that:
  - An object (value) of type X could be used when one of type Y is acceptable, or equivalently
  - X conforms to Y
  - In Java this means that X extends Y
- Define a relation ≤ on classes

  X ≤ X

  X ≤ Y if X inherits from Y

  X ≤ Z if X ≤ Y and Y ≤ Z

Dynamic and Static Types

```
class A: …
class B extends A: …
…………
  x: A
  x =  A()
  …
  x = B()
  …
```

x has static type A

Here, x's value has dynamic type A

Here, x's value has dynamic type B

- A variable of static type A can hold values of static type B at runtime, if B ≤ A

Dynamic and Static Types

Soundness theorem:

$\forall$ E.   dynamic_type(E) ≤ static_type(E)

Why is this Ok?

- For E, compiler uses static_type(E) (call it C)
- All operations that can be used on an object of type C can also be used on an object of type C' ≤ C
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can *only add* attributes or methods
- Methods can be redefined but with same type !

Type Checking Overview

- Three kinds of languages:
  - *Statically typed*: All or almost all checking of types is done as part of compilation (C#, Java). Static type system is rich.
  - *Dynamically typed*: Almost all checking of types is done as part of program execution (Scheme, Python). Static type system is trivial.
  - *Untyped*: No type checking (machine code). Static and dynamic type systems trivial.

The Type Wars

- Competing views on static vs. dynamic typing
- Static typing proponents say:
  - Static checking catches many programming errors at compile time
  - Avoids overhead of runtime type checks
- Dynamic typing proponents say:
  - Static type systems are restrictive
  - Rapid prototyping easier in a dynamic type system

The Type Wars (Cont.)

- In practice, most code is written in statically typed languages with an "escape" mechanism
  - Unsafe casts in C, native methods in Java, unsafe modules in Modula-3
- Within the strongly typed world, are various devices, including subtyping, coercions, and type parameterization.
- Of course, each such wrinkle introduces its own complications.

Conversion

- In Java, can write

```
int x = 'c';
float y = x;
```

- But relationship between **char** and **int,** or **int** and **float** not usually called subtyping, but rather *conversion* (or *coercion).*
- In general, might be a change of value or representation. Indeed **int**→**float** can lose information—a *narrowing conversion.*

Conversions: Implicit vs. Explicit

- Conversions, when automatic (implicit), another way to ease the pain of static typing.
- Typical rule (from Java):
  - Widening conversions are implicit; narrowing conversions require explicit cast.
- *Widening conversions* convert "smaller" types to "larger" ones (those whose values are a superset).
- *Narrowing conversions* go in opposite direction (and thus may lose information).

Examples

- Thus,

```
Object x = ...;   String y = ...
int a = ...;  short b = 42;
x = y; a = b;         // OK
y = x; b = a;  // ERRORS
x = (Object) y;        // OK
a = (int) b;        // OK
y = (String) x;  // OK but may cause exception
b = (short) a;          // OK but may lose information
```

- Possibility of implicit coercion complicates type-matching rules (see C++).

Type Inference

- *Type Checking* is the process of checking that the program obeys the type system
- Often involves inferring types for parts of the program
  - Some people call the process *type inference* when inference is necessary

Rules of Inference

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions (for the lexer)
  - Context-free grammars (for the parser)
- The appropriate formalism for type checking is logical rules of inference having the form
  - *If Hypothesis is true, then Conclusion is true*
- For type checking, this becomes:
  - *If E_1 and E_2 have certain types, then E_3 has a certain type*
  - *(eg) if E_1 and E_2 have type int, then $E_1 + E_2$ has a certain type*

Why Rules of Inference?

- Rules of inference are a compact notation for “If-Then” statements
- Given proper notation, easy to read (with practice), so easy to check that the rules are accurate.
- Can even be mechanically translated into programs.

Type Judgments

- The type judgment:

  |– E : T

  is read:

  "E is a well-typed construct of type T"

- Type checking program P is demonstrating the validity of the type judgment |– P : T for some type T

- Sample valid type judgments for program fragments:

  |– 2 : int |– 2 * (3 + 4) : int

  |– true : bool |– (true ? 2 : 3) : int

Deriving a Type Judgment

- Consider the judgment:

  |– (b ? 2 : 3) : int

- What do we need in order to decide that this is a valid type judgment?

- b must be a bool (|– b: bool)
- 2 must be an int (|– 2: int)
- 3 must be an int (|– 3: int)

Hypothetical Type Judgments

- The hypothetical type judgment

  A |– E : T

  is read:

  "In type context A expression E is well-typed with type T"

- A type context is a mapping of identifiers to types (i.e., a symbol table). It's a set of assumptions about the types of identifiers.

- Sample valid hypothetical type judgments:

  b: bool |– b : bool

  |– 2 + 2 : int

  b: bool, x: int |– (b ? 2 : x) : int

  b: bool, x: int |– b : bool

  b: bool, x: int |– 2 + 2 : int

- Type checking program P is demonstrating the validity of A|– P : T for some type T and the language's standard environment A

Deriving a Type Judgment

- To show:

  b: bool, x: int |– (b ? 2 : x) : int

- Need to show:

  b: bool, x: int |– b : bool

  b: bool, x: int |– 2 : int

  b: bool, x: int |– x : int

General Rule

- For any type environment A, expressions E, E_1 and E_2, the judgment

$$A \mid\!- (E\ ?\ E_1 : E_2) : T$$

is valid if:

$$A \mid\!- E : \text{bool}$$
$$A \mid\!- E_1 : T$$
$$A \mid\!- E_2 : T$$

Inference Rule Schema

$$\frac{\overbrace{A \mid\!- E : \text{bool} \qquad A \mid\!- E_1 : T \qquad A \mid\!- E_2 : T}^{\text{Premises (a.k.a., antecedant)}}}{\underbrace{A \mid\!- (E\ ?\ E_1 : E_2) : T}_{\text{Conclusion (a.k.a., consequent)}}}\ \text{(if-rule)}$$

- Holds for any choice of A, E, E_1, E_2, and T
- An inference rule schema defines an infinite number of inference rules

Axioms

- An axiom is an inference rule (schema) with no premises

$$\frac{}{A \mid\!- \textbf{true} : \text{bool}}$$

Why Inference Rules?

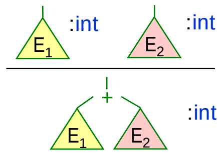
- Inference rules: compact, precise language for specifying static semantics (can specify languages in ~20 pages vs. 100's of pages of Java Language Specification)
- Inference rules are to type inference systems as productions are to context-free grammars
- Type judgments are to type inference systems as nonterminals are to context-free grammars
- Type checking is an attempt to prove that a type judgment is $A \mid\!- E : T$ is valid

Meaning of Inference Rule

- Inference rule says:
  given that the antecedent judgments are derivable
  - with a uniform substitution for meta-variables (i.e., A, E_1, E_2)

  then the consequent judgment is derivable
  - with the same uniform substitution for the meta-variables

$$\frac{\begin{array}{c} A \mid\!- E_1 : \text{int} \\ A \mid\!- E_2 : \text{int} \end{array}}{A \mid\!- E_1 + E_2 : \text{int}} \ (+)$$



Proof Tree

- A construct is well-typed if there exists a type derivation for a type judgment for the construct
- Type derivation is a proof tree where all the leaves are axioms
- Example: if A1 = b: bool, x: int, then:

$$\dfrac{\dfrac{\dfrac{}{A1 \mid\!- b : \text{bool}}}{A1 \mid\!- \,!b : \text{bool}} \quad \dfrac{\dfrac{}{A1 \mid\!- 2 : \text{int}} \ \dfrac{}{A1 \mid\!- 3 : \text{int}}}{A1 \mid\!- 2+3 : \text{int}} \quad \dfrac{}{A1 \mid\!- x : \text{int}}}{A1 \mid\!- (\,!b\ ?\ 2+3 : x\,) : \text{int}}$$

Proof Tree, cont.

- Axioms are analogous to production with epsilon on the right hand side
- A complete proof of A |– E : T is like a derivation of epsilon from A |– E : T

Type Judgments for Statements

- Statements that have no value are said to have type void, i.e., judgment
  |– S : void
  means "S is a well-typed statement with no result type"
- ML uses unit instead of void

While Statements

- Rule for while statements:

$$\frac{A \vdash E : bool \qquad A \vdash S : T}{A \vdash \textbf{while}\ (E)\ S : void} \ \text{(while)}$$

Assignment (Expression) Statements

$$\frac{A,\ id : T \vdash E : T}{A,\ id : T \vdash id = E : T} \ \text{(variable-assign)}$$

$$\frac{A \vdash E_3 : T \qquad A \vdash E_2 : int \qquad A \vdash E_1 : array[T]}{A \vdash E_1[E_2] = E_3 : T} \ \text{(array-assign)}$$

Statement Sequences

- Rule: A sequence of statements is well-typed if the first statement is well-typed, and the remaining are well-typed too:

$$\frac{A \vdash S_1 : T_1 \qquad A \vdash (S_2 ; \ldots ; S_n) : T_n}{A \vdash (S_1 ; S_2 ; \ldots ; S_n) : T_n} \ \text{(sequence)}$$

Identifier Declaration List

- What about variable declarations (with initialization)?
- Declarations add entries to the type environment in which the scope of the declared variable must type check

$$\frac{A \vdash E : T \qquad A,\ id : T \vdash (S_2 ; \ldots ; S_n) : T'}{A \vdash (id : T = E ; S_2 ; \ldots ; S_n) : T'} \ \text{(declaration)}$$

Function Calls

- If expression E is a function value, it has a type $T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r$
- T_i are argument types; T_r is return type
- How to type-check function call $E(E_1,\ldots,E_n)$?

$$\frac{A \vdash E : T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r \qquad A \vdash E_i : T_i \quad (i \in 1..n)}{A \vdash E(E_1,\ldots,E_n) : T_r} \text{ (function-call)}$$

Function Declarations

- Consider a function declaration of the form

  $T_r \ f\ (T_1\ a_1,\ldots,\ T_n\ a_n)\ \ \{ E; \}$

- The body of the function must type check in an environment containing the type bindings for the formal parameters

$$\frac{A,\ a_1 : T_1,\ldots,\ a_n : T_n \vdash E : T_r}{A \vdash T_r\ \ f\ (T_1\ a_1,\ldots,\ T_n\ a_n)\ \ \{ E; \} : \text{void}} \text{ (function-body)}$$

But what about recursion?

- Example:

```
int fact(int x)  {
    if (x==0) return 1;
    else return x * fact(x - 1);
}
```

- Need to prove: $A \vdash x * \text{fact}(x-1) : \text{int}$
  where: $A = \{ \text{fact}: \text{int} \rightarrow \text{int},\ x : \text{int} \}$

And mutual recursion?

- Example:

```
int f(int x) { return g(x) + 1; }
int g(int x) { return f(x) – 1; }
```

- Need environment containing at least

  $f: \text{int} \rightarrow \text{int},\ g: \text{int} \rightarrow \text{int}$

  when checking both f and g
- Two-pass approach needed:
  - First pass: collect all function signatures into a type environment A
  - Second pass: type-check each function declaration using this global environment A
  - How do we express this in our type inference notation?

Solution

- Intuition:
  - Make one pass over program to add top level function signatures to symbol table
  - Use these signatures in a second pass to type-check program
  - Slight complication for object-oriented programs with methods inside classes:
    - functions are named using pair (Class, method name)
- Formalization:
  - Split the type environment into two parts, one for functions and one for variables
  - Type environment for functions does not change during the second pass
- We will not show this to keep the notation simple.

How to Check Return?

$$\frac{A \vdash E : T}{A \vdash \textbf{return}\ E : void} \quad \text{(return1)}$$

- A return statement produces no value for its containing context to use
- Does not return control to containing context
- Suppose we use type void...
- ...then how to make sure T is the return type of the current function?

Put return type in environment

- Add a special entry { return_fun : T } when we start checking the function "f", look up this entry when we hit a return statement.
- To check T_r f (T_1 a_1,..., T_n a_n) { **return** S; } in environment A, need to check:

$$\frac{A,\ a_1 : T_1, \ldots,\ a_n : T_n,\ return\_f : T_r \vdash E : void}{A \vdash T_r\ \ f\ (T_1\ a_1, \ldots, T_n\ a_n)\ \ \{\ E;\ \}\ :\ void} \text{(function-body)}$$

$$\frac{A,\ return\_f : T \vdash E : T}{A,\ return\_f : T \vdash \textbf{return}\ E : void} \quad \text{(return)}$$

Example

$$\cfrac{\cfrac{\{f{:}int \rightarrow int,\ x : int,\ return\_f : int\} \vdash x{:}int}{\{f{:}int \rightarrow int,\ x : int,\ return\_f : int\} \vdash return\ x;\ :\ void} \text{(return)}}{\{f{:}int \rightarrow int\} \vdash int\ \ f\ (x{:}int)\ \ \{\ return\ x;\ \}\ :\ void} \text{(function definition)}$$

Arrays

- Arrays:
  - array types are of form int[], float[] etc.

$$\frac{A \vdash E_0 : T[\,] \quad A \vdash E_1 : \text{int}}{A \vdash E_0[E_1] : T}$$

[redacted]

$$\frac{A \vdash E : T[\,]}{A \vdash E\,.\,\text{length} : \text{int}}$$

$$\frac{A \vdash E : \text{int}}{A \vdash \text{new } T[E] : T[\,]}$$

Classes

- Class would be represented in the type environment by a list of (name:type) pairs which has one entry for each field and method

```
class C1 {
  int x,y;
  int get_x() {return x;}
}
C1: {x:int,y:int,get_x:void→int}
```

Inference rules

- Constructors:

$$\frac{T \in C}{A \vdash \text{new } T() : T}$$

- Field accesses:

$$\frac{A \vdash E : T \quad T \in C \quad (id : T') \in T}{A \vdash E\,.\,id : T'}$$

- Method invocations

$$\frac{\begin{array}{c} A \vdash E_0 : T_1 \times \ldots \times T_n \to T \\ A \vdash E_i : T_i,\ 1 \leq i \leq n \end{array}}{A \vdash E_0(E_1, \ldots, E_n) : T}$$

Static Semantics Summary

- Type inference system = formal specification of typing rules
- Concise form of static semantics: typing rules expressed as inference rules
- Expression and statements are well-formed (or well-typed) if a typing derivation (proof tree) can be constructed using the inference rules