CS 377P Fall 2024
Assignment 3
Due: 9AM, October 9th, 2024

October 2, 2024

No late submissions will be accepted for this assignment.

Some of the problems in this assignment ask you to write code in MATLAB, which is available on the public machines in the CS department. You can also use Octave, which is free software and is similar to MATLAB, or Mathematica.

1. (Iterative solution of linear systems, 20 points) Consider the linear system

   $4x + 2y = 6$
   $x - 3y = -2$

   (a) (5 points) Write down the recurrence relation that corresponds to solving this system using the Jacobi method, starting with the initial approximation ($x_1 = 0, y_1 = 0$). Use the first equation to refine the approximation for x and the second equation to refine the approximation for y. Express this recurrence as a computation involving matrices and vectors.

   (b) (5 points) Compute the first 10 approximations (x_i, y_i) and plot a 3D plot (x, y, i) in which the z-axis is the iteration number i. Give an intuitive explanation of this 3D plot. You do not need to turn in any code but turn in your plot and explanation.

   (c) (10 points) Repeat these two parts for the Gauss-Seidel method. You can find a description of the Gauss-Seidel method online. It was also described in class.

2. (ODE's, 25 points) Consider the second-order differential equation

   $\frac{d^2y}{dx^2} = -y$

   with initial conditions $y(0) = 1, y'(0) = 0$. The exact solution of this equation is $y = cos(x)$.

   (a) What is the difference equation if we use the forward-Euler method to discretize derivatives? Assume the step size is h.

(b) Discretize the initial conditions to find expressions for the first two terms in the solution to the difference equation.

(c) Calculate the solutions to the difference equation in the interval $x = [0, 2\pi]$ for $h = 0.01, 0.1, 0.5, 1.0, 2.0$. Graph each solution together with the exact solution, using a separate graph for each value of h. What trends do you see in your plots? No need to turn in code for the calculations.

Hint: I used Mathematica on the web (for free) to graph the solutions quickly and make sure that my difference equations were correct. Feel free to use any other way to generate the graphs.

3. (PDE's, 40 points) In this problem, we will solve the one-dimensional diffusion equation, which models how heat spreads through a material of uniform conductivity and similar problems. The diffusion equation is usually written as follows

$\frac{\delta f}{\delta t} = D\frac{\delta^2 f}{\delta x^2}$

The solution $f(x,t)$ depends on both x and t. Assume that we have a rod of length 10 meters, and that the two ends of the rod are kept at a fixed temperature of 0^oC, so $f(0,t) = 0.0$ and $f(10,t) = 0.0$. Assume that initially the temperature in the interior of the rod is $f(x,0) = 5.0 * e^{-(x-5)^2}$.

The approximate solution $\hat{f}$ can be thought of a two-dimensional array in which $\hat{f}(i,j) \approx f(i\Delta x, j\Delta t)$. Intuitively, $\hat{f}(i,j)$ is the computed solution after j time steps at a spatial position i spatial steps away from the origin.

Compute the array $\hat{f}$ using the following discretization scheme, which discretizes time using Forward-Euler and discretizes space using centered-differences.

$$\begin{aligned} \Delta x &= 0.25 \\ \Delta t &= 0.020 \\ D &= 1.5 \\ 0 \le x &\le 10 \\ 0 \le t &\le 10 \end{aligned}$$

$\frac{\delta f}{\delta t}|_{(i*\Delta x, j*\Delta t)} \approx \frac{\hat{f}(i,j+1) - \hat{f}(i,j)}{\Delta t}$ (Forward-Euler)

$\frac{\delta^2 f}{\delta x^2}|_{(i*\Delta x, j*\Delta t)} \approx \frac{\hat{f}(i+1,j) - 2*\hat{f}(i,j) + \hat{f}(i-1,j)}{(\Delta x)^2}$ (Centered differences)

(a) Draw the stencil for this discretization scheme. You will find it useful to think of this problem in terms of filling in a matrix whose rows correspond to time-steps and columns correspond to spatial regions in the grid as shown in Figure 1. Notice that the values in row 0 of this matrix are given to you by the initial condition and the values in the first and last columns for all time steps are given by the boundary condition, so the problem is to fill in the interior elements of this matrix.

(b) Use the recurrence equation to find an approximate solution to this problem. Plot the temperature distribution along the rod for different time steps on the same graph (so the x-axis is the distance from the origin in the rod and the y-axis is the temperature). Does this graph jive with your intuition? No need to turn in code.

(c) Increase the time step to 0.075 and repeat the previous steps. Explain your observations briefly.

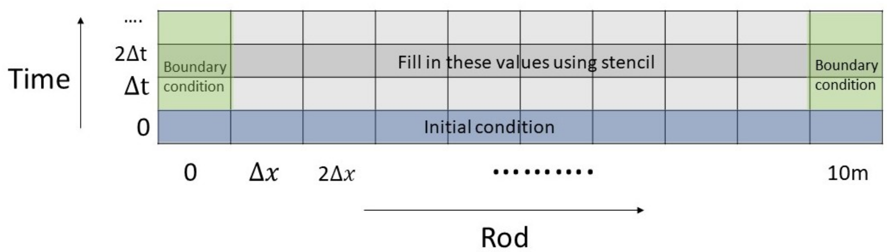

Figure 1: Solve the pde by filling this array

4. (15 points) Explain the following terms in a few sentences each.

   (a) (4 points) Spatial and temporal locality in memory references

   (b) (6 points) The 3 C's: cold misses, capacity misses, conflict misses

   (c) (4 points) Direct-mapped cache, set-associative cache

   (d) (1 points) Explain briefly why the set size in a set-associate cache does not have to be a power of 2.