Live variables and copy propagation

1

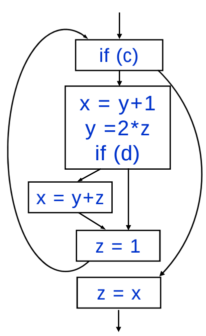
Control Flow Graphs

- Control Flow Graph (CFG) = graph representation of computation and control flow in the program
  - framework to statically analyze program control-flow
- In a CFG:
  - Nodes are basic blocks; they represent computation
  - Edges characterize control flow between basic blocks
- Can build the CFG representation either from the high IR or from the low IR

2

Build CFG from High IR

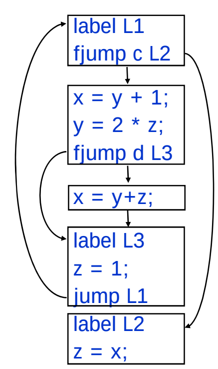
```
while (c) {
    x = y + 1;
    y = 2 * z;
    if (d)  x = y+z;
    z = 1;
}
z = x;
```



3

Build CFG from Low IR

```
label L1
fjump c L2
x = y + 1;
y = 2 * z;
fjump d L3
x = y+z;
label L3
z = 1;
jump L1
label L2
z = x;
```



4

Using CFGs

- Next: use CFG representation to statically extract information about the program
  - Reason at compile-time
  - About the run-time values of variables and expressions in all program executions
- Extracted information example: live variables
- Idea:
  - Define program points in the CFG
  - Reason statically about how the information flows between these program points

Program Points

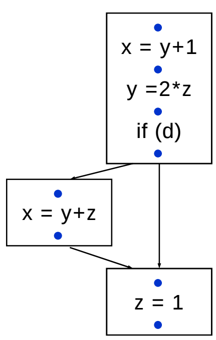
- Two program points for each instruction:
  - There is a program point before each instruction
  - There is a program point after each instruction

Point before → •
x = y+1
Point after → •

- In a basic block:
  - Program point after an instruction = program point before the successor instruction

Program Points: Example

- Multiple successor blocks means that point at the end of a block has multiple successor program points
- Depending on the execution, control flows from a program point to one of its successors
- Also multiple predecessors
- How does information propagate between program points?

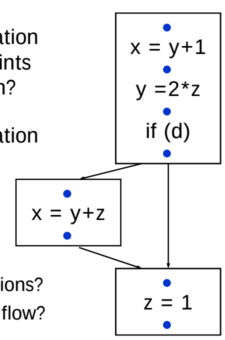


Flow of Extracted Information

- Question 1: how does information flow between the program points before and after an instruction?
- Question 2: how does information flow between successor and predecessor basic blocks?
- ... in other words:

  Q1: what is the effect of instructions?

  Q2: what is the effect of control flow?



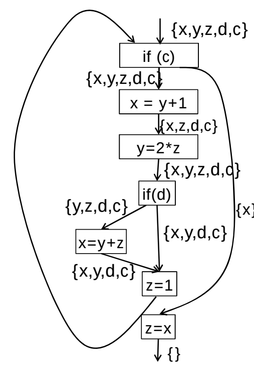
Using CFGs

- To extract information: reason about how it propagates between program points
- Rest of this lecture: how to use CFGs to compute information at each program point for:
  - Live variable analysis, which computes which variables are live at each program point
  - Copy propagation analysis, which computes the variable copies available at each program point

9

Live variables

- A statement is a *definition* of a variable v if it may write to v.
- A statement is a *use* of variable v if it may read from v.
- A variable v is live at a point p in a CFG if
  - there is a path from p to a use of v, and
  - that path does not contain a definition of v



10

Computing Use/Def

- Compute use[I] and def[I] for each instruction I:

if I is x = y OP z : use[I] = {y, z} def[I] = {x}
if I is x = OP y : use[I] = {y} def[I] = {x}
if I is x = y : use[I] = {y} def[I] = {x}
if I is x = addr y : use[I] = {} def[I] = {x}
if I is if (x) : use[I] = {x} def[I] = {}
if I is return x : use[I] = {x} def[I] = {}
if I is x = f(y_1 ,..., y_n) : use[I] = {y_1, ..., y_n}
def[I] = {x}

(For now, ignore load and store instructions)

11

Part 1: Analyze Instructions

- Question: what is the relation between sets of reaching definitions before and after an instruction?

in[I]
I
out[I]

- Examples:

conclude	in[I] = {y,z}	in[I] = {y,z,t}	in[I] = {x,t}
	x = y+z;	x = y+z;	x = x+1;
assume	out[I] = {z}	out[I] = {x,t}	out[I] = {x,t}

- ... is there a general rule?

12

Live Variable Analysis

- Computes live variables at each program point
  - I.e., variables holding values that may be used later (in some execution of the program)
- For an instruction I, consider:
  - in[I] = live variables at program point before I
  - out[I] = live variables at program point after I
- For a basic block B, consider:
  - in[B] = live variables at beginning of B
  - out[B] = live variables at end of B
- If I = first instruction in B, then in[B] = in[I]
- If I' = last instruction in B, then out[B] = out[I']

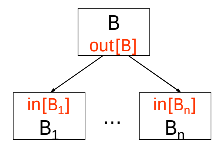
13

How to Compute Liveness?

- Answer question 1: for each instruction I, what is the relation between in[I] and out[I] ?

in[I]
I
out[I]

- Answer question 2: for each basic block B with successor blocks B_1, ..., B_n, what is the relation between out[B] and in[B_1], ..., in[B_n]?



14

Part 1: Analyze Instructions

- Question: what is the relation between sets of live variables before and after an instruction?

in[I]
I
out[I]

- Examples:

conclude	in[I] = {y,z}	in[I] = {y,z,t}	in[I] = {x,t}
	x = y+z;	x = y+z;	x = x+1;
assume	out[I] = {z}	out[I] = {x,t}	out[I] = {x,t}

- ... is there a general rule?

15

Analyze Instructions

- Yes: knowing variables live after I, can compute variables live before I:
  - Each variable live after I is also live before I, unless I defines (writes) it
  - Each variable that I uses (reads) is also live before instruction I

in[I]
I
out[I]

- Mathematically:

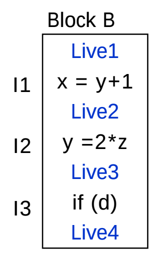
$$in[I] = (out[I] - def[I]) \cup use[I]$$

where:
  - def[I] = variables defined (written) by instruction I
  - use[I] = variables used (read) by instruction I

16

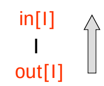
Example

- Example: block B with three instructions I1, I2, I3:
  - Live1 = in[B]   = in[I1]
  - Live2 = out[I1] = in[I2]
  - Live3 = out[I2] = in[I3]
  - Live4 = out[I3] = out[B]
- Relation between Live sets:
  - Live1 = (Live2-{x}) ∪ {y}
  - Live2 = (Live3-{y}) ∪ {z}
  - Live3 = (Live4-{}) ∪ {d}

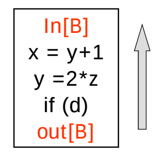


Backward Flow

- Relation:
  in[I] = (out[I] – def[I]) ∪ use[I]

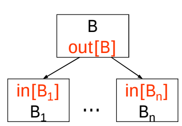


- The information flows backward!
- Instructions: can compute in[I] if we know out[I]
- Basic blocks: information about live variables flows from out[B] to in[B]

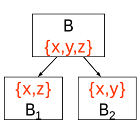


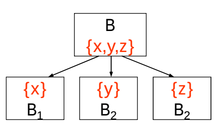
Part 2: Analyze Control Flow

- Question: for each basic block B with successor blocks B_1, …, B_n, what is the relation between out[B] and in[B_1], …, in[B_n]?



- Examples:



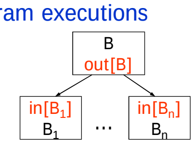


- What is the general rule?

Analyze Control Flow

- Rule: A variables is live at end of block B if it is live at the beginning of one (or more) successor blocks
- Characterizes all possible program executions
- Mathematically:

$$out[B] = \bigcup_{B' \in succ(B)} in[B']$$



- Again, information flows backward: from successors B’ of B to basic block B

Constraint System

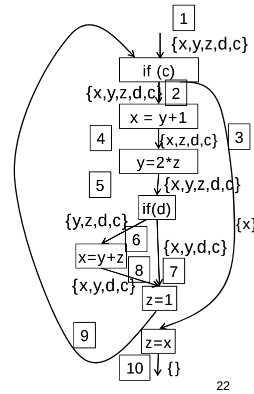
- Put parts together: start with CFG and derive a system of constraints between live variable sets:

$$\begin{cases} in[I] = (out[I] - def[I]) \cup use[I] & \text{for each instruction I} \\ out[B] = \bigcup_{B' \in succ(B)} in[B'] & \text{for each basic block B} \end{cases}$$

- Solve constraints:
  - Start with empty sets of live variables
  - Iteratively apply constraints
  - Stop when we reach a fixed point

Live variables

L10 = { }
L3 = {x} U (L10- {z})
L9 = L2 U L3 U {c}
L8 = L9 – {z}
L7 = L9 – {z}
L6 = {y,z} U (L8 – {x})
L5 = L6 U L7 U {d}
L4 = {z} U (L5 – {y})
L2 = {y} U (L4 – {x})
L1 = L2 U L3 U {c}



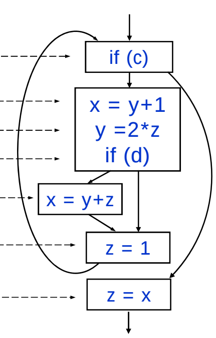
Constraint Solving Algorithm

for all instructions I **do** in[I] = out[I] = ∅;
repeat
  select an instuction I (or a basic block B) such that
    in[I] ≠ (out[I] – def[I]) ∪ use[I]
  or (respectively)
    $out[B] \neq \bigcup_{B' \in succ(B)} in[B']$

  and update in[I] (or out[B]) accordingly
until no such change is possible

Example

def = {}, use = {c}
def = {x}, use = {y}
def = {y}, use = {z}
def = {}, use = {d}
def = {x}, use = {y,z}
def = {z}, use = {}
def = {z}, use = {x}



Example

def = {}, use = {c}

def = {x}, use = {y}

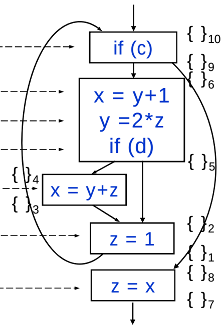
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

def = {z}, use = {x}



25

Example

def = {}, use = {c}

def = {x}, use = {y}

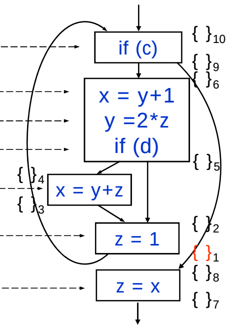
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

def = {z}, use = {x}



26

Example

def = {}, use = {c}

def = {x}, use = {y}

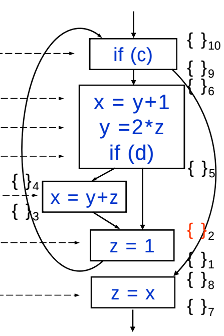
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

def = {z}, use = {x}



27

Example

def = {}, use = {c}

def = {x}, use = {y}

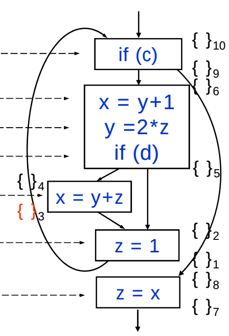
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

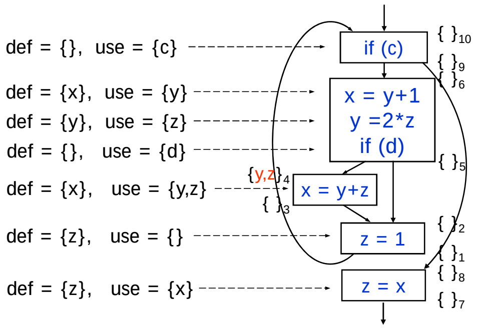
def = {z}, use = {}

def = {z}, use = {x}



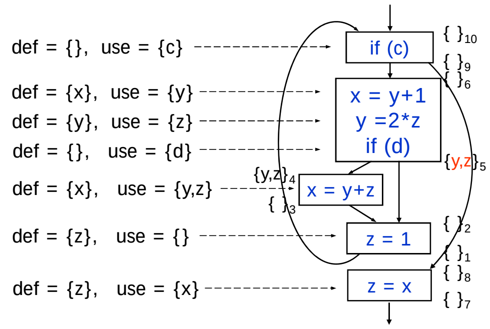
28

Example



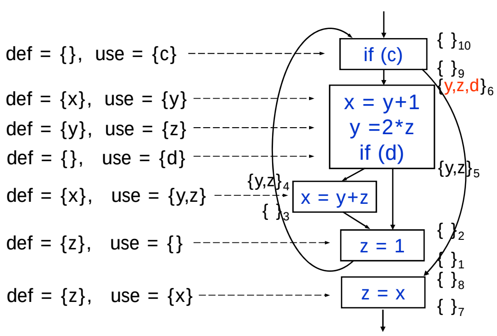
29

Example



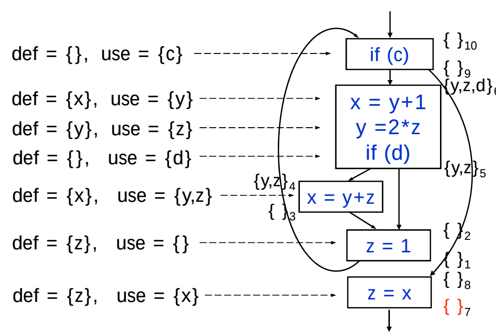
30

Example



31

Example



32

Example

def = {}, use = {c}

def = {x}, use = {y}

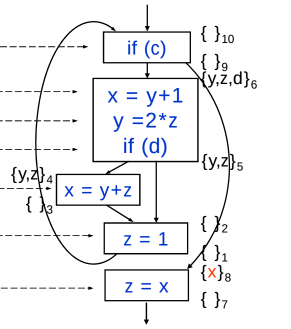
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

def = {z}, use = {x}



33

Example

def = {}, use = {c}

def = {x}, use = {y}

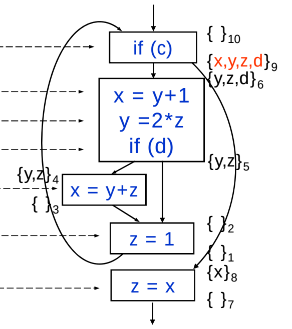
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

def = {z}, use = {x}



34

Example

def = {}, use = {c}

def = {x}, use = {y}

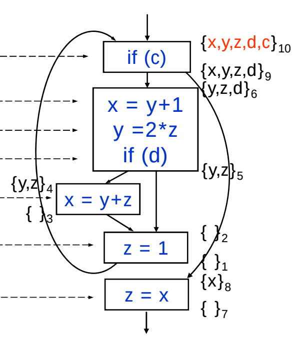
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

def = {z}, use = {x}



35

Example

def = {}, use = {c}

def = {x}, use = {y}

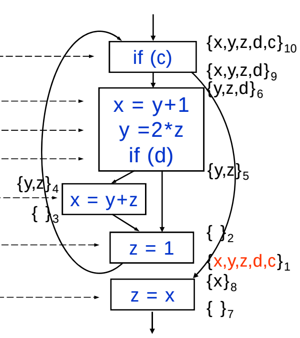
def = {y}, use = {z}

def = {}, use = {d}

def = {x}, use = {y,z}

def = {z}, use = {}

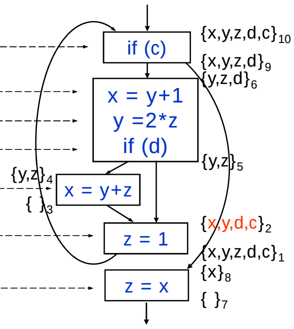
def = {z}, use = {x}



36

Example

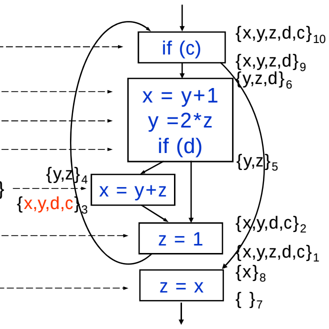
def = {}, use = {c}
def = {x}, use = {y}
def = {y}, use = {z}
def = {}, use = {d}
def = {x}, use = {y,z}
def = {z}, use = {}
def = {z}, use = {x}



37

Example

def = {}, use = {c}
def = {x}, use = {y}
def = {y}, use = {z}
def = {}, use = {d}
def = {x}, use = {y,z}
def = {z}, use = {}
def = {z}, use = {x}



38

Example

def = {}, use = {c}
def = {x}, use = {y}
def = {y}, use = {z}
def = {}, use = {d}
def = {x}, use = {y,z}
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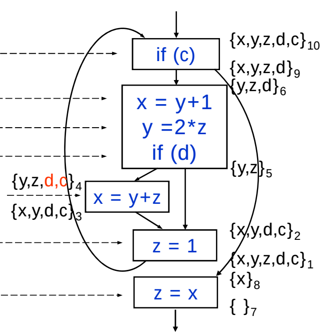
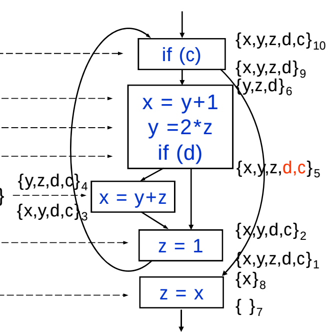
def = {z}, use = {x}



39

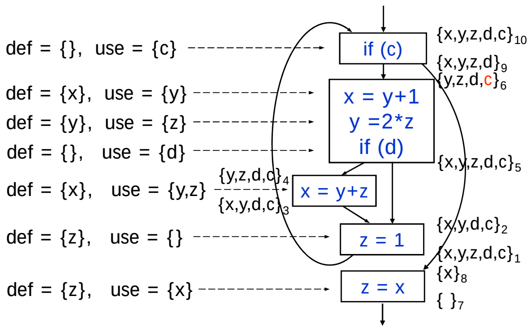
Example

def = {}, use = {c}
def = {x}, use = {y}
def = {y}, use = {z}
def = {}, use = {d}
def = {x}, use = {y,z}
def = {z}, use = {}
def = {z}, use = {x}



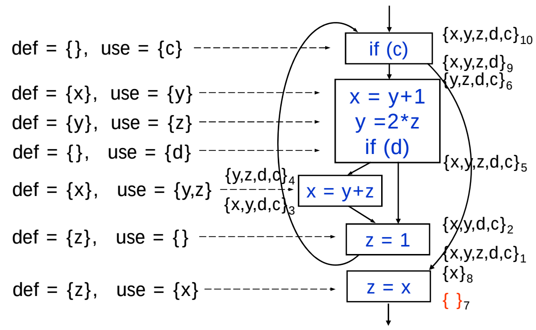
40

Example



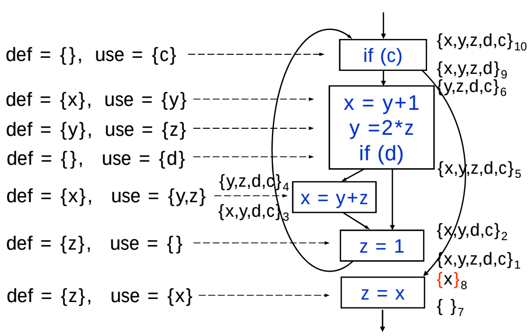
41

Example



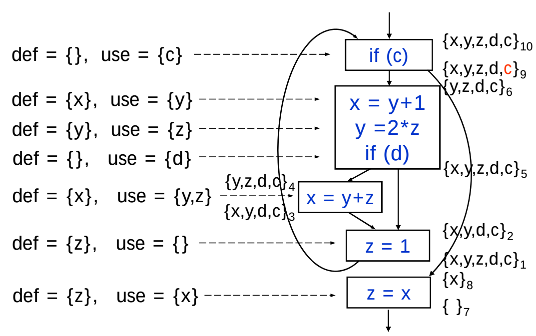
42

Example



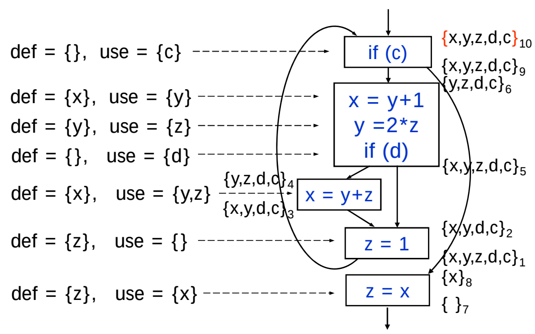
43

Example



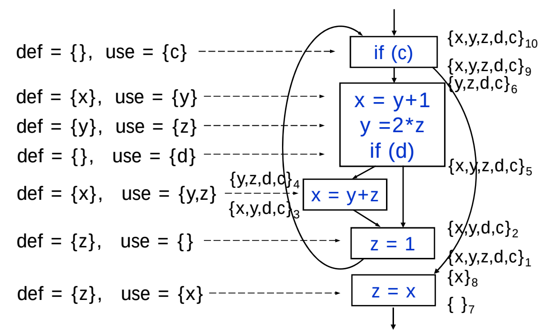
44

Example



45

Fixed Point Reached



46

General questions

- Do systems of equations of this sort always have solutions?
- If so, do they have unique solutions?
- If there are multiple solutions, which one is the "right" one?
- How do we solve such systems of equations in general?
- If we use the iterative method, does it always terminate and if so, does it always produce a unique answer?

47

Copy Propagation

- Goal: determine copies available at each program point
- Information: set of copies <x=y> at each point
- For each instruction I:
  - in[I] = copies available at program point before I
  - out[I] = copies available at program point after I
- For each basic block B:
  - in[B] = copies available at beginning of B
  - out[B] = copies available at end of B
- If I = first instruction in B, then in[B] = in[I]
- If I' = last instruction in B, then out[B] = out[I']

48

Same Methodology

1. Express flow of information (i.e., available copies):
   - For points before and after each instruction (in[I], out[I])
   - For points at exit and entry of basic blocks (in[B], out[B])
2. Build constraint system using the relations between available copies
3. Solve constraints to determine available copies at each point in the program

49

Analyze Instructions

- Knowing in[I], can compute out[I]:
  - Remove from in[I] all copies <u=v> if variable u or v is written by I
  - Keep all other copies from in[I]
  - If I is of the form x=y, add it to out[I]

in[I]
I
out[I]

- Mathematically:

$$out[I] = (in[I] – kill[I]) \cup gen[I]$$

where:
- kill[I] = copies "killed" by instruction I
- gen[I] = copies "generated" by instruction I

50

Computing Kill/Gen

- Compute kill[I] and gen[I] for each instruction I:

if I is x = y OP z :	gen[I] = {}	kill[I] = {u=v|u or v is x}
if I is x = OP y :	gen[I] = {}	kill[I] = {u=v|u or v is x}
if I is x = y :	gen[I] = {x=y}	kill[I] = {u=v|u or v is x}
if I is x = addr y :	gen[I] = {}	kill[I] = {u=v|u or v is x}
if I is if (x) :	gen[I] = {}	kill[I] = {}
if I is return x :	gen[I] = {}	kill[I] = {}
if I is x = f(y_1 ,..., y_n) :	gen[I] = {}	kill[I] = {u=v| u or v is x}

(again, ignore load and store instructions)

51

Forward Flow

- Relation:

  out[I] = (in[I] – kill[I]) ∪ gen[I]

in[I]
I
out[I]

- The information flows forward!
- Instructions: can compute out[I] if we know in[I]
- Basic blocks: information about available copies flows from in[B] to out[B]

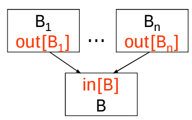
In[B]
x = y
y =2*z
if (d)
out[B]

52

Analyze Control Flow

- Rule: A copy is available at beginning of block B if it is available at the end of all predecessor blocks
- Characterizes all possible program executions
- Mathematically:

$$in[B] = \bigcap_{B' \in pred(B)} out[B']$$



- Information flows forward: from predecessors B' of B to basic block B

Constraint System

- Build constraints: start with CFG and derive a system of constraints between sets of available copies:

$$\begin{cases} out[I] = (in[I] - kill[I]) \cup gen[I] & \text{for each instruction I} \\ in[B] = \bigcap_{B' \in pred(B)} out[B'] & \text{for each basic block B} \end{cases}$$

- Solve constraints:
  - Start with empty set of available copies at start and universal set of available copies everywhere else
  - Iteratively apply constraints
  - Stop when we reach a fixed point

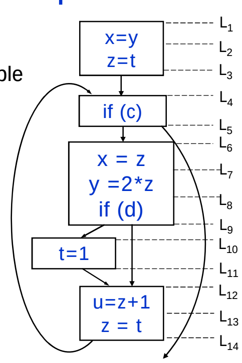
Example

- What are the available copies at the end of the program?

x=y?

z=t?

x=z?



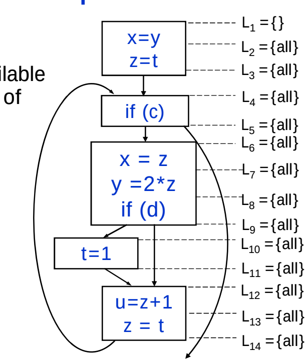
Example

- What are the available copies at the end of the program?

x=y?

z=t?

x=z?



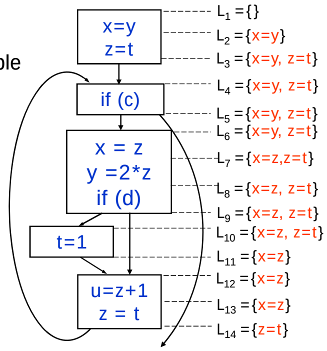
Iteration 1

- What are the available copies at the end of the program?

x=y?

z=t?

x=z?



57

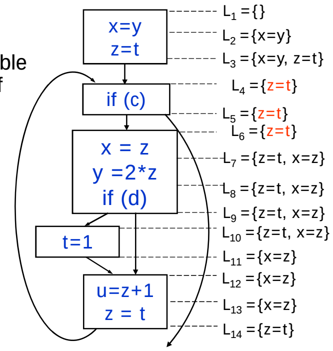
Iteration 2

- What are the available copies at the end of the program?

x=y?

z=t?

x=z?



58

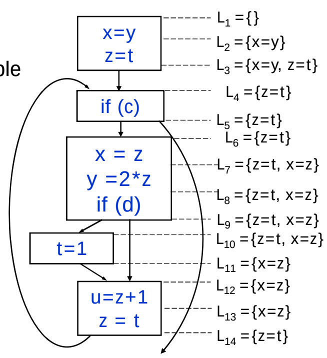
Fixed Point Reached!

- What are the available copies at the end of the program?

x=y? NO

z=t? YES

x=z? NO



59

Summary

- Extracting information about live variables and available copies is similar
  - Define the required information
  - Define information before/after instructions
  - Define information at entry/exit of blocks
  - Build constraints for instructions/control flow
  - Solve constraints to get needed information
- ...is there a general framework?
  - Yes: dataflow analysis!

60