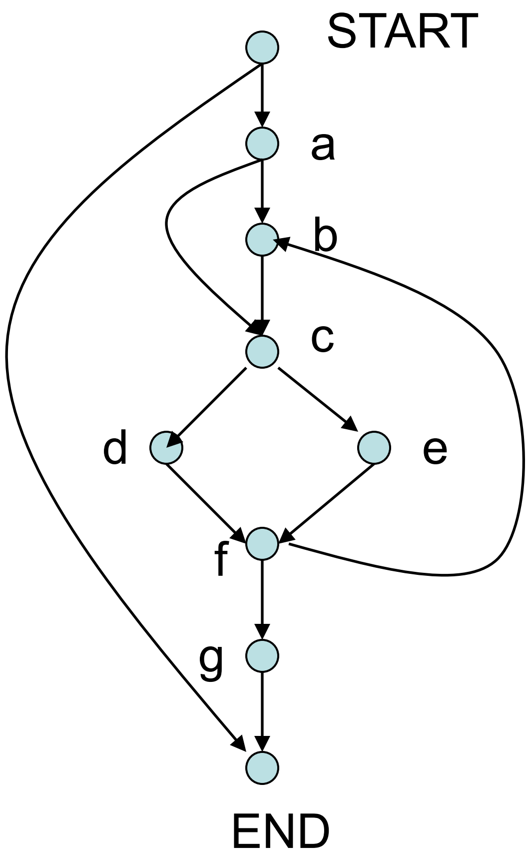
Dominators, control-dependence and SSA form

Organization

- Dominator relation of CFGs
  - postdominator relation
- Dominator tree
- Computing dominator relation and tree
  - Dataflow algorithm
  - Lengauer and Tarjan algorithm
- Control-dependence relation
- SSA form

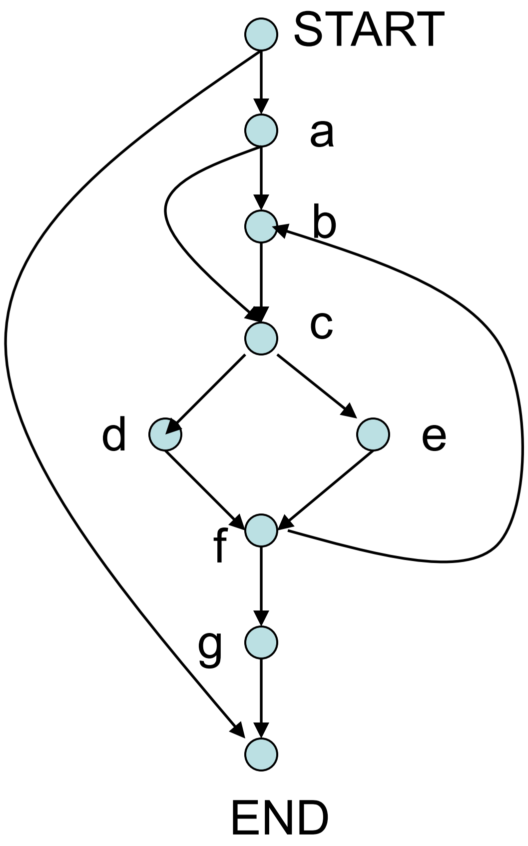
Control-flow graphs

- CFG is a directed graph
- Unique node START from which all nodes in CFG are reachable
- Unique node END reachable from all nodes
- Dummy edge to simplify discussion START → END
- Path in CFG: sequence of nodes, possibly empty, such that successive nodes in sequence are connected in CFG by edge
  - If x is first node in sequence and y is last node, we will write the path as x →* y
  - If path is non-empty (has at least one edge) we will write x →+ y

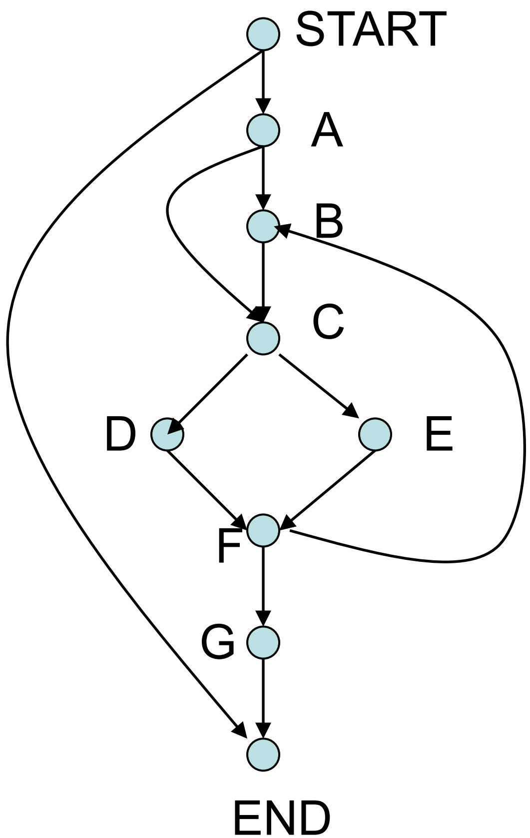


Dominators

- In a CFG G, node a is said to dominate node b if every path from START to b contains a.
- Dominance relation: relation on nodes
  - We will write a dom b if a dominates b



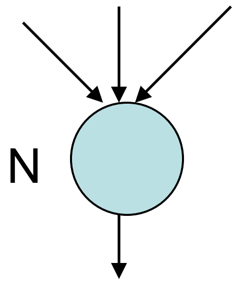
Example



	START	A	B	C	D	E	F	G	END
START	x	x	x	x	x	x	x	x	x
A		x	x	x	x	x	x	x	
B			x						
C				x	x	x	x	x	
D					x				
E						x			
F							x	x	
G								x	
END									x

Computing dominance relation

- Dataflow problem:



Domain: powerset of nodes in CFG

$$Dom(N) = \{N\} \cup \bigcap_{M \in pred(N)} Dom(M)$$

Find greatest solution.

Work through example on previous slide to check this.
Question: what do you get if you compute least solution?

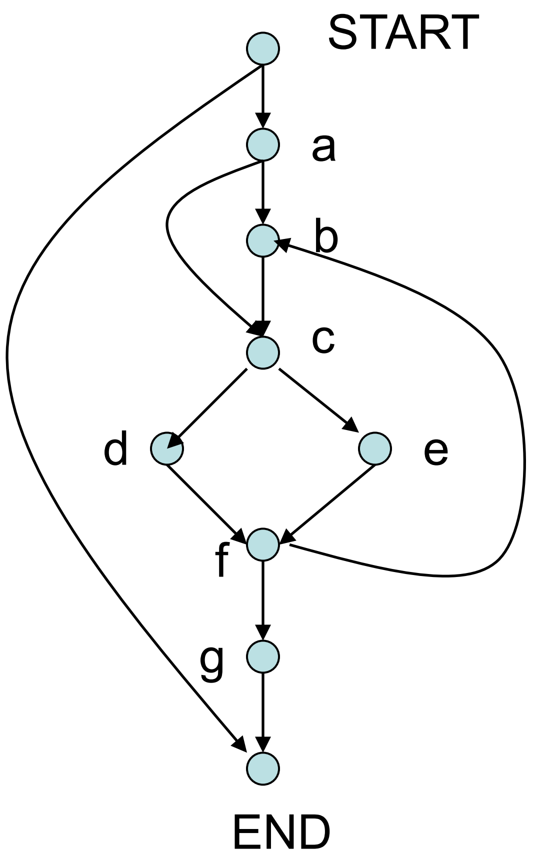
Properties of dominance

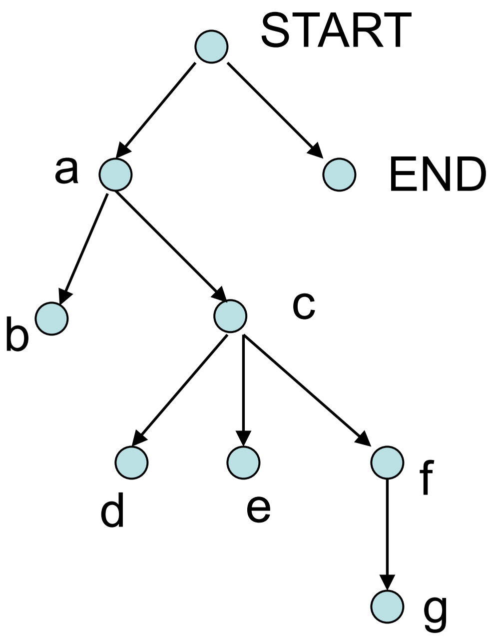
- Dominance is
  - reflexive: a dom a
  - anti-symmetric: a dom b and b dom a ➔ a = b
  - transitive: a dom b and b dom c ➔ a dom c
  - tree-structured:
    - a dom c and b dom c ➔ a dom b or b dom a
    - intuitively, this means dominators of a node are themselves ordered by dominance

Example of proof

- Let us prove that dominance is transitive.
  - Given: a dom b and b dom c
  - Consider any path P: START $\rightarrow^+$ c
  - Since b dom c, P must contain b.
  - Consider prefix of P = Q: START $\rightarrow^+$ b
  - Q must contain a because a dom b.
  - Therefore P contains a.

Dominator tree example





Check: verify that from dominator tree, you can generate full relation

Computing dominator tree

- Inefficient way:
  - Solve dataflow equations to compute full dominance relation
  - Build tree top-down
    - Root is START
    - For every other node
      - Remove START from its dominator set
      - If node is then dominated only by itself, add node as child of START in dominator tree
    - Keep repeating this process in the obvious way

Building dominator tree directly

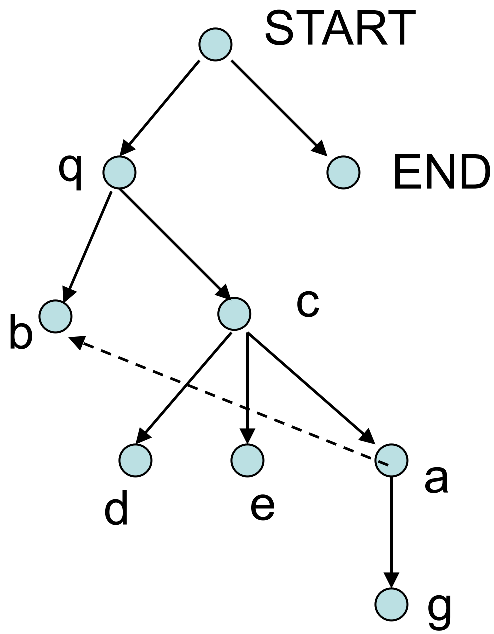
- Algorithm of Lengauer and Tarjan
  - Based on depth-first search of graph
  - $O(E^*\alpha(E))$ where E is number of edges in CFG
  - Essentially linear time
- Linear time algorithm due to Buchsbaum et al
  - Much more complex and probably not efficient to implement except for very large graphs

Immediate dominators

- Parent of node b in tree, if it exists, is called the immediate dominator of b
  - written as idom(b)
  - idom not defined for START
- Intuitively, all dominators of b other than b itself dominate idom(b)
  - In our example, idom(c) = a

Useful lemma

- Lemma: Given CFG G and edge a→b, idom(b) dominates a
- Proof: Otherwise, there is a path P: START →+ a that does not contain idom(b). Concatenating edge a→b to path P, we get a path from START to b that does not contain idom(b) which is a contradiction.



a→b is edge in CFG
idom(b) = q which dominates f

Postdominators

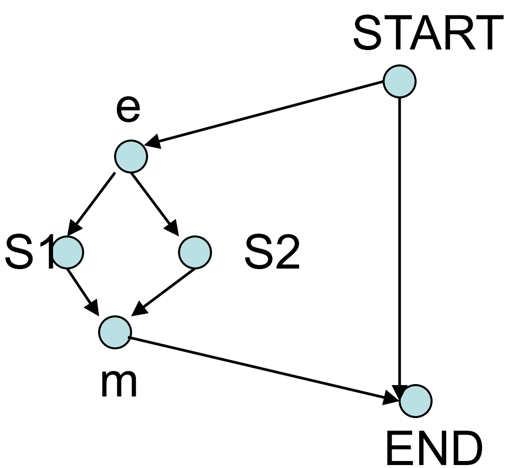
- Given a CFG G, node b is said to postdominate node a if every path from a to END contains b.
  – we write b pdom a to say that b postdominates a
- Postdominance is dominance in reverse CFG obtained by reversing direction of all edges and interchanging roles of START and END.
- Caveat: a dom b does not necessarily imply b pdom a.
  – See example: a dom b but b does not pdom a

Obvious properties

- Postdominance is a tree-structured relation
- Postdominator relation can be built using a backward dataflow analysis.
- Postdominator tree can be built using Lengauer and Tarjan algorithm on reverse CFG
- Immediate postdominator: ipdom
- Lemma: if a → b is an edge in CFG G, then ipdom(a) postdominates b.

Control dependence

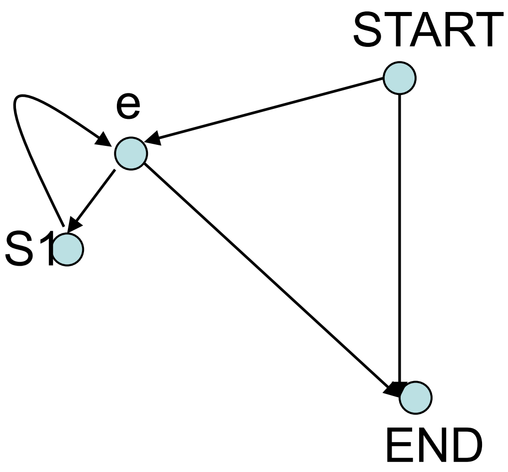
- Intuitive idea:
  - node w is control-dependent on a node u if node u determines whether w is executed
- Example:



START
…..
if e then S1 else S2
….
END

We would say S1 and S2 are control-dependent on e

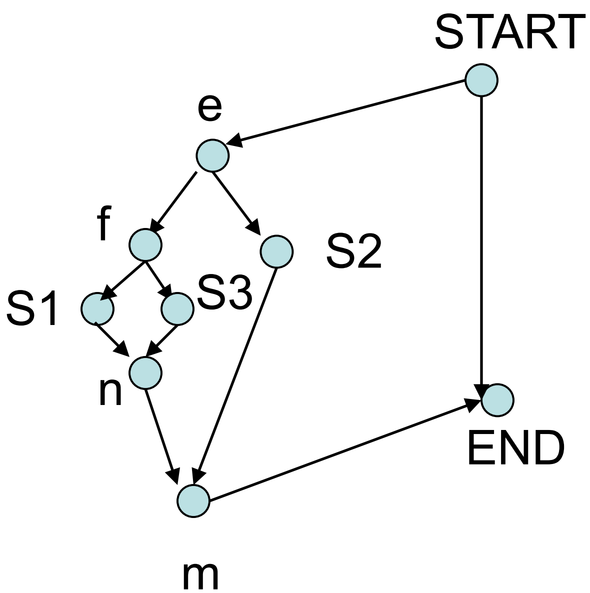
Examples (contd.)



START
…..
while e do S1;
….
END

We would say node S1 is control-dependent on e.
It is also intuitive to say node e is control-dependent on itself:
- execution of node e determines whether or not e is executed again.

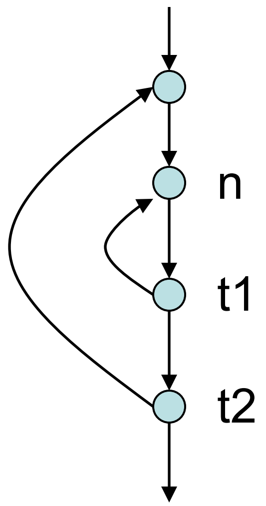
Example (contd.)



- S1 and S3 are control-dependent on f
- Are they control-dependent on e?
- Decision at e does not fully determine if S1 (or S3 is executed) since there is a later test that determines this
- So we will NOT say that S1 and S3 are control-dependent on e
  - Intuition: control-dependence is about “last” decision point
- However, f is control-dependent on e, and S1 and S3 are transitively (iteratively) control-dependent on e

Example (contd.)

- Can a node be control-dependent on more than one node?
  - yes, see example
  - nested repeat-until loops
    - n is control-dependent on t1 and t2 (why?)
- In general, control-dependence relation can be quadratic in size of program



Formal definition of control dependence

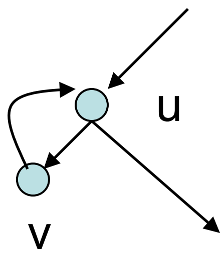
- Formalizing these intuitions is quite tricky
- Starting around 1980, lots of proposed definitions
- Commonly accepted definition due to Ferrane, Ottenstein, Warren (1987)
- Uses idea of postdominance
- We will use a slightly modified definition due to Bilardi and Pingali which is easier to think about and work with

Control dependence definition

- First cut: given a CFG G, a node w is control-dependent on an edge (u→v) if
  - w postdominates v
  - ……. w does not postdominate u
- Intuitively,
  - first condition: if control flows from u to v it is guaranteed that w will be executed
  - second condition: but from u we can reach END without encountering w
  - so there is a decision being made at u that determines whether w is executed

Control dependence definition

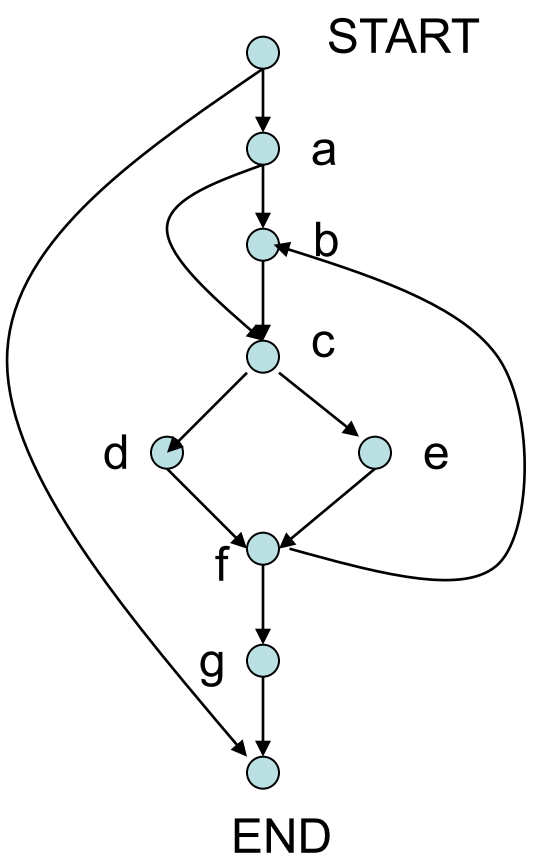
- Small caveat: what if w = u in previous definition?
  - See picture: is u control-dependent on edge u→v?
  - Intuition says yes, but definition on previous slides says "u should not postdominate u" and our definition of postdominance is reflexive
- Fix: given a CFG G, a node w is control-dependent on an edge (u→v) if
  - w postdominates v
  - if w is not u, w does not postdominate u



Strict postdominance

- A node w is said to strictly postdominate a node u if
  - w != u
  - w postdominates u
- That is, strict postdominance is the irreflexive version of the postdominance relation
- Control dependence: given a CFG G, a node w is control-dependent on an edge (u→v) if
  - w postdominates v
  - w does not strictly postdominate u

Example

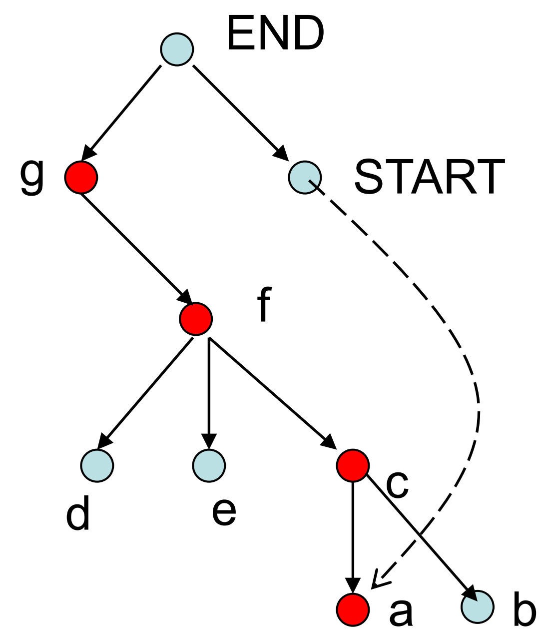


	a	b	c	d	e	f	g
START→a	x		x			x	x
f→b		x	x			x	
c→d				x			
c→e					x		
a→b		x					

Computing control-dependence relation

- Control dependence: given a CFG G, a node w is control-dependent on an edge (u→v) if
  - w postdominates v
  - w does not strictly postdominate u

- Nodes control dependent on edge (u→v) are nodes on path up the postdominator tree from v to ipdom(u), excluding ipdom(u)
  - We will write this as [v,ipdom(u))
    - half-open interval in tree



	a	b	c	d	e	f	g
START→a	x		x			x	x
f→b		x	x			x	
c→d				x			
c→e					x		
a→b		x					

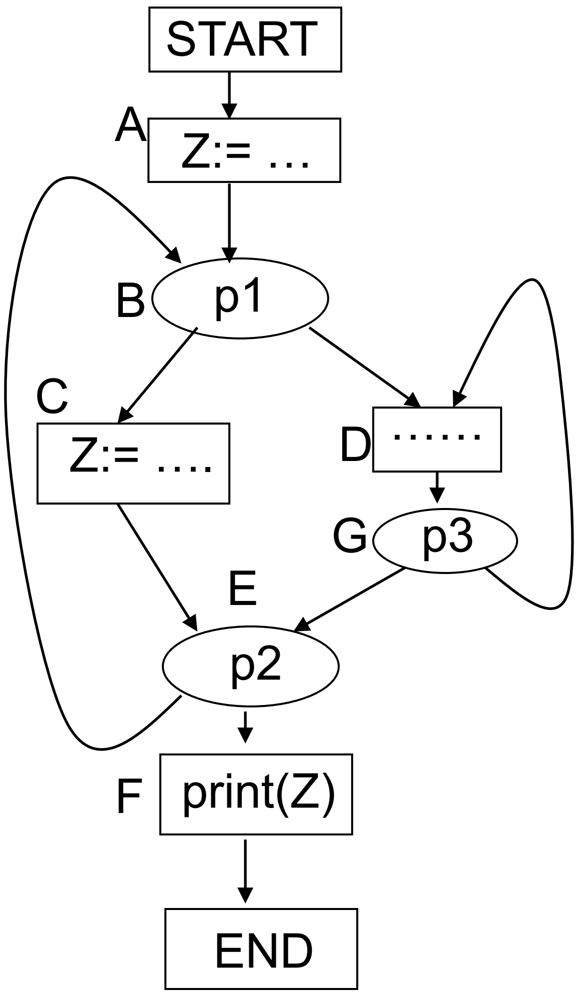
Computing control-dependence relation

- Compute the postdominator tree
- Overlay each edge u→v on pdom tree and determine nodes in interval [v,ipdom(u))
- Time and space complexity is O(EV).
- Faster solution: in practice, we do not want the full relation, we only make queries
  - cd(e): what are the nodes control-dependent on an edge e?
  - conds(w): what are the edges that w is control-dependent on?
  - cdequiv(w): what nodes have the same control-dependences as node w?
- It is possible to implement a simple data structure that takes O(E) time and space to build, and that answers these queries in time proportional to output of query (optimal) (Pingali and Bilardi 1997).

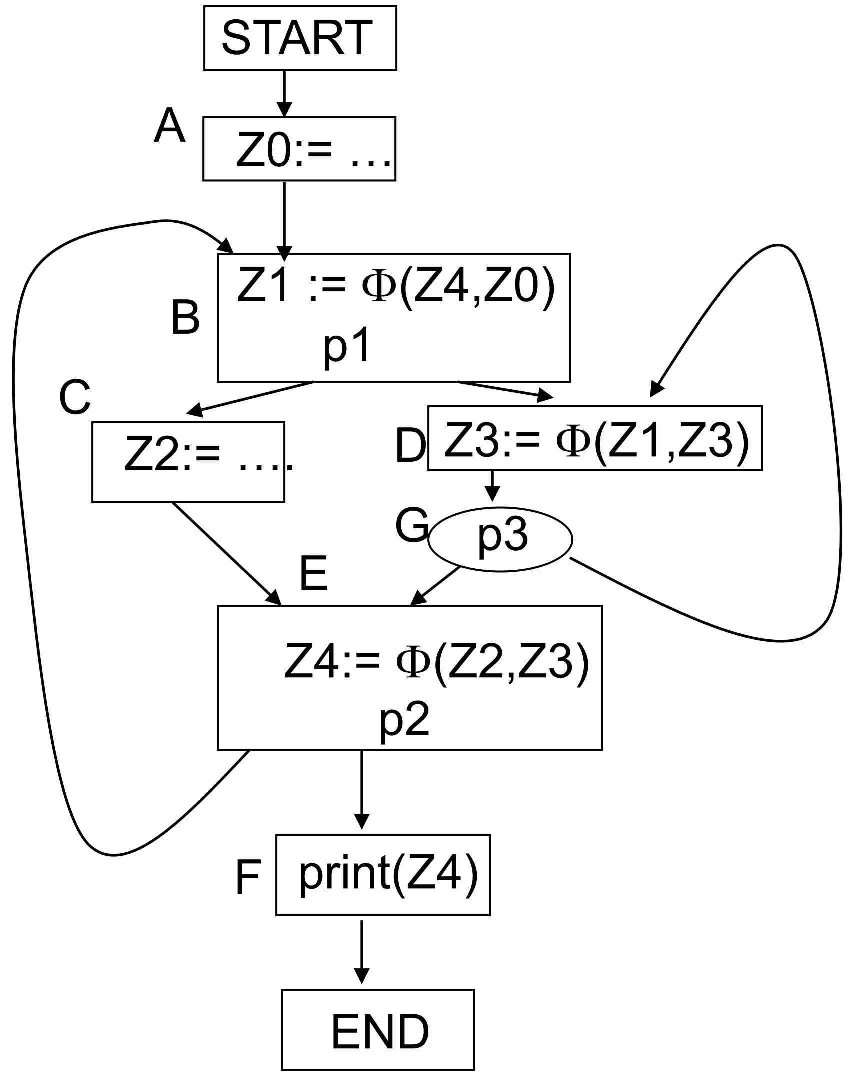
SSA form

- Static single assignment form
  - Intermediate representation of program in which every use of a variable is reached by exactly one definition
  - Most programs do not satisfy this condition
    - (eg) see program on next slide: use of Z in node F is reached by definitions in nodes A and C
  - Requires inserting dummy assignments called Φ-functions at merge points in the CFG to “merge” multiple definitions
  - Simple algorithm: insert Φ-functions for all variables at all merge points in the CFG and rename each real and dummy assignment of a variable uniquely
    - (eg) see transformed example on next slide

SSA example



START
A
Z:= …
B
p1
C
Z:= ….
D
……
G
p3
E
p2
F
print(Z)
END

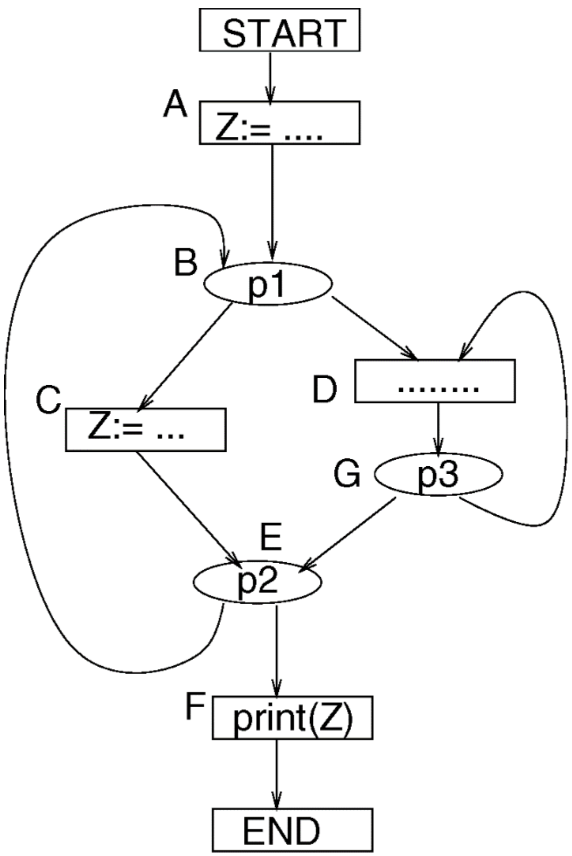


START
A
Z0:= …
B
Z1 := Φ(Z4,Z0)
p1
C
Z2:= ….
D
Z3:= Φ(Z1,Z3)
G
p3
E
Z4:= Φ(Z2,Z3)
p2
F
print(Z4)
END

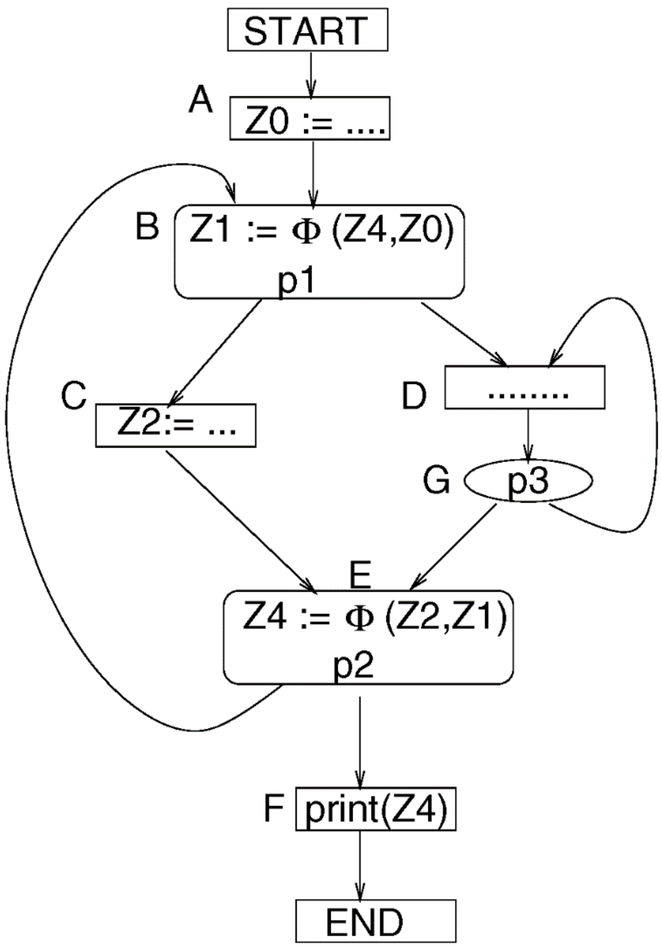
Minimal SSA form

- In previous example, dummy assignment Z3 is not really needed since there is no actual assignment to Z in nodes D and G of the original program.
- Minimal SSA form
  - SSA form of program that does not contain such “unnecessary” dummy assignments
  - See example on next slide
- Question: how do we construct minimal SSA form directly?

Minimal-SSA form Example



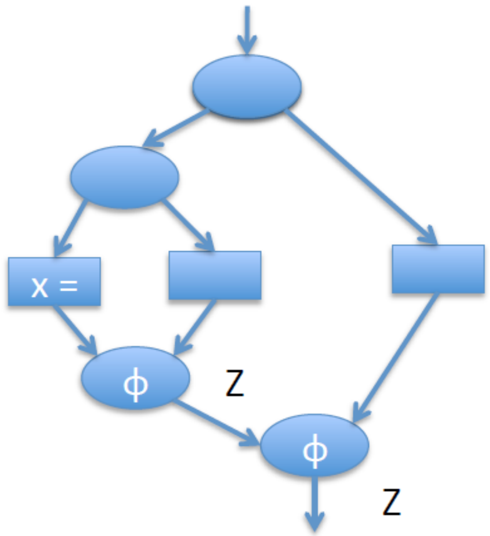
(a) Original Control Flow Graph



(b) Control Flow Graph with Φ-functions

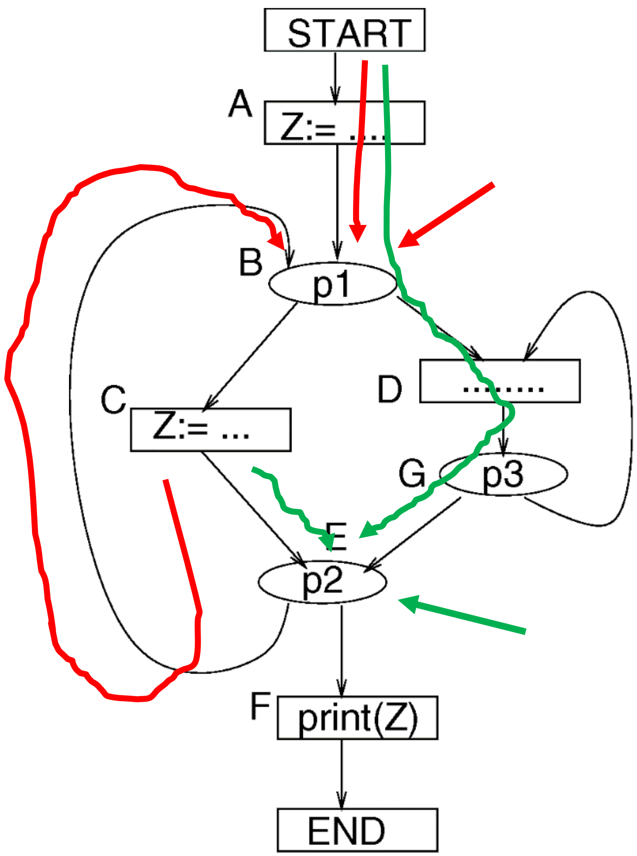
Minimal SSA form

- Compute Merge relation M: V → P(V)
- If node N contains an assignment to a variable x, then node Z is in M(N) if:
  1. There is a non-null path P1 := N $\rightarrow^+$ Z
     - The value computed at X reaches Z
  2. There is a non-null path P2 := START $\rightarrow^+$ Z
  3. P1 and P2 are disjoint except for Z

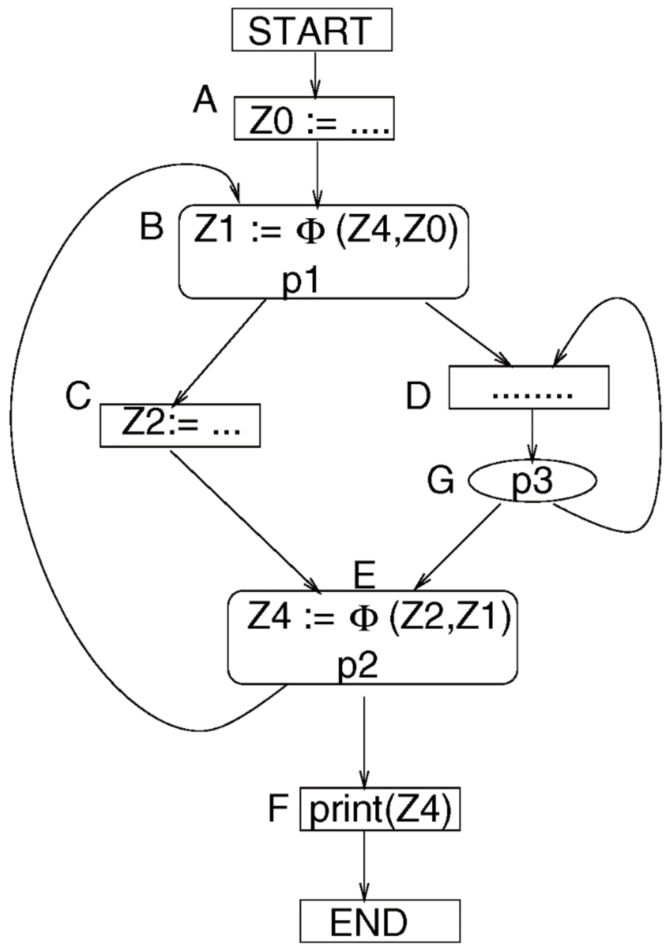


- If S ⊆ V where there are assignments to variable x, then place ϕ functions for x in nodes $\bigcup_{N \in S} M(N)$

Minimal-SSA form Example



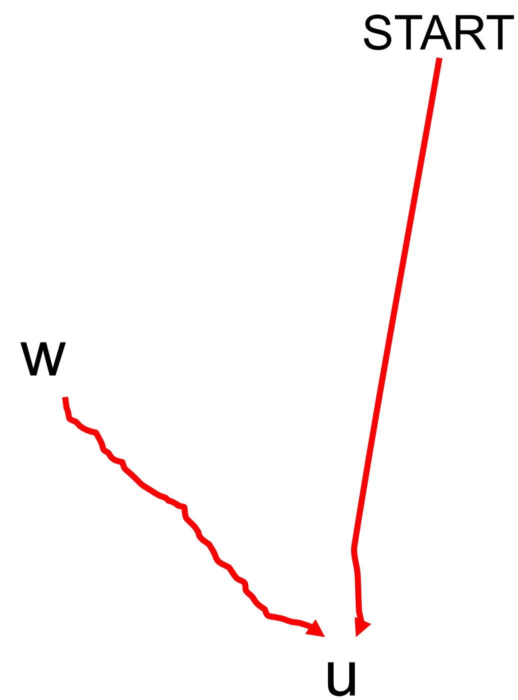
(a) Original Control Flow Graph



(b) Control Flow Graph with Φ-functions

Computing Merge(v)

- If u ε Merge(w), w does not strictly dominate u
  - Proof: there is a path from START to v that does not contain w
- Conversely
  - if w dominates u, u $\not\varepsilon$ Merge(w)
- Idea:
  - compute nodes on the dominance frontier of w
    - w does not strictly dominate u but dominates some CFG predecessor of u
  - iterate



Dominance frontier

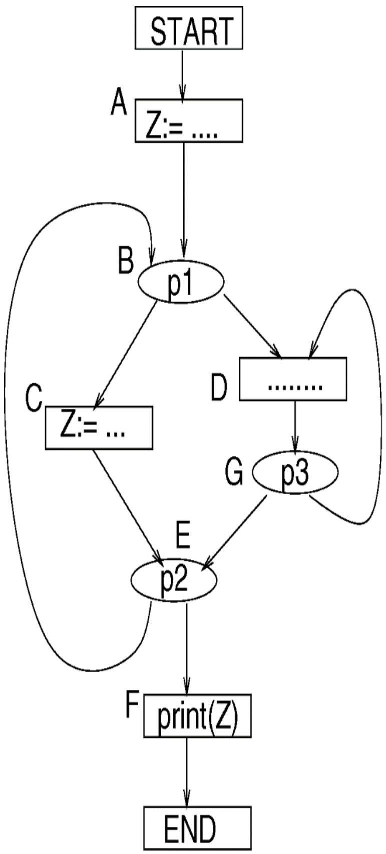
- Dominance frontier of node w
  - Node u is in dominance frontier of node w if w
    - dominates a CFG predecessor v of u, but
    - does not strictly dominate u
- Dominance frontier = control dependence in reverse graph

Running example:

	A	B	C	D	E	F	G
A							
B		x					
C					x		
D				x			
E		x					
F							
G				x	x		

Iterated dominance frontier

- Irreflexive closure of dominance frontier relation
- Related notion: iterated control dependence in reverse graph
- Where to place Φ-functions for a variable Z
  - Let Assignments = {START} U {nodes with assignments to Z in original CFG}
  - Find set I = iterated dominance frontier of nodes in Assignments
  - Place Φ-functions in nodes of set I
- For example
  - Assignments = {START,A,C}
  - DF(Assignments) = {E}
  - DF(DF(Assignments)) = {B}
  - DF(DF(DF(Assignments))) = {B}
  - So I = {E,B}
  - This is where we place Φ-functions, which is correct



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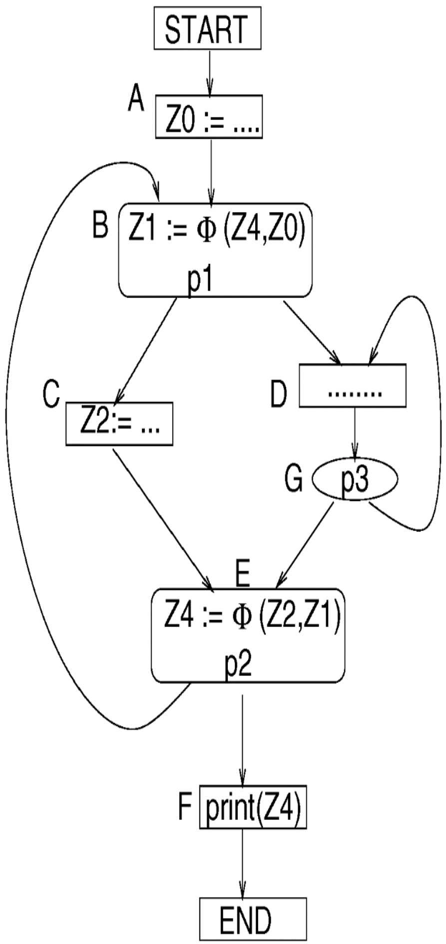


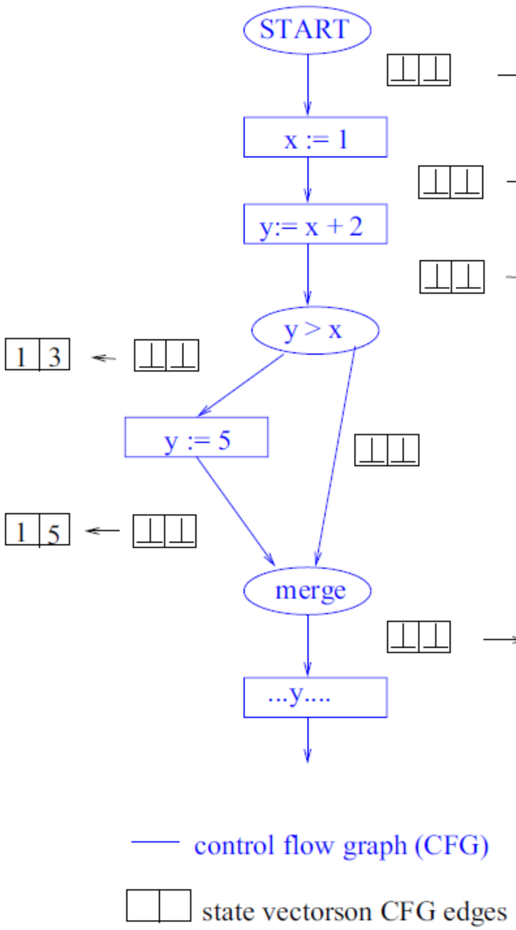
(b) Control Flow Graph with Φ-functi

Why is SSA form useful?

- For many dataflow problems, SSA form enables "sparse" dataflow analysis that
  - yields the same precision as bit-vector CFG-based dataflow analysis
  - but is asymptotically faster since it permits the exploitation of sparsity
  - Example: constant propagation (see following slides)
- SSA has two distinct features
  - factored def-use chains
  - renaming
  - you do not have to perform renaming to get advantage of SSA for many dataflow problems

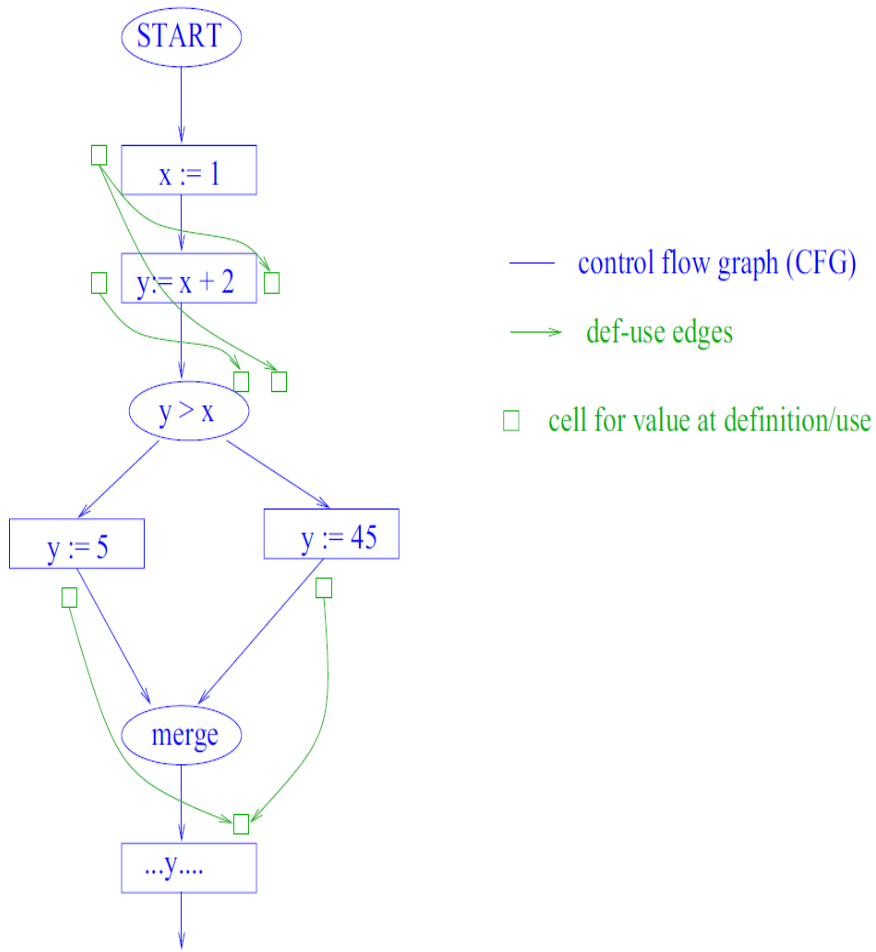
Constant propagation

- Dataflow algorithm described earlier will determine that the last use of y is constant
- Intuition: it discovers that the false side of the conditional is dead



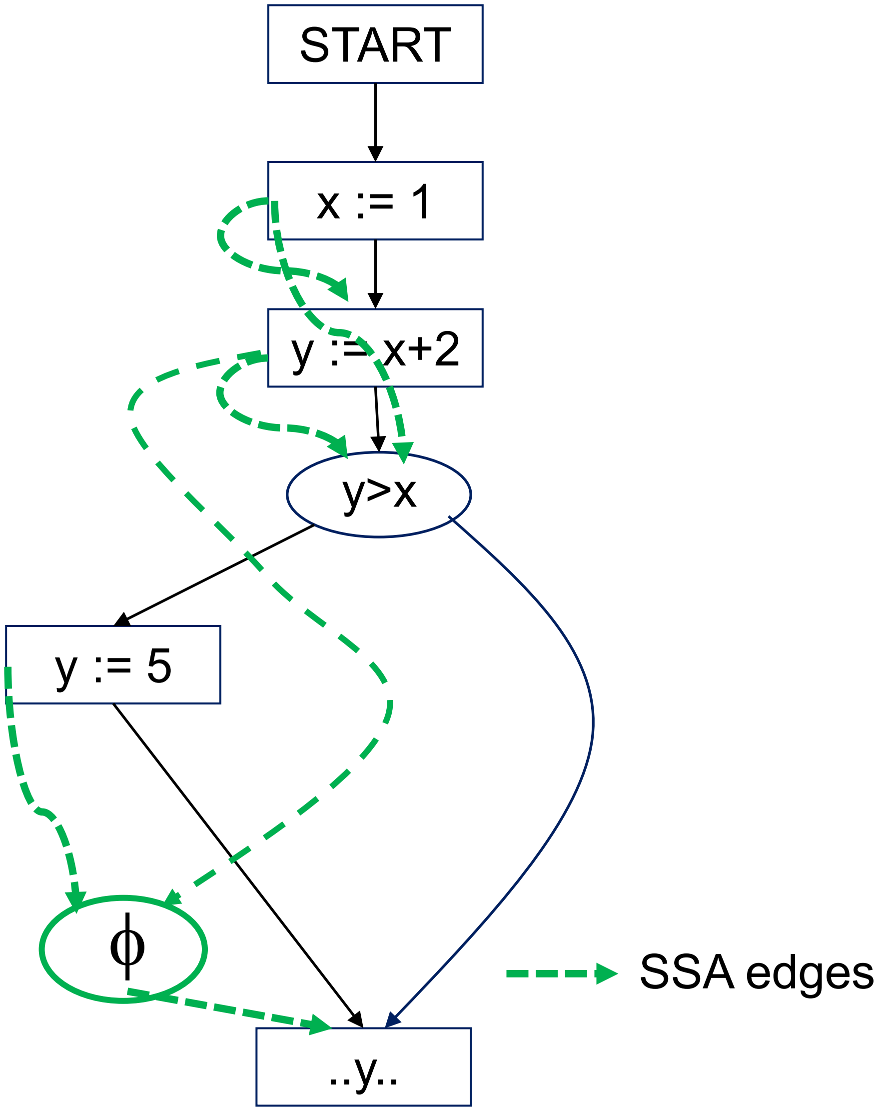
Def-use chains algorithm

- Algorithm:
  - Compute reaching definitions
  - Add def-use chains to CFG
  - Cell for each definition and use, initialized to ⊥
  - Propagate lattice values from definitions to uses, using confluence operator to merge values from multiple definitions that reach a given use
- Algorithm will not find all the constants found by CFG dataflow algorithm



SSA algorithm

- Cells for each def and use of SSA edges initialized to $\perp$
- Cell per edge and statement to mark liveness
- Propagate liveness along CFG edges to mark live edges
- Statement is live if any incoming edge is live
- Propagate constants along SSA edges from live statements
- At conditional, evaluate condition using propagated values to mark liveness on outgoing edges
- At ϕ function, merge values only from live CFG edges using confluence operator
- Will find all constants found by CFG dataflow algorithm



Computing SSA form

- Cytron et al algorithm
  - compute DF relation (see slides on computing control-dependence relation)
  - find irreflexive transitive closure of DF relation for set of assignments for each variable
- Computing full DF relation
  - Cytron et al algorithm takes O(|V| +|DF|) time
  - |DF| can be quadratic in size of CFG
- Faster algorithms
  - O(|V|+|E|) time per variable: see Bilardi and Pingali

Dependences

- We have seen control-dependences.
- What other kind of dependences are there in programs?
  - Data dependences: dependences that arise from reads and writes to memory locations
- Think of these as constraints on reordering of statements

Data dependences

- Flow-dependence (read-after-write): S1→S2
  - Execution of S2 may follow execution of S1 in program order
  - S1 may write to a memory location that may be read by S2
  - Example:

```
 …..
x := 3        flow-dependence
…x..
 …….
```

```
while e do
   …x…
   x: = …     flow-dependence
   ……
```

This is called a loop-carried dependence

Anti-dependences

- Anti-dependence (write-after-read): S1→S2
  - Execution of S2 may follow execution of S1 in program order
  - S1 may read from a memory location that may be (over)written by S2
  - Example:

```
x := …
..x….
 x:= …
```

anti-dependence

Output-dependence

- Output-dependence (write-after-write): S1→S2
  - Execution of S2 may follow execution of S1 in program order
  - S1 and S2 may both write to same memory location

Summary of dependences

- Dependence
  - Data-dependence: relation between nodes
    - Flow- or read-after-write (RAW)
    - Anti- or write-after-read (WAR)
    - Output- or write-after-write (WAW)
  - Control-dependence: relation between nodes and edges