CS 357 Assignment #2

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There are two parts to this assignment, which are described in Sections 1 and 2, respectively.

1 Programming & Problem Solving

This part of the assignment consists of a programming task and a collection of related exercises. The programming task is due by 8pm on Wednesday, September 14, and the solutions to the related exercises are due at the start of class on Wednesday, September 14. Students are allowed to work on this part of the assignment with a partner, and are strongly encouraged to do so. Each team should turn in only one program, and only one set of solutions to the related exercises. If you are having trouble finding a partner, send me an email and I will match you up with someone else in the same situation.

1.1 Exercises

Let G be a CBG and let M be a matching of G. We define *satisfied*(G, M) as the set of all edges (u, v) in G such that either (1) u is matched in M to a pong v' such that $v' \leq v$, or (2) v is matched in M to a ping u' such that $u' \leq u$. We say that matching M is *stable* if *satisfied*(G, M) is equal to the set of all edges in G.

1. Let $G = (U, V)$ be a CBG such that V is nonempty. Let v be the minimum pong in V. Let U' denote the set of all pings u in U such that (u, v) is an edge of G. If U' is empty then it is easy to prove that v is unmatched in any matching of G, and that the set of all stable matchings of G is equal to the set of all stable matchings of the CBG $(U, V - v)$. In this question, we assume that U' is nonempty. Let u' denote the minimum ping in U'. Let G' denote the CBG $(U - u', V - v)$. Prove that if M is a stable matching of G', then $M + (u', v)$ is a stable matching of G. Remark: Using similar reasoning, one can show that if M is a stable matching of G, then edge (u', v) belongs to M and $M - (u', v)$ is a stable matching of G'.

2. Prove that any CBG has a unique stable matching. Hint: Make use of Exercise 1 above, including the claims asserted without proof in Exercise 1. You are not required to prove the latter claims.

3. Briefly describe a polynomial-time algorithm to compute the unique stable matching of a given CBG.

4. Let G be a CBG and let M be the unique stable matching of G. Prove that M is an MCM of G.

1.2 Programming Task

Your program will read input from standard input, and write output to standard output. The first line of the input contains a nonnegative integer k that specifies the number of instances to follow. The integer k is followed by k "input blocks". Your program will produce k "output blocks", one for each input block. Each input block specifies a CBG G in exactly the same manner as in Assignment 1. The corresponding output block consists of a single line specifying the unique stable matching of the CBG G. The output matching should be printed in the same format as we used to print out each MWMCM in Assignment 1.

Your program should implement the algorithm described in your solution to Exercise 3 in Section 1.1, and is required to run in polynomial time.

2 Textbook Exercises

This part of the assignment consists of textbook exercises, and is due at the beginning of class on Friday, September 23. Each student should work on this part separately.

1. Problem 1.4, page 22. Hint: Generalize the proposal algorithm.

2. Problem 1.7, page 26. Hint: Give a reduction to the stable marriage problem.

3. Problem 3.6, page 108. In solving this problem, you should assume that the graph G has no self-loops or parallel edges.

4. Problem 3.12, page 112.