CS 357 Assignment #6

Greg Plaxton

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There are two parts to this assignment, which are described in Sections 1 and 2, respectively.

1 Programming & Problem Solving

This part of the assignment consists of a programming task and a collection of related exercises. The programming task is due by 8pm on Monday, November 21, and the solutions to the related exercises are due at the start of class on Monday, November 21. Students are allowed to work on this part of the assignment with a partner, and are strongly encouraged to do so. Each team should turn in only one program, and only one set of solutions to the related exercises. If you are having trouble finding a partner, send me an email and I will try to match you up with someone else in the same situation.

1.1 Exercises

Let $G = (V, E)$ be an undirected graph. Assume that V is partitioned into a set L of "left" vertices and a set R of "right" vertices, and that every edge in E connects a left vertex to a right vertex. (Thus, G is bipartite.) We say that a set of vertices A is *good* if there is an MCM of G that matches every vertex in A.

1. Prove that if X is a good subset of L and Y is a good subset of R, then $X \cup Y$ is good.
2. In Assignment 4, we developed a polynomial-time algorithm to compute an MWMCM of a ping-weighted CBG. In Assignment 5, we developed a polynomial-time algorithm to compute an MWMCM of a pong-weighted CBG. Use these two algorithms, together with the result of Exercise 1 above, to develop a polynomial-time algorithm for computing an MWMCM of an arbitrary CBG.

1.2 Programming Task

Your program will read input from standard input, and write output to standard output. The first line of the input contains a nonnegative integer k that specifies the number of instances to follow. The integer k is followed by k "input blocks". Your program will produce k "output blocks", one for each input block. Each input block specifies a CBG G in exactly the same manner as in Assignment 1. The corresponding output block consists of a single line specifying a particular MWMCM of G, as specified below. The output matching should be printed in the same format as we used to print out each MWMCM in Assignment 1.

A CBG can have many MWMCMs. For ease of grading, you are asked to produce as output a specific MWMCM that we now describe. Let the input CBG G be (U, V), let k denote the cardinality of an MCM of G, and let $\mathcal{U}$ denote the set of all subsets U_0 of U such that U_0 is the set of pings matched by some MWMCM of G. Define a total order over the sets in $\mathcal{U}$ in the same manner as we did in Assignment 4, and let U' denote the minimum set in $\mathcal{U}$. Let G' denote the CBG (U', V). Let $\mathcal{V}$ denote the set of all subsets V_0 of V such that V_0 is the set of pongs matched by some MWMCM of G'. Define a total order over the sets in $\mathcal{V}$ in the same manner as we did in Assignment 5, and let V' denote the maximum set in $\mathcal{V}$. Let G'' denote the CBG (U', V'). Then your program should produce as output the stable matching M of G''. Below we argue that M is an MWMCM of G.

Since the set of pings matched by any MCM of G' is U', the definition of V' implies that M is a perfect matching of G'', that $|U'| = |V'| = k$, and that M is an MWMCM of G'. Since M is an MWMCM of G' that matches all of the pings in U', and since U' belongs to $\mathcal{U}$, we conclude that M is an MWMCM of G, as required.

2 Textbook Exercises

This part of the assignment is due at the beginning of class on Wednesday, November 30.

1. Problem 8.13, page 511. Hint: Use a reduction from *Independent Set*.
2. Problem 8.17, page 513. Hint: Use a reduction from *Subset Sum*.
3. Problem 9.1, page 550.
4. Problem 11.5, page 653.
5. Problem 11.8, page 655.
6. Problem 13.9, page 788.