CS311H

Prof: Peter Stone

Department of Computer Science
The University of Texas at Austin

Challenge

- Prove that a graph with exactly two vertices with odd degree must contain a path between these two vertices.

Good Morning, Colleagues

Good Morning, Colleagues

Are there any questions?

Logistics

- Honors modules for next week

Logistics

- Honors modules for next week
- Midterm back at end of class

Logistics

- Honors modules for next week
- Midterm back at end of class
- Out of 70:

Logistics

- Honors modules for next week
- Midterm back at end of class
- Out of 70:
  - Max: 67
  - Mean: 46

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?
- Are all trees 2-colorable?

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?
- Are all trees 2-colorable?
- Definitions
  - Tree: a connected graph with no cycles.
  - Leaf: a vertex of degree one.

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?
- Are all trees 2-colorable?
- Definitions
  - Tree: a connected graph with no cycles.
  - Leaf: a vertex of degree one.
- Fact: For any tree there exists a leaf

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?

- Are all trees 2-colorable?

- Definitions
  - Tree: a connected graph with no cycles.
  - Leaf: a vertex of degree one.
- Fact: For any tree there exists a leaf
  - Problem?

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?
- Are all trees 2-colorable?
- Definitions
  - Tree: a connected graph with no cycles.
  - Leaf: a vertex of degree one.
- Fact: For any tree there exists a leaf
  - Problem?
  - One node tree has no leaves

Some questions

- Why do bipartite graphs not need to be all connected, but trees do?

- Are all trees 2-colorable?

- Definitions
  - Tree: a connected graph with no cycles.
  - Leaf: a vertex of degree one.
- Fact: For any tree there exists a leaf
  - Problem?
  - One node tree has no leaves
  - Why not a problem for his proof that tree with n nodes has $n-1$ edges?

Quiz

- FACT: All finite trees with $|V| > 1$ have at least one leaf

Quiz

- FACT: All finite trees with $|V| > 1$ have at least one leaf
- Prove: Any tree with n vertices has $n-1$ edges.

Definitions

For $G = (\{a, b, c, d, e\}, \{(a, b), (e, d), (a, c), (b, c), (e, c), (d, c)\})$

1. Identify all simple paths from a to e.

Definitions

For $G = (\{a, b, c, d, e\}, \{(a, b), (e, d), (a, c), (b, c), (e, c), (d, c)\})$

1. Identify all simple paths from a to e.

2. Identify all simple circuits starting and ending at a.

3. Identify all cycles starting and ending at a.

Definitions

For $G = (\{a, b, c, d, e\}, \{(a, b), (e, d), (a, c), (b, c), (e, c), (d, c)\})$

1. Identify all simple paths from a to e.
   (a,c,e), (a,b,c,e), (a,c,d,e), (a,b,c,d,e)

2. Identify all simple circuits starting and ending at a.
   (a,b,c,a), (a,c,b,a), (a,b,c,d,e,c,a), (a,b,c,e,d,c,a), (a,c,e,d,c,b,a), (a,c,d,e,c,b,a)

3. Identify all cycles starting and ending at a.
   Subset of the simple circuits: (a,b,c,a), (a,c,b,a)

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.
  2. Then there must be a repeated vertex v so the circuit is $(s, ..., v, ..., v, ..., s)$

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.
  2. Then there must be a repeated vertex v so the circuit is $(s, ..., v, ..., v, ..., s)$
  3. Since the circuit is "simple", no repeated edges.

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.
  2. Then there must be a repeated vertex v so the circuit is $(s, ..., v, ..., v, ..., s)$
  3. Since the circuit is "simple", no repeated edges.
  4. Therefore the vertices preceding and following v in each case must be distinct.

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.
  2. Then there must be a repeated vertex v so the circuit is $(s, ..., v, ..., v, ..., s)$
  3. Since the circuit is "simple", no repeated edges.
  4. Therefore the vertices preceding and following v in each case must be distinct.
  5. So the circuit is $(s, ..., a, v, b, ..., x, v, y, ..., s)$ (a or y could equal s, but not both)

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.
  2. Then there must be a repeated vertex v so the circuit is $(s, ..., v, ..., v, ..., s)$
  3. Since the circuit is "simple", no repeated edges.
  4. Therefore the vertices preceding and following v in each case must be distinct.
  5. So the circuit is $(s, ..., a, v, b, ..., x, v, y, ..., s)$ (a or y could equal s, but not both)
  6. Then the degree of v is at least 4.

Prove

- For a graph G, if MAX-DEGREE(G) = 3, then any simple circuit is actually a cycle.
  Proof by contradiction:
  1. Assume the simple circuit $(s, ..., s)$ is not a cycle.
  2. Then there must be a repeated vertex v so the circuit is $(s, ..., v, ..., v, ..., s)$
  3. Since the circuit is "simple", no repeated edges.
  4. Therefore the vertices preceding and following v in each case must be distinct.
  5. So the circuit is $(s, ..., a, v, b, ..., x, v, y, ..., s)$ (a or y could equal s, but not both)
  6. Then the degree of v is at least 4.
  7. $4 > 3 =$MAX-DEGREE(G) is a contradiction.

Find a Counterexample

Suppose all vertices of a graph G have been colored. Now suppose that all cycles are found, and it turns out that for each cycle $(v_1, v_2, ..., v_n, v_1)$ that $v_1, ..., v_n$ all have distinct colors. In this case, the coloring must be valid.

Find a Counterexample

Suppose all vertices of a graph G have been colored. Now suppose that all cycles are found, and it turns out that for each cycle $(v_1, v_2, ..., v_n, v_1)$ that $v_1, ..., v_n$ all have distinct colors. In this case, the coloring must be valid.

Create a counterexample using a vertex that doesn't appear in ANY cycles. Take the graph
$G = (\{a, b, c, d\}, \{(a, b), (b, c), (c, a), (a, d)\})$.
Then the cycles are $(a, b, c, a), (b, c, a, b), (c, a, b, c)$, none of which contain d, so assign the colors: a:RED, b:BLUE, c:GREEN, d:RED. Colors are distinct within each cycle, but the color of d clashes with a.

Solve Alone then Pair Up

1. Prove that a graph with exactly two vertices with odd degree must contain a path between these two vertices.

Solve Alone then Pair Up

1. Prove that a graph with exactly two vertices with odd degree must contain a path between these two vertices.

2. Prove that every graph with vertices that each have degree at least 2 contains a cycle.

Assignments for Thursday

- Module 14