CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Are there any questions?

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  - Handwritten notes allowed
  - No book, nothing printed, nothing electronic
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- HW4 review

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$$f(x) = \begin{cases} 1/2 & \text{if } x = 0 & \text{(case 1)} \\ x/(1+x) & \text{if } \exists n \in \mathbb{N}[x = 1/n] & \text{(case 2)} \\ x & \text{otherwise} & \text{(case 3)} \end{cases}$$

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   $P(x)$: x has blue eyes
   $Q(x)$: x has black eyes
   Statement: There exist people with blue eyes and with black eyes, but one cannot have blue and black eyes at the same time.

2. Domain: all UT student
   $P(x)$: x is a computer science student
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   Answer: All computer science students in UT must take 313

Pair up activity

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- Sponge: Suppose $A \subseteq B \subseteq C$. Prove $C - B \subseteq C - A$.

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Assignments for Thursday

- Modules 10 and 11