# CS344M
# Autonomous Multiagent Systems

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# Good Afternoon, Colleagues

Are there any questions?

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- Can you handle mutual exclusivity of actions?
- What's the theory of Nash Eq.?
  - Probabilistic strategies

# Logistics

- Progress reports due in 2 weeks

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- FAI talk on Friday - Andrew McCallum - NLP

# Game Theory Premises

- Simultaneous actions: (mutual exclusivity?)

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- Simultaneous actions: (mutual exclusivity?)
- No communication
- Outcome depends on **combination** of actions
- Utility (payoff) encapsulates **everything** about preferences over outcomes

# Solution Concepts

- Dominant strategy
- Nash equilibrium
- Pareto optimality
- Maximum social welfare
- Maximin strategy

# Prisoner's Dilemma

| | | Column | |
|---|---|---|---|
| | | C(1) | D(2) |
| Row | C(1) | 3,3 | 0,5 |
| | D(2) | 5,0 | 1,1 |

# Chicken

| | | Column | |
|---|---|---|---|
| | | C(1) | D(2) |
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  - If not, so distraught we don't care what we're listening to
- Propose a payoff matrix

# Bach/Stravinsky

| | | Wife | |
|---|---|---|---|
| | | S | B |
| Me | S | 2,1 | 0,0 |
| | B | 0,0 | 1,2 |

# Nash Equilibrium

- Does every game have a pure strategy Nash equilibrium?

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- We each put a penny down covered
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  - Nobel prize **and** academy award!
- Not known if complexity of finding one is NP-complete or in P

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- Is the outcome of a Nash equilibrium necessarily Pareto optimal?
- Is a Pareto optimal outcome necessarily the result of Nash equilibrium strategies?
- Is the maximum social welfare outcome necessarily Pareto optimal?
- If both players play maximin, is it necessarily a Nash equilibrium?

# Mixed strategy equilibrium

| | | Player 2 | |
|---|---|---|---|
| | | Action 1 | Action 2 |
| Player 1 | Action 1 | 4,8 | 2,0 |
| | Action 2 | 6,2 | 0,8 |

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Do actual numbers matter?

# Rock/Paper/Scissors

- Nash equilibrium?

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- Nash equilibrium?
- Why is anything else **not** an equilibrium?

# Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky

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Want only S,S or B,B - 50% each