## CS378

# Autonomous Multiagent Systems Spring 2005 

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The University of Texas at Austin
Week 10a: Thursday, March 31st

## Good Afternoon, Colleagues

Are there any questions?

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- What if you don't know outcomes ahead of time?
- Can strategies in the iterated case be automated?


## Logistics

- Project progress reports due next week


## T-test vs. Paired T-test

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## T-test vs. Paired T-test

- Test: Your team better than UvA vs. CMUnited
- Test: Your team better than UvA vs. a set of 20 opponents
- What if neither is significant?


## Student-led discussion

- Zac on real-world uses of game theory


## Mixed strategy equilibrium

## Player 2 <br> Action 1 Action 2

|  | Action 1 | 2,2 | 2,0 |
| :--- | :--- | :--- | :--- |
| Player 1 | Action 2 | 3,1 | 0,2 |

## Mixed strategy equilibrium

## Player 2 <br> Action 1 Action 2

$$
\begin{array}{llll} 
& \text { Action 1 } & 2,2 & 2,0 \\
\text { Player 1 } & & \\
& \text { Action 2 } & 3,1 & 0,2
\end{array}
$$

- Pure strategy Nash equilibrium?


## Mixed strategy equilibrium

$$
\begin{array}{lll}
\hline & \text { Player } & 2 \\
\text { Action } 1 & \text { Action } 2
\end{array}
$$

Action $1 \quad 2,2 \quad 2,0$

Player 1
Action 2
3,1
0,2

- Pure strategy Nash equilibrium?
- Mixed strategy Nash equilibrium?


## Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky

> Wife

S
2,1
0,0
Me
B
0,0
1,2

## Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky
Wife
S B
$S \quad 2,1 \quad 0,0$

Me
B
0,0
1,2

Want only S,S or B,B-50\% each

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- We have no way of getting in touch.
- When and where?
- What are the Nash equilibria?


## Incomplete Information Games

- We each get one of 3 cards: 1,2,3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum


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Card ?

$$
\begin{array}{ccc} 
& \mathrm{R} & \mathrm{~F} \\
\mathrm{R} & 5,-5 & 1,-1
\end{array}
$$

Card 3

$$
\begin{array}{ll}
F & -1,1
\end{array}
$$

## Incomplete Information Games

|  | Card ? |  |
| :---: | :---: | :---: |
|  | R |  |
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Card 3

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$$

$$
0,0
$$

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| R | $5,-5$ | $1,-1$ |

Card 3

F $\quad-1,1 \quad l$|  |  |
| :--- | :--- |
|  |  |
|  | Card ? |

R F

R $\quad-5,5$
$1,-1$
Card 1
F
$-1,1$
0,0

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With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium

