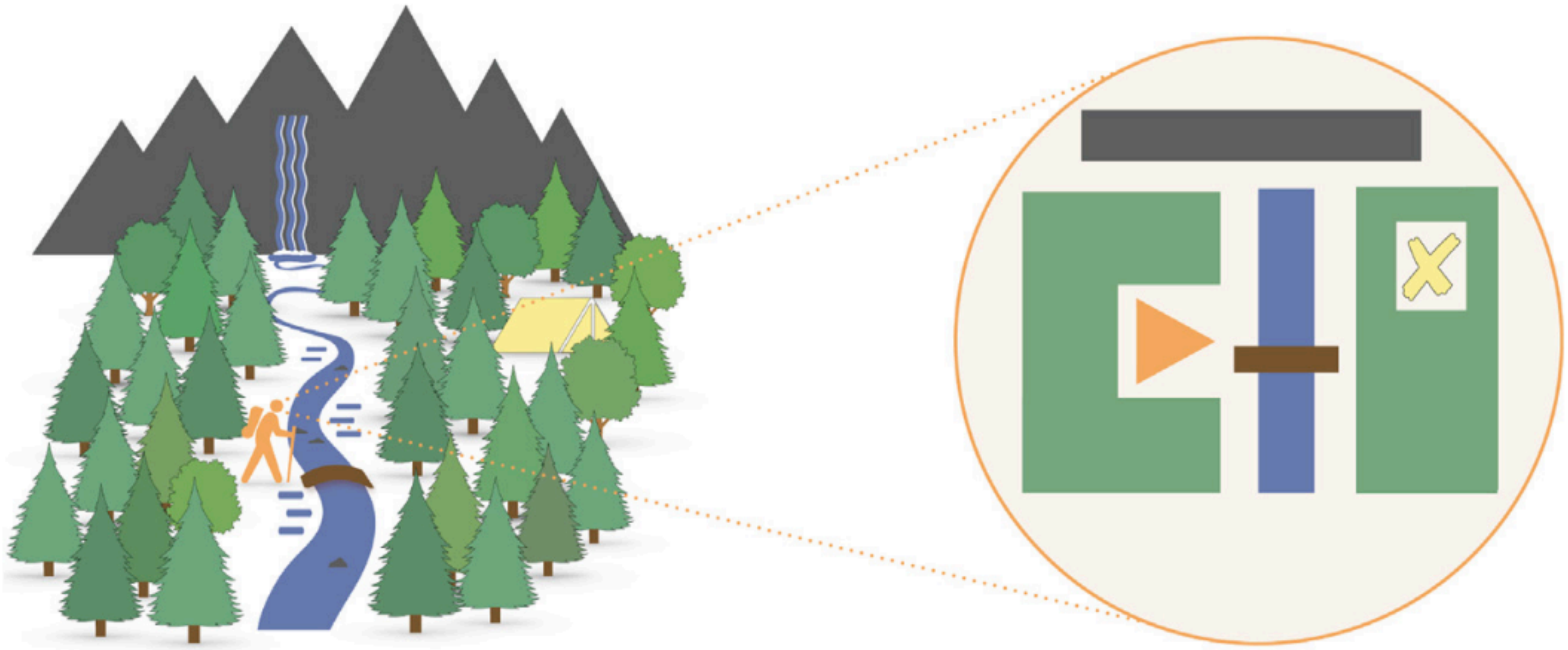
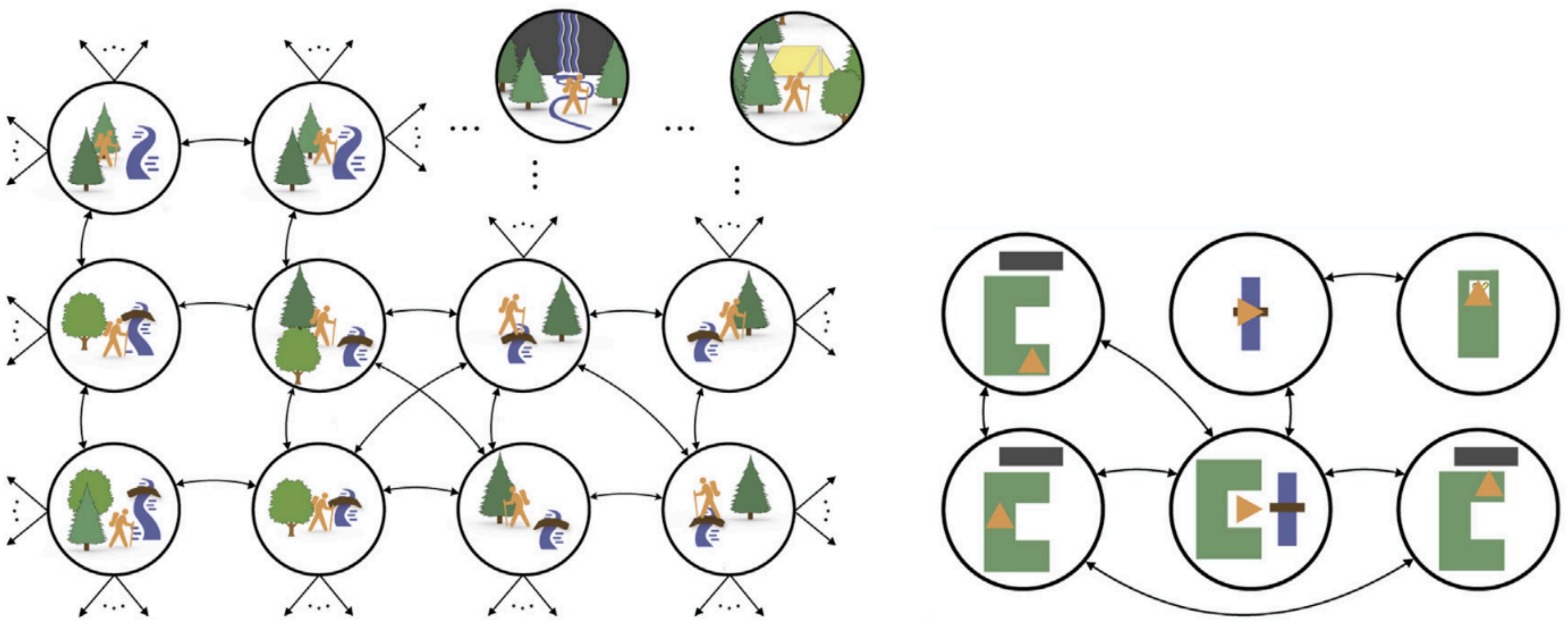
State abstractions



The Value of Abstraction Ho, M., Abel, D., Griffiths, T., and Littman, M.

State + temporal abstractions



(a) (b)

[The Value of Abstraction](#) Ho, M., Abel, D., Griffiths, T., and Littman, M.

Hierarchical Reinforcement Learning

George Konidaris
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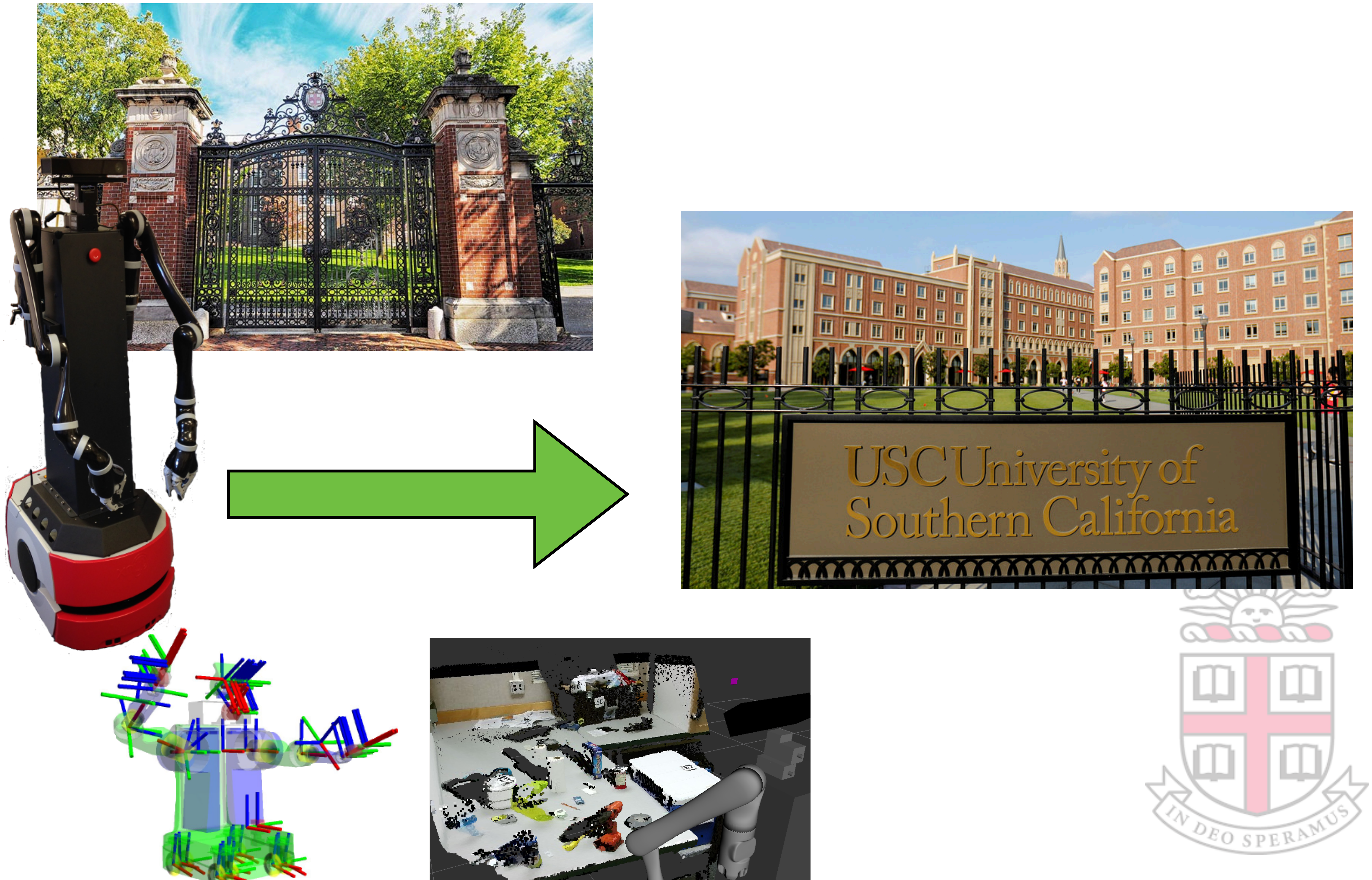
Why Hierarchies?



Skill Hierarchies

Hierarchical RL: base hierarchical control on *skills*.

- Component of behavior.
- Performs continuous, low-level control.
- Can treat as discrete action.

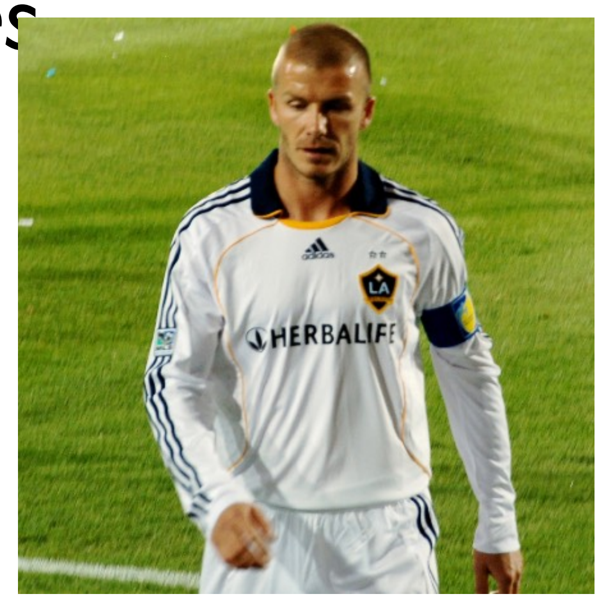
Behavior is modular and compositional.

Skills are like *subroutines*

```
d
 ):
  x

  return -x
```

Development



Specialization

[Wilkes, Wheeler and Gill 1951]

Simplification

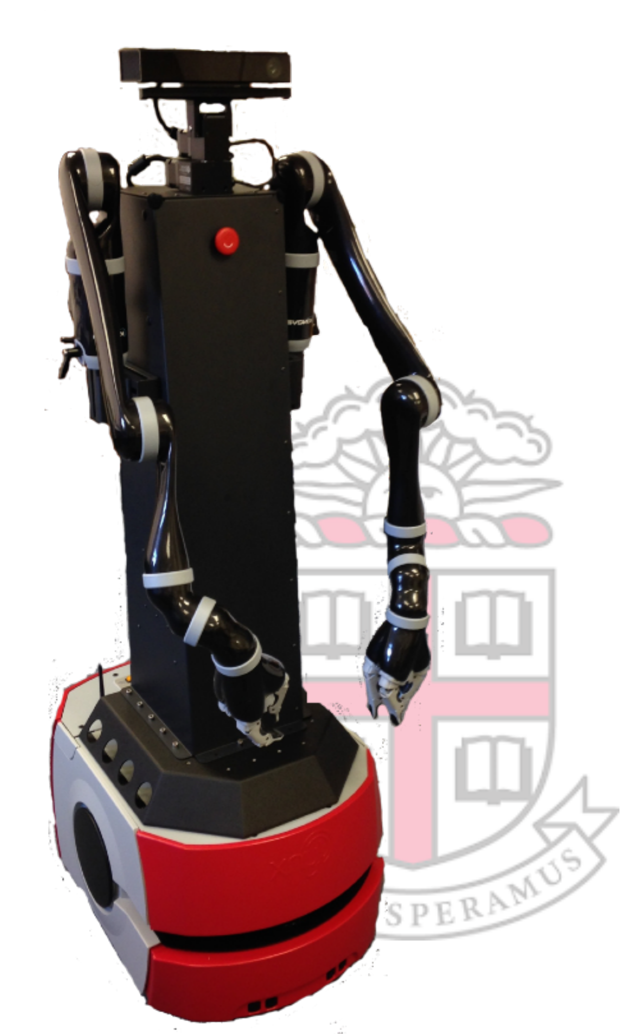
Forms of Abstraction

$$\langle \bar{S}, \bar{A}, R, T, \gamma \rangle$$

state abstraction

action abstraction

$$\langle S, A, R, T, \gamma \rangle$$



The Options Framework

Options

Options Framework: theoretical basis for skill acquisition, learning and planning using higher-level actions (options).

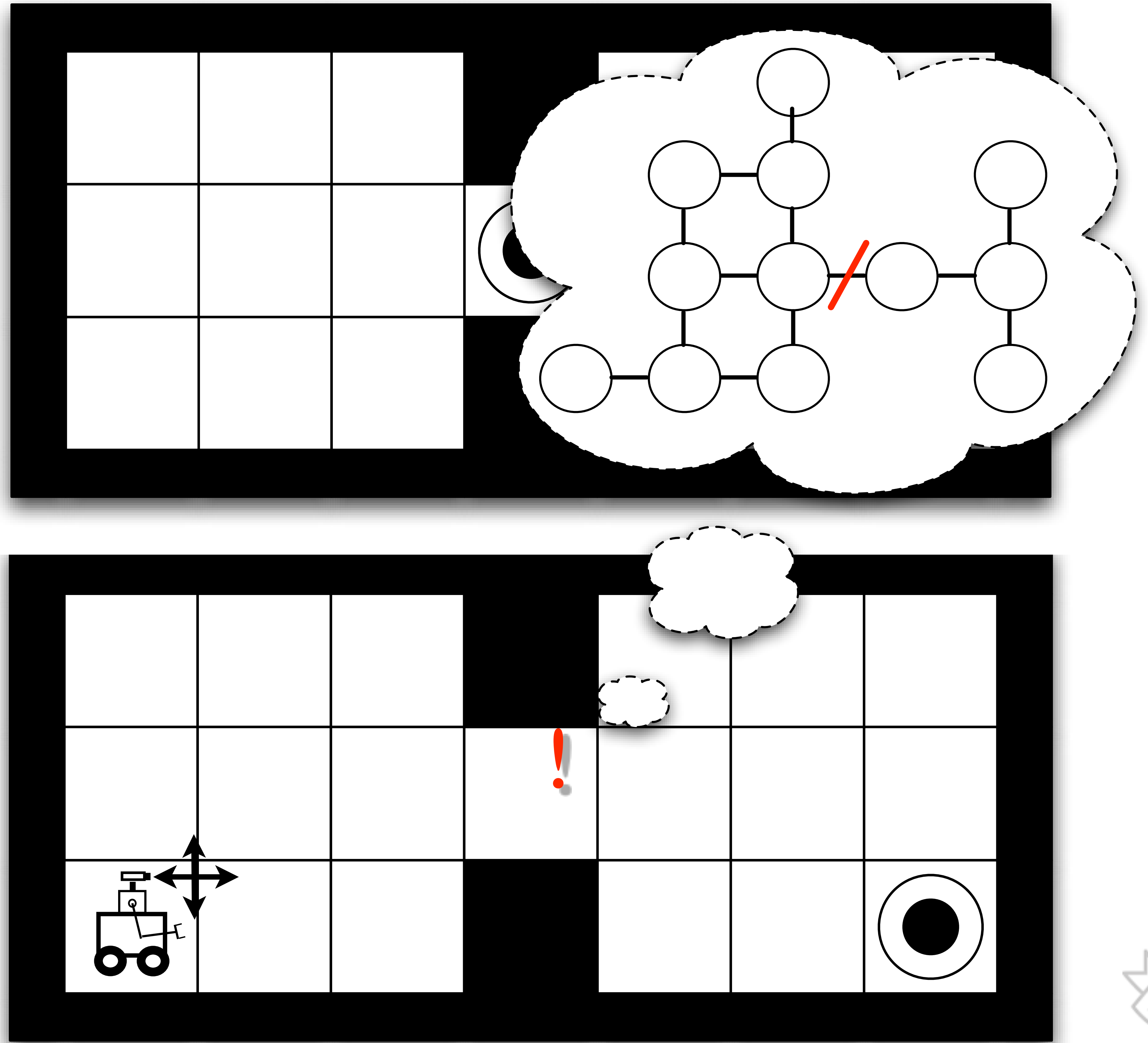
RL typically solves a *single* problem *monolithically*.

Action abstraction:

- Create and use higher-level macro-actions.
- Problem now contains subproblems.
- Each subproblem is also an RL problem.

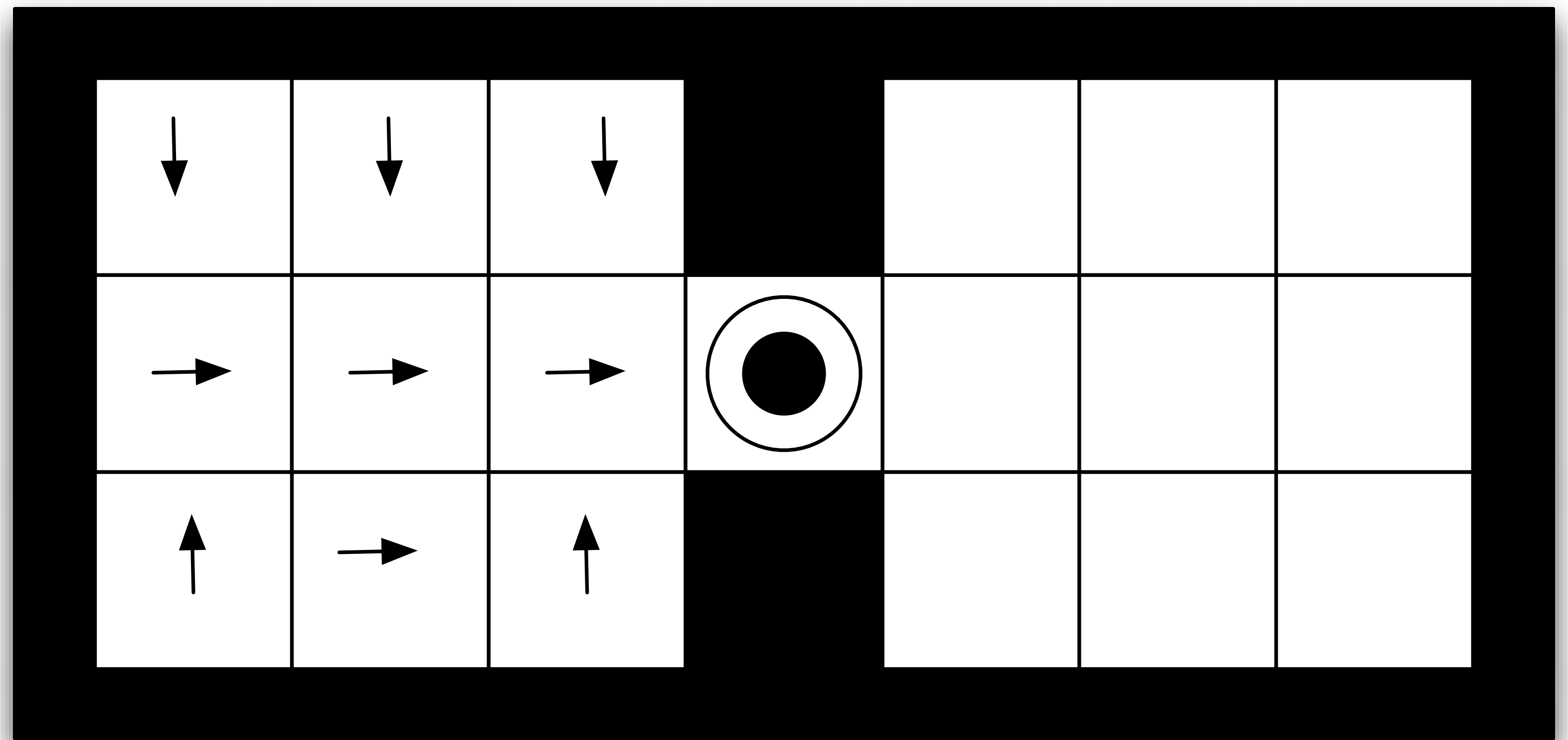
[Sutton, Precup, and Singh, 1999]

Hierarchical RL

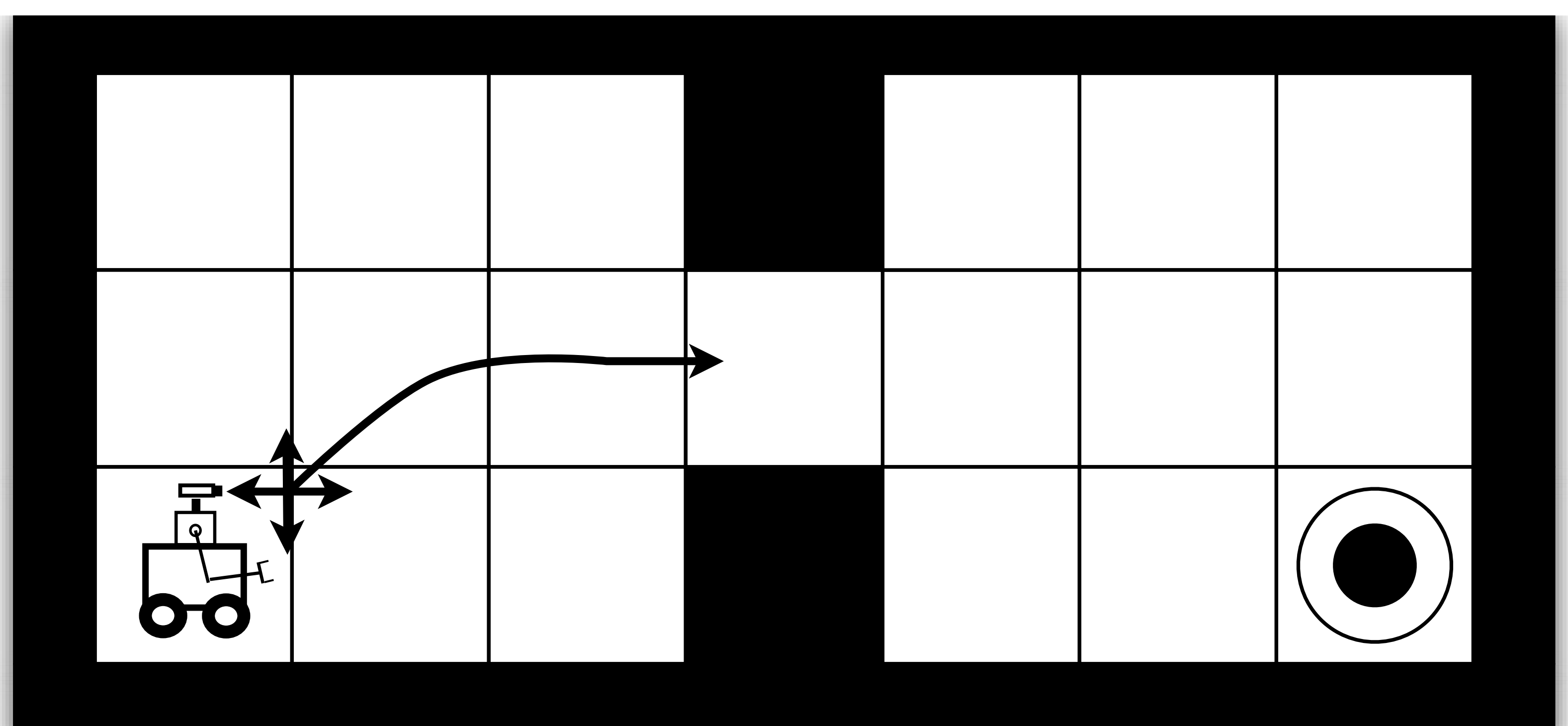


Hierarchical RL

Skill



Problem



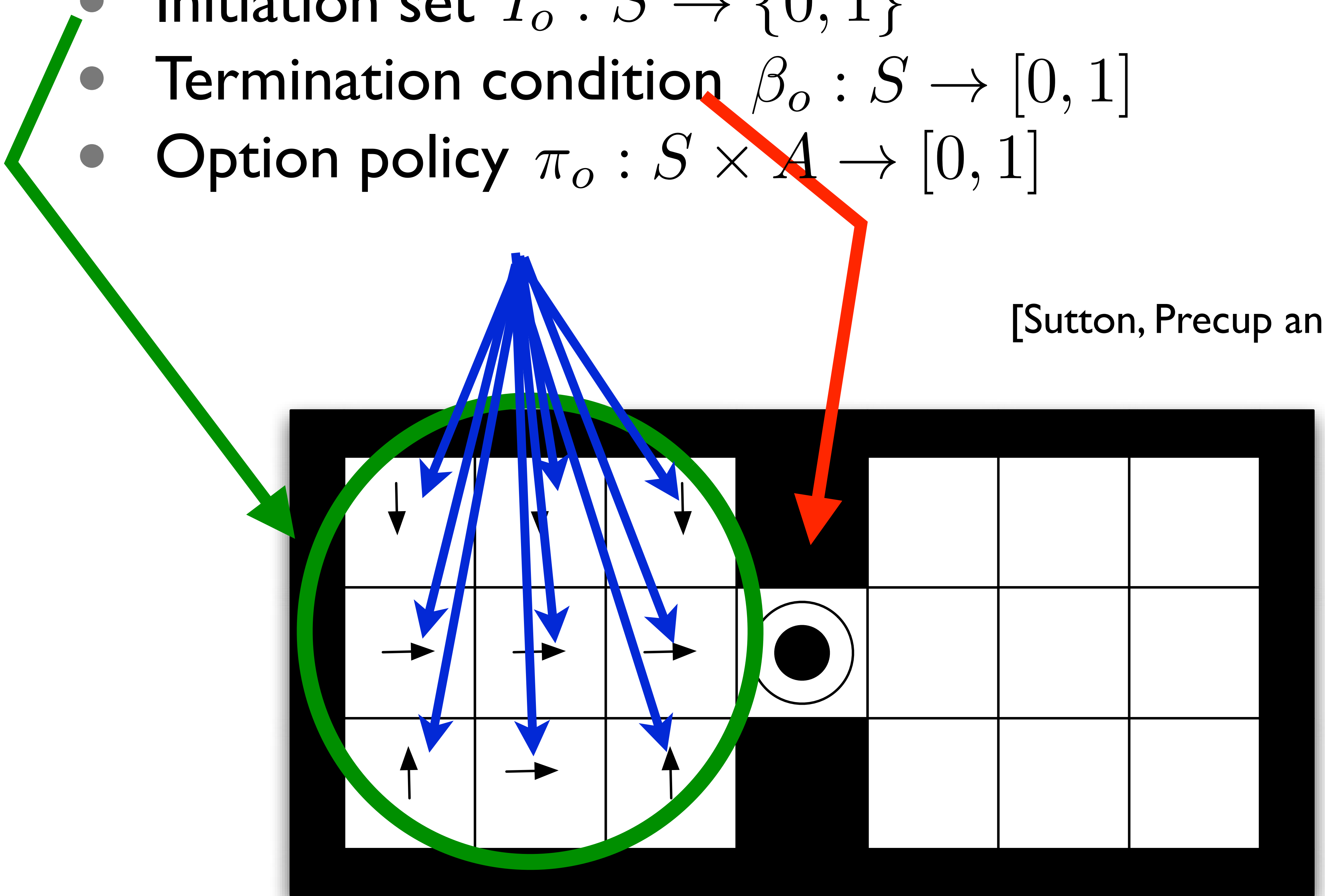
The Options Framework

An option is one formal model of a skill.

An option *o* is a policy unit:

- Initiation set $I_o : S \to \{0, 1\}$
- Termination condition $\beta_o : S \to [0, 1]$
- Option policy $\pi_o : S \times A \to [0, 1]$

[Sutton, Precup and Singh 1999]



Actions as Options

A primitive action *a* can be represented by an option:

- $I_a(s) = 1, \forall s \in S$
- $\beta_a(s) = 1, \forall s \in S$
- $\pi_a(s,b) = \begin{cases} 1 & a = b \\ 0 & \text{otherwise} \end{cases}$

A primitive action can be executed anywhere, lasts exactly one time step, and always chooses action *a*.

Questions

Given an MDP:

$$(S, A, R, T, \gamma)$$

... let's replace A with a set of options O (some of which may be primitive actions).

- How do we characterize the resulting problem?
- How do we plan using options?
- How do we learn using options?
- How do we characterize the resulting policies?

SMDPs

The resulting problem is a *Semi-(Markov Decision Process)*.
This consists of:

- S — Set of states
- O — Set of options
- $P(s', t|o, s)$ — Transition model
- $R(s', s, t)$ — Reward function
- γ — Discount factor (per step)

In this case:

- All times are natural numbers.
- "Semi" here means transitions can last *t* timesteps.
- Transition and reward function involve time taken for option to execute.

Planning?

option model

Easy

$$Q^\pi(s,o) = \mathbb{E}_{t,s'}[R(s',s,t)] + \mathbb{E}_{t,s'}[\gamma^t \pi(s',o')Q^\pi(s',o')]$$

where

$$\mathbb{E}_{t,s'}[R(s',s,t)] = \sum_{t,s'} P(s',t|o,s)R(s',s,t)$$

$$\mathbb{E}_{t,s'}[\gamma^t \pi(s',o')Q^\pi(s',o')] = \sum_{t,s'} P(s',t|o,s)\gamma^t \pi(s',o')Q^\pi(s',o')$$

All things flow from Bellman.

Learning and Planning

$$Q^\pi(s,o) = \mathbb{E}_{t,s'}[R(s',s,t)] + \mathbb{E}_{t,s'}[\gamma^t \pi(s',o') Q^\pi(s',o')]$$

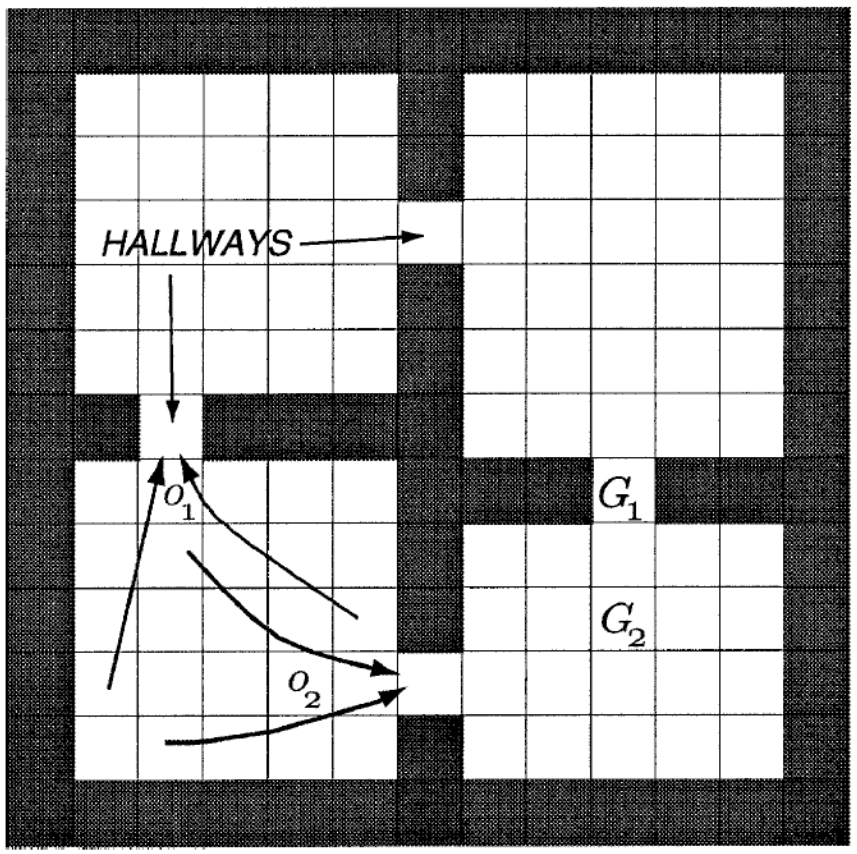
For learning:

- Stochastic samples.
- Use SMDP Bellman equation.

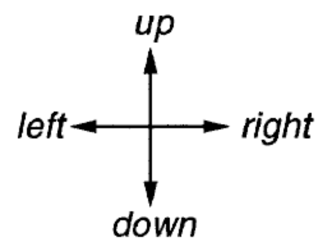
For planning:

- Synchronous Value Iteration via SMDP Bellman eqn

Example



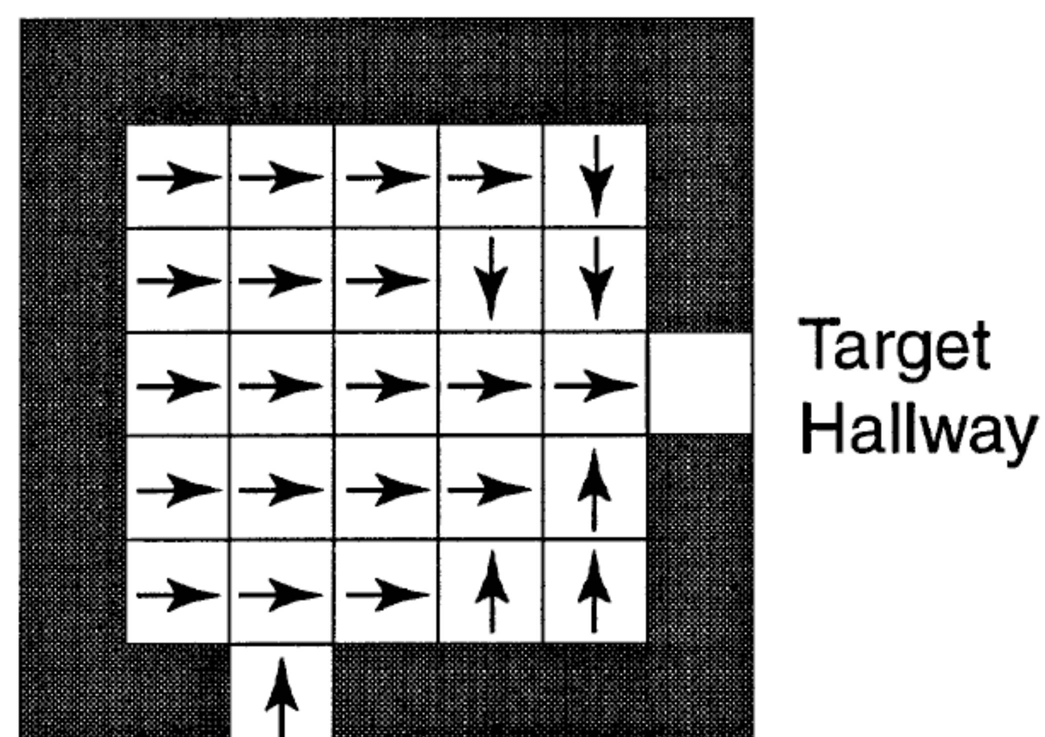
4 stochastic primitive actions



Fail 33% of the time

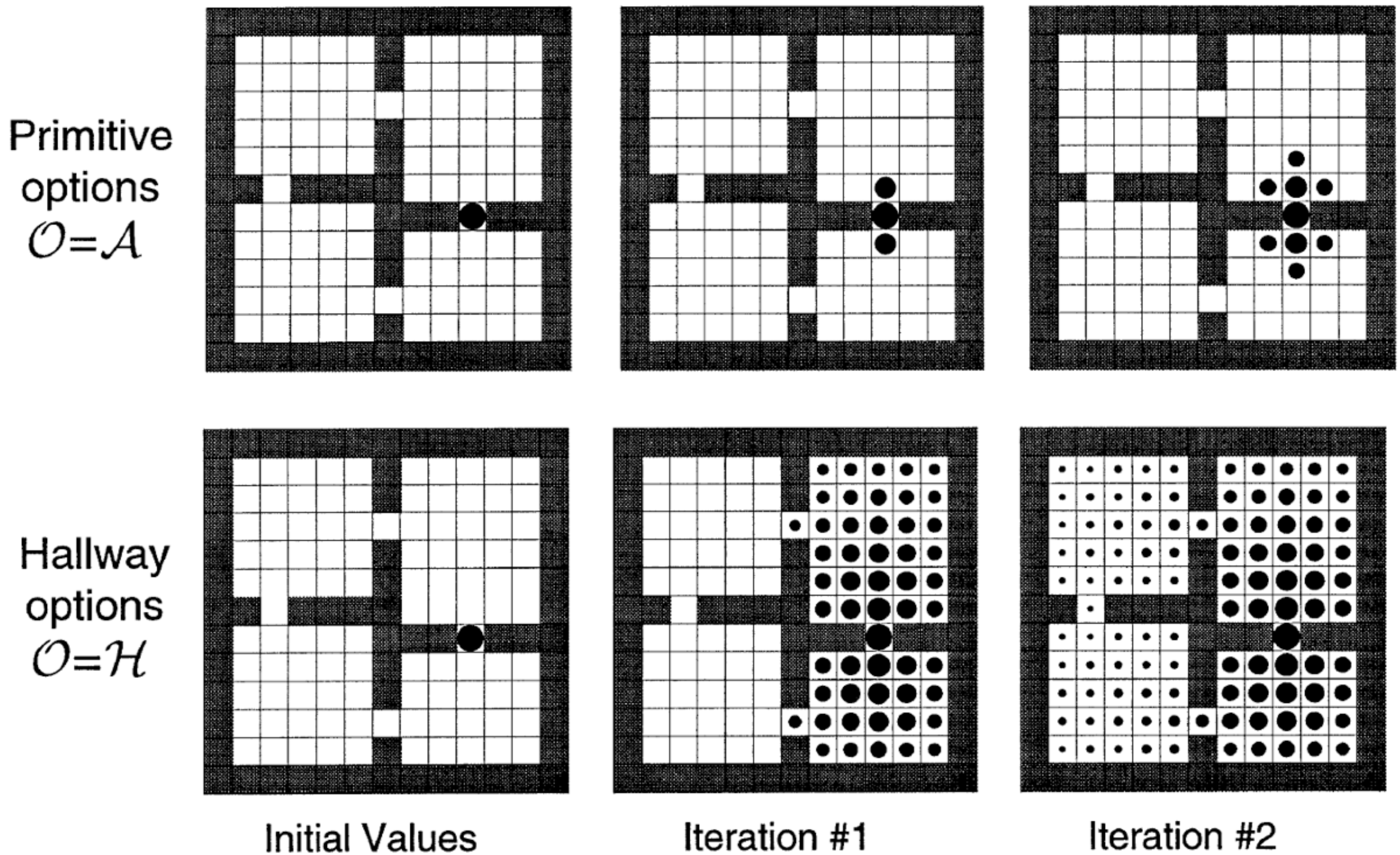
8 multi-step options

(to each room's 2 hallways)



(Sutton, Precup and Singh, AIJ 1999)

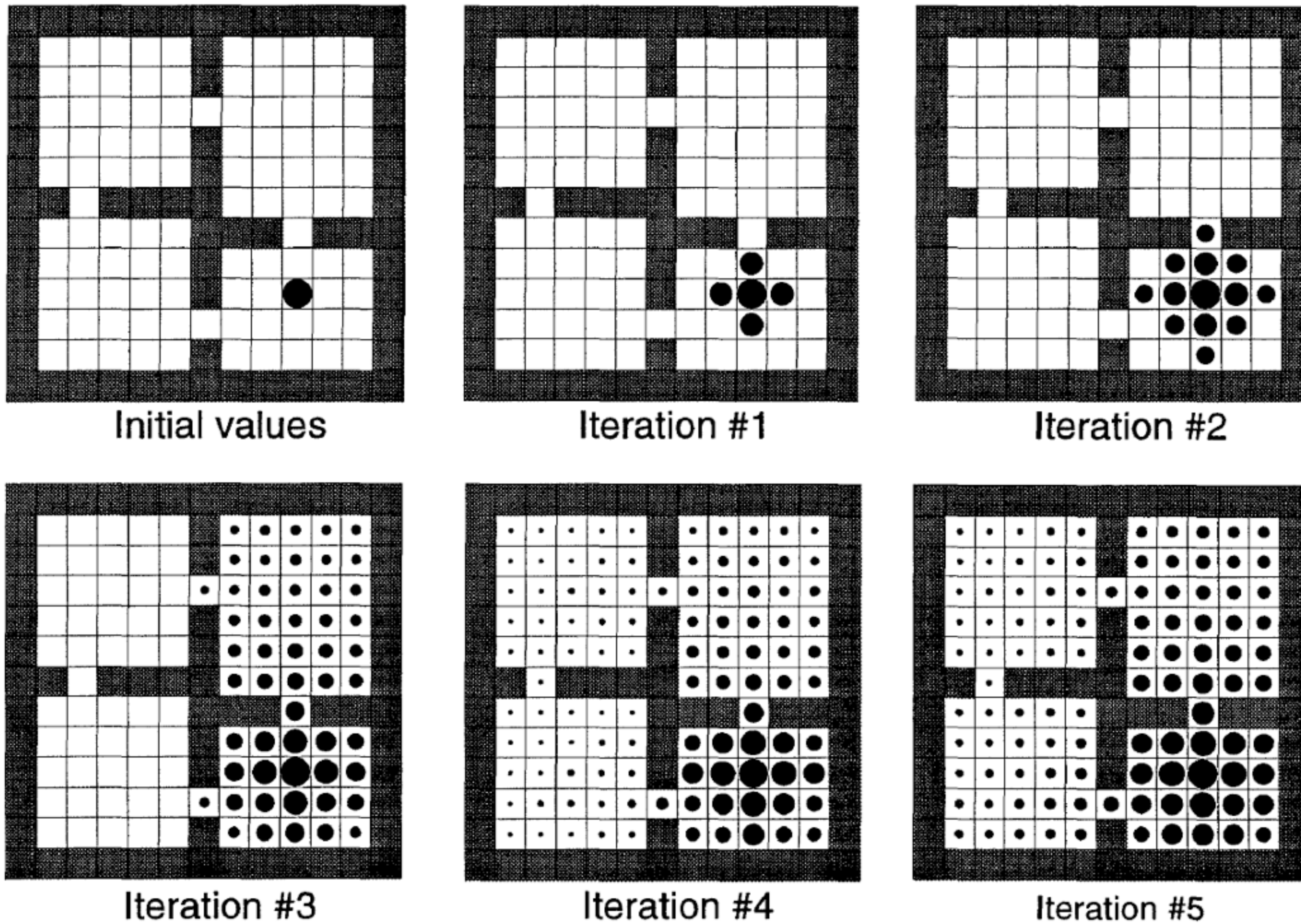
Example



(Sutton, Precup and Singh, AIJ 1999)

Example

Primitive and hallway options $\mathcal{O}=\mathcal{A}\cup\mathcal{H}$



(Sutton, Precup and Singh, AIJ 1999)

Final note: policies.

A policy over an MDP with primitive actions is a *Markov policy*:

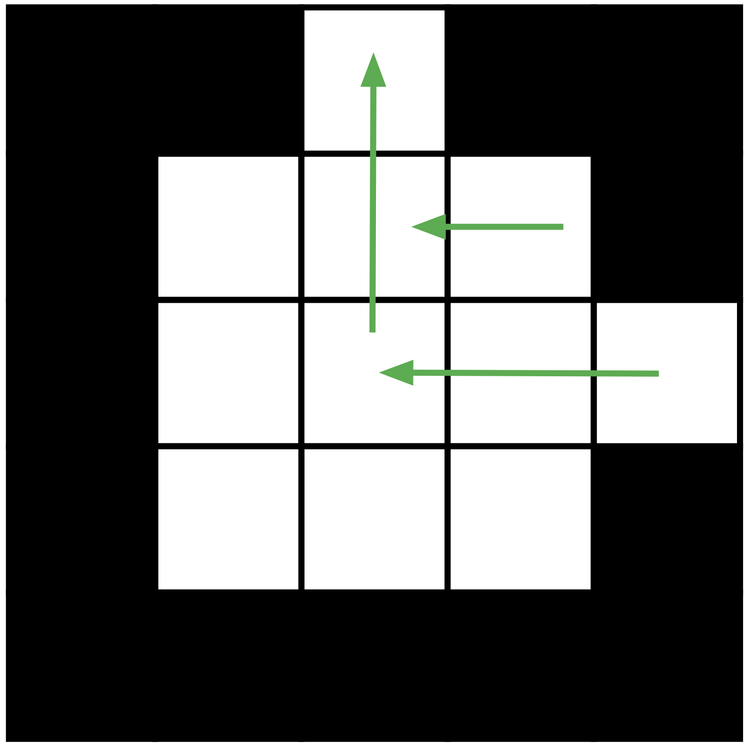
$$\pi : S \times A \rightarrow [0, 1]$$

A policy over an MDP with options could also be Markov:

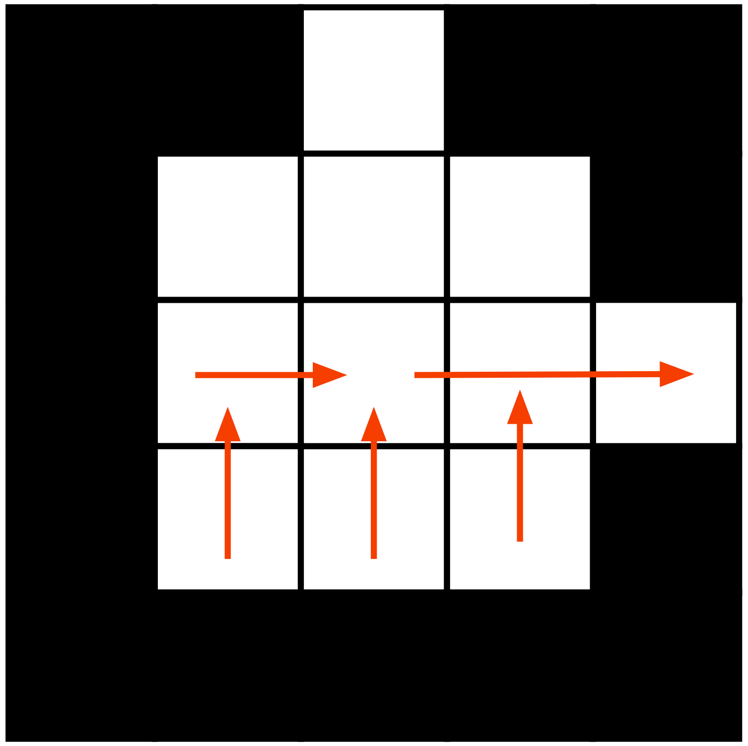
$$\pi : S \times O \rightarrow [0, 1]$$

... but this could imply a policy in the original MDP that is not, because the probability of taking an action at a state *depends on the option currently running*.

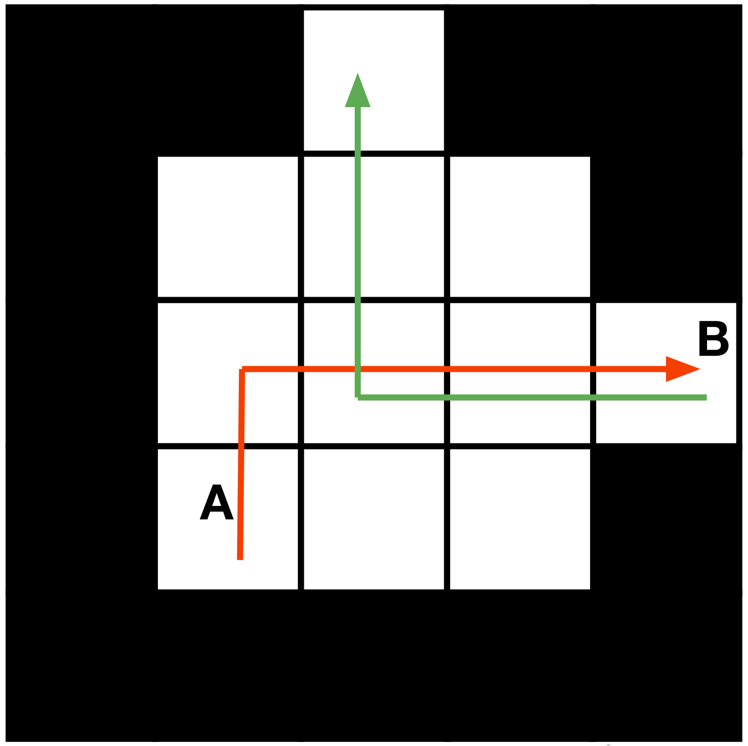
Example



Option A



Option B



Policy

So

A Markov policy for an SMDP may result in a *semi-Markov* policy for the underlying MDP.

(Even if the options are Markov options!)

Here, semi-Markov means that the probability of taking a primitive action at each step depends on more than the current state.

What are Options For?

Lots of things!

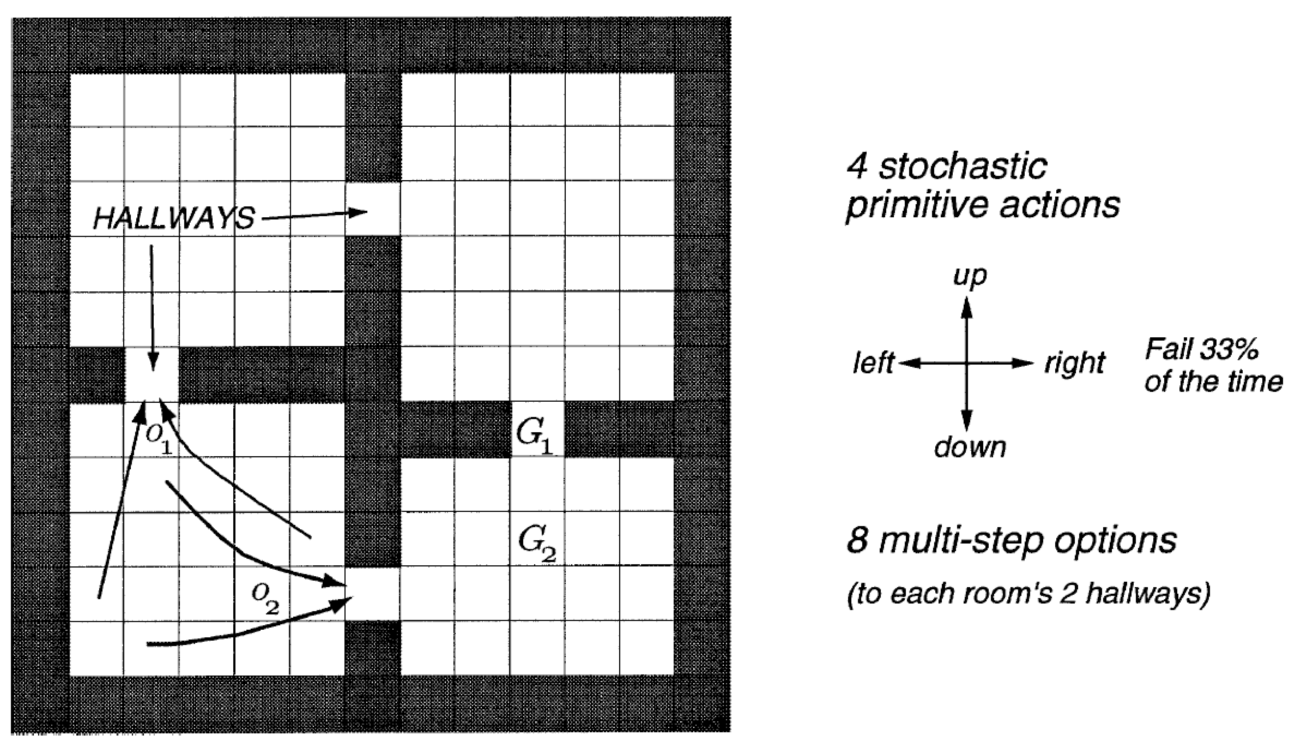
A few salient points:

- Rewiring.
- Transfer.
- Skill-Specific State Abstractions.

Rewiring

Adding an option changes the connectivity of the MDP.
This affects:

- Learning and planning.
- Exploration.
- State-visit distribution.
- *Diameter of problem.*



(Sutton, Precup and Singh, AIJ 1999)

Transfer

Use experience gained while solving one problem to improve performance in another.

Skill transfer:

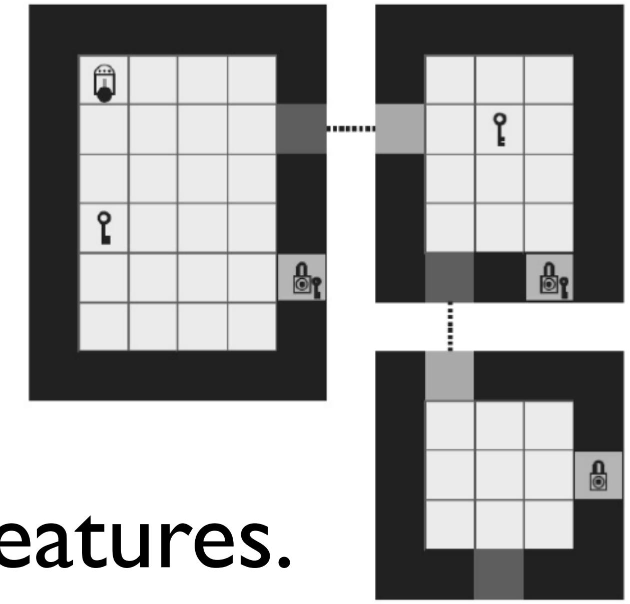
- Use options as mechanism for transfer.
- Transfer *components* of solution.
- Can drastically improve performance
- ... even if it takes a lot of effort to learn them.

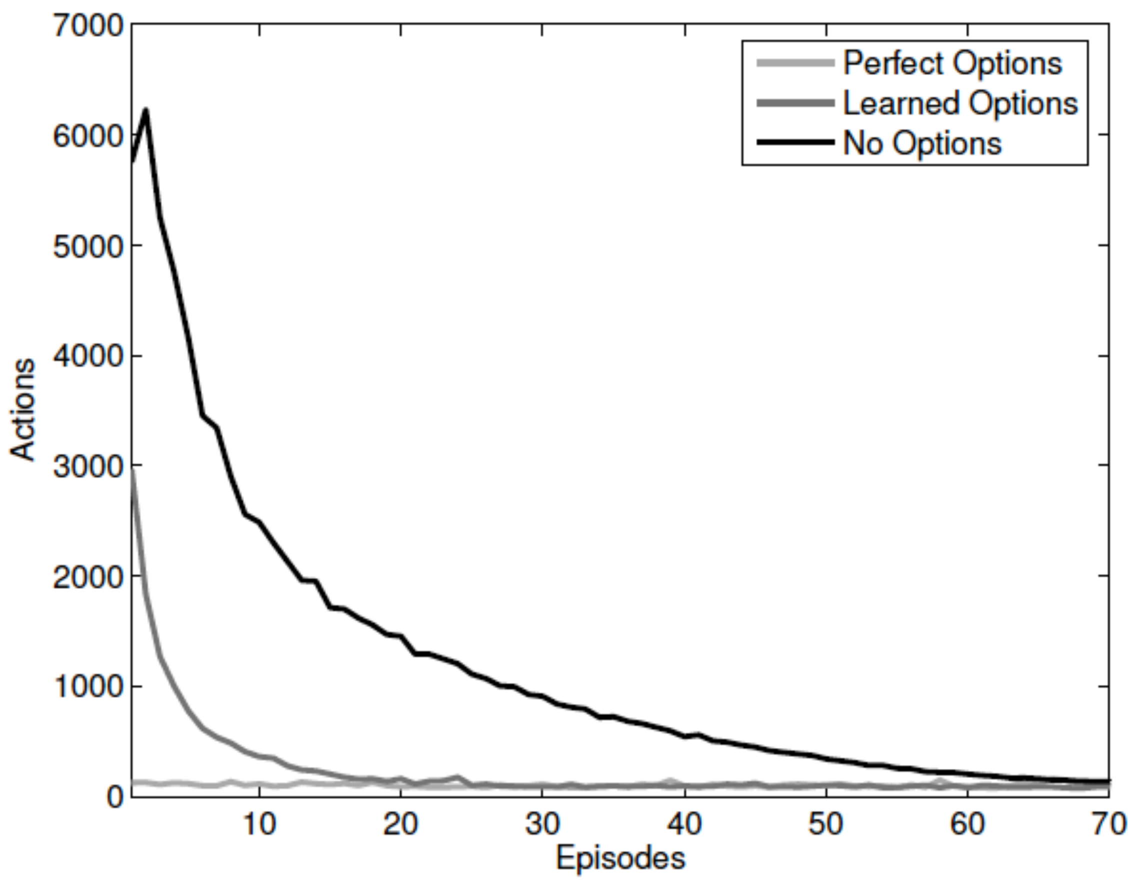
General principle: **subtasks recur.**

Transfer

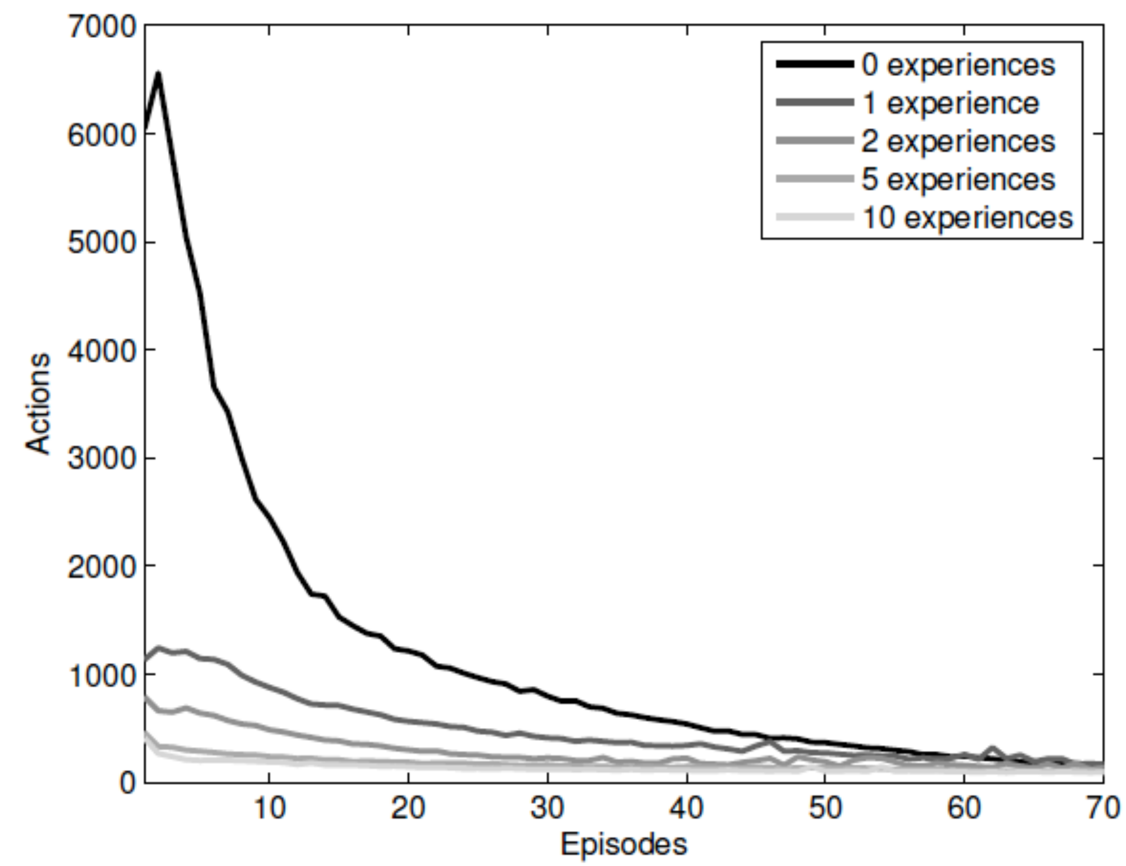
Tasks drawn from parametrized family.

- Common features present.
- Options defined using only common features.





(a) Learning curves for agents with problem-space options.



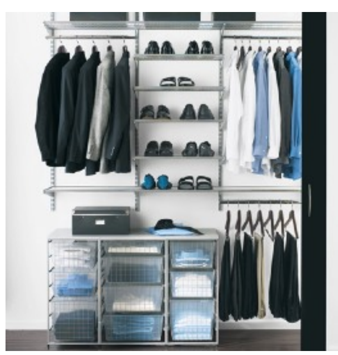
(b) Learning curves for agents with agent-space options, with varying numbers of training experiences.

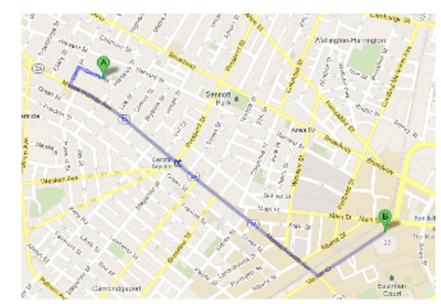
[Konidaris and Barto, IJCAI 2007]

Skill-Specific Abstractions

Options provide opportunities for abstraction

- Split high-dimensional problem into subproblems ...
- ... such that each one supports a solution using an abstraction.


Working hypothesis: *behavior is modular and compositional* ***and piecewise low-dimensional.***

Skill Acquisition

Skill Discovery

Where do skills come from?

Research goal: discover options autonomously, through interaction with an environment.

- Typically *subgoal options*.
- This means that we must determine β_o.
- Sometimes also R_o.

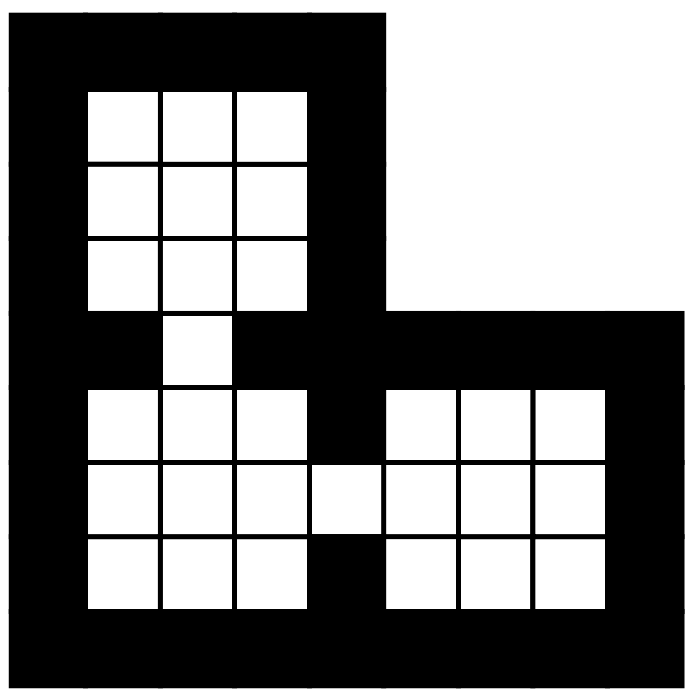
The question then becomes:

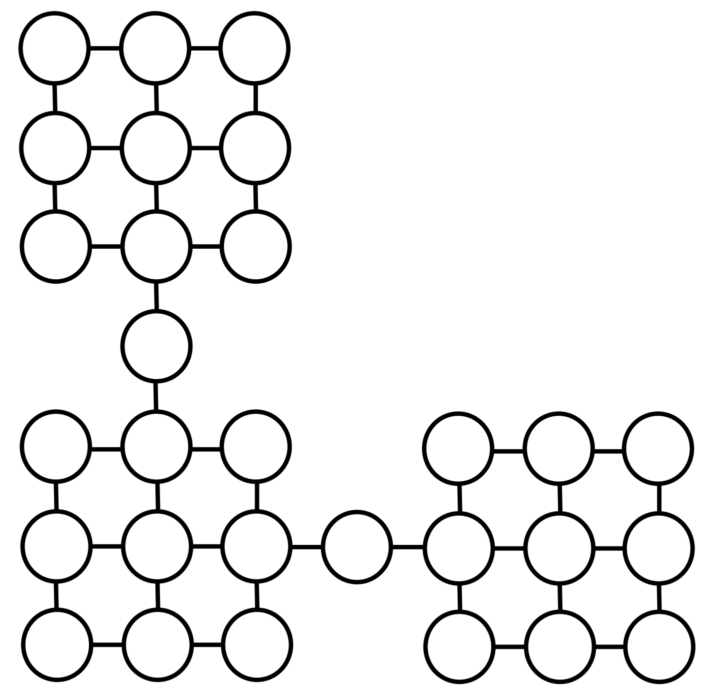
- Which states are good subgoals?

Betweenness Centrality

Consider an MDP as a graph.

- States are vertices.
- Edges indicate possible transition between two states.





Further, let us assume a task distribution over start states and goal pairs:

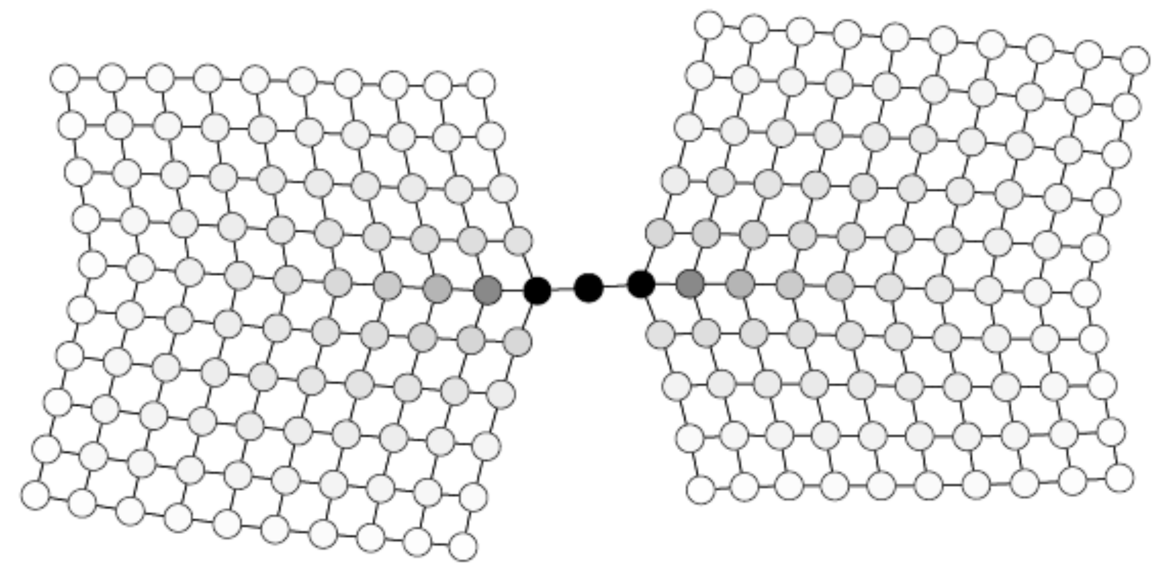
- $P_T(s, e)$

(Simsek and Barto, 2008)

Betweenness Centrality

We can define the *betweenness centrality* of a vertex (state) as:

$$\sum_{s,e} \frac{\sigma_{se}(v)}{\sigma_{se}} w_{se}$$



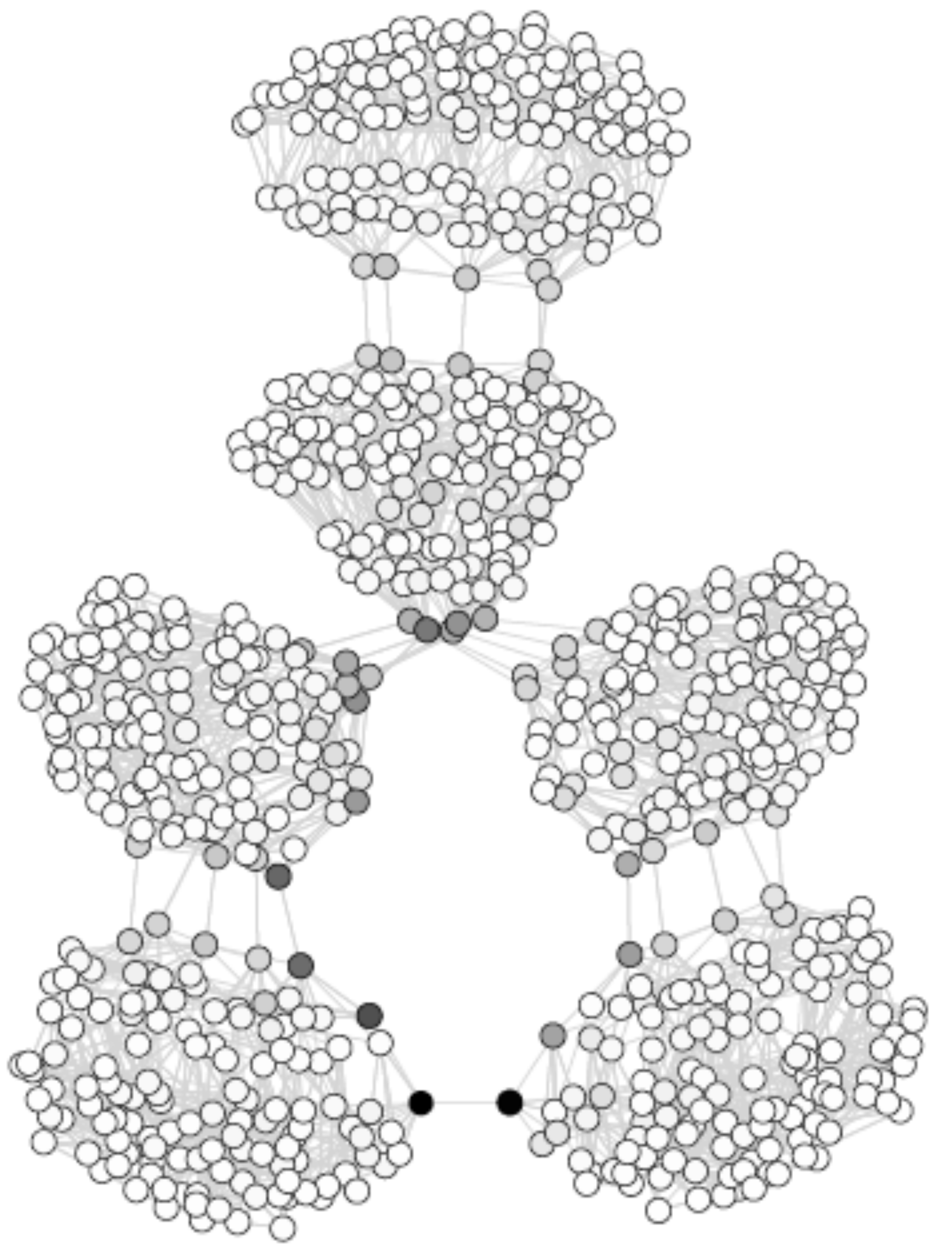
This indicates it probability of being on a shortest path from s to e; if we define:

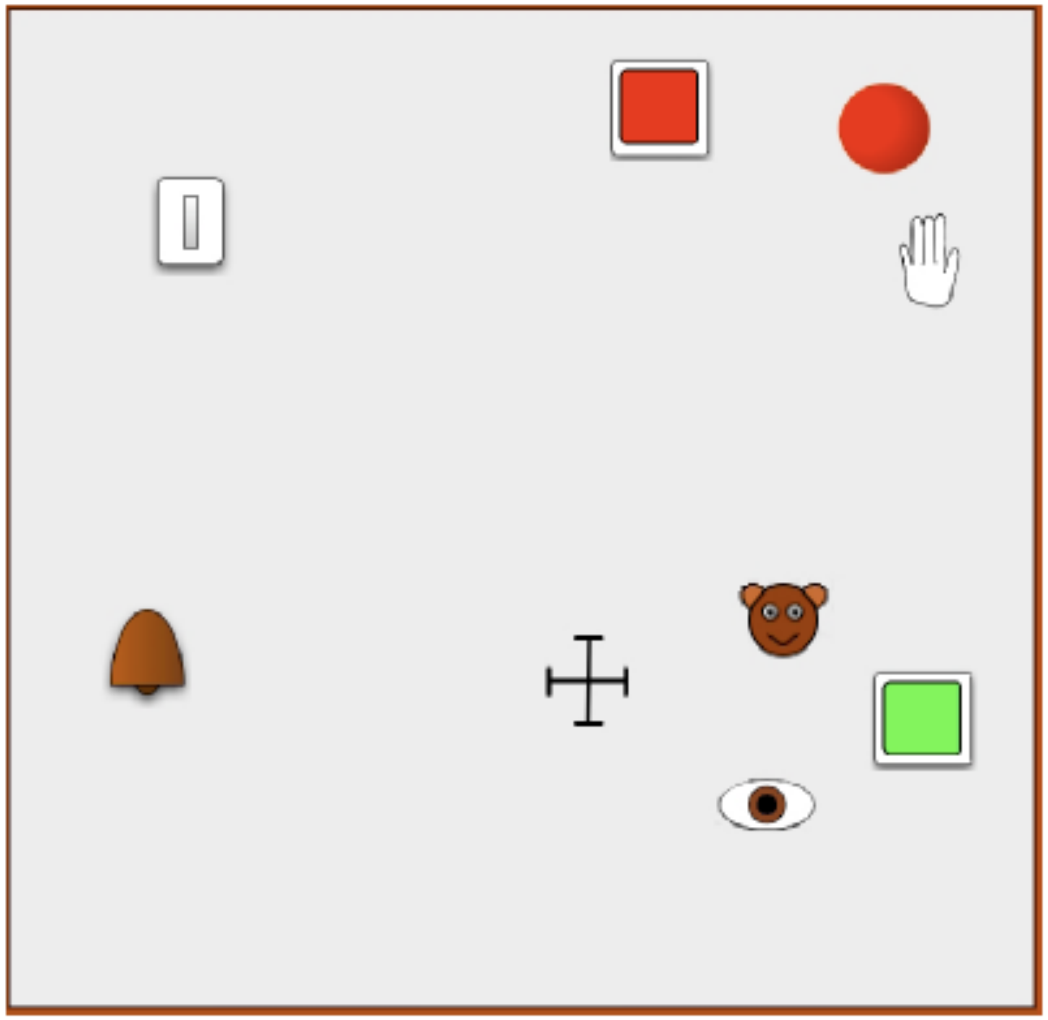
- *Shortest path* as *optimal solution*.
- $w_{se} = P_T(s, e)$

... then we get something sensible for RL.

(Simsek and Barto, 2008)

Betweeness Centrality





(Simsek and Barto, 2008)

Covering Options

More modern:

- Formulate a specific objective
- Find options with formal link to objective

E.g., finding options to aid with exploration:

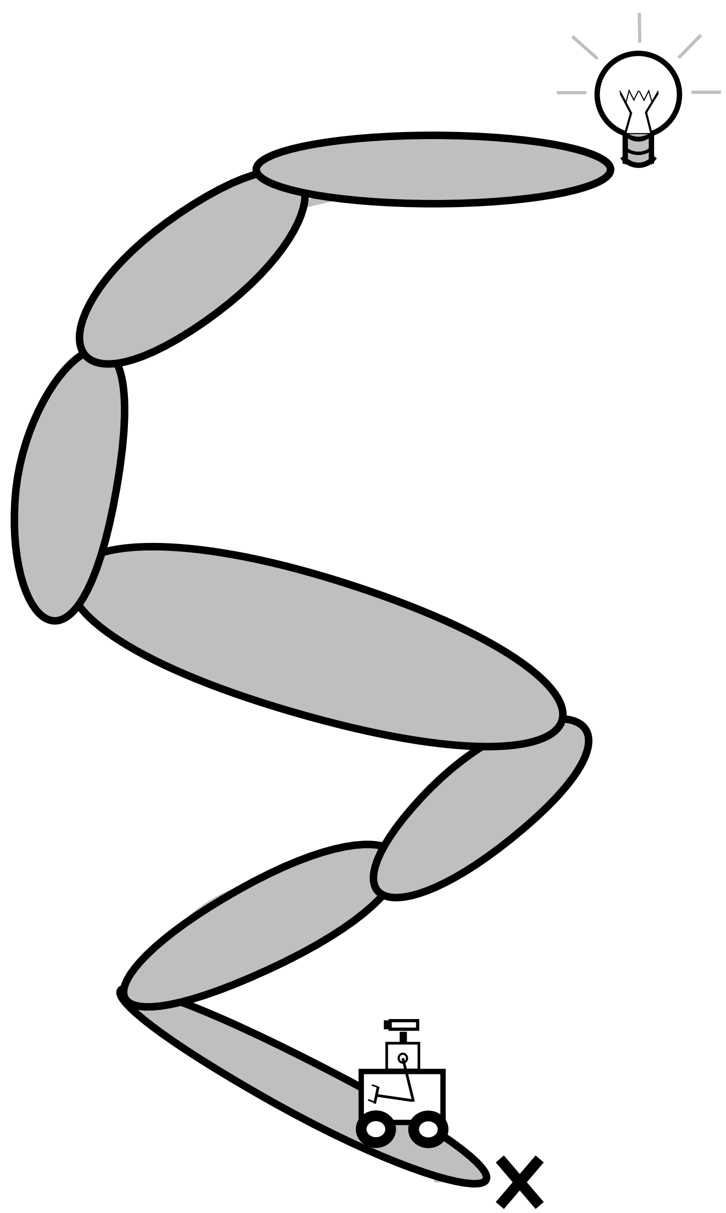
- *"the difficulty of discovering a distant rewarding state in an MDP is bounded by the expected cover time of a random walk over the graph induced by the MDP's transition dynamics"*

- Therefore, find options to minimize cover time.
- This is NP-Hard.
- Bounded-suboptimal approximation algorithm.

[Jinnai et al., 2019]

What About Continuous Domains?

Skill Chaining



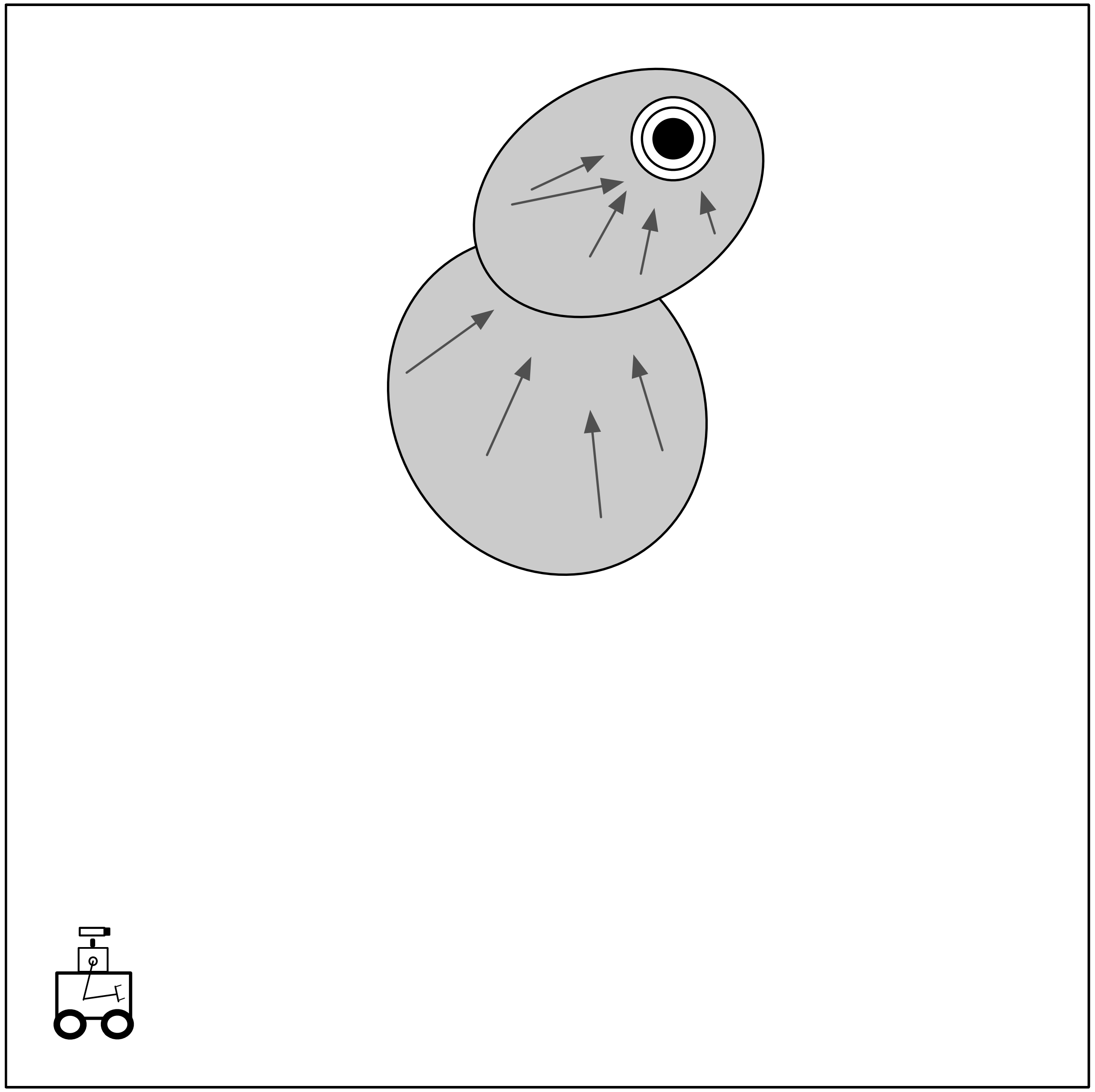
Executing one skill should either:

- Solve the problem.
- Let you execute another skill that could solve the problem.

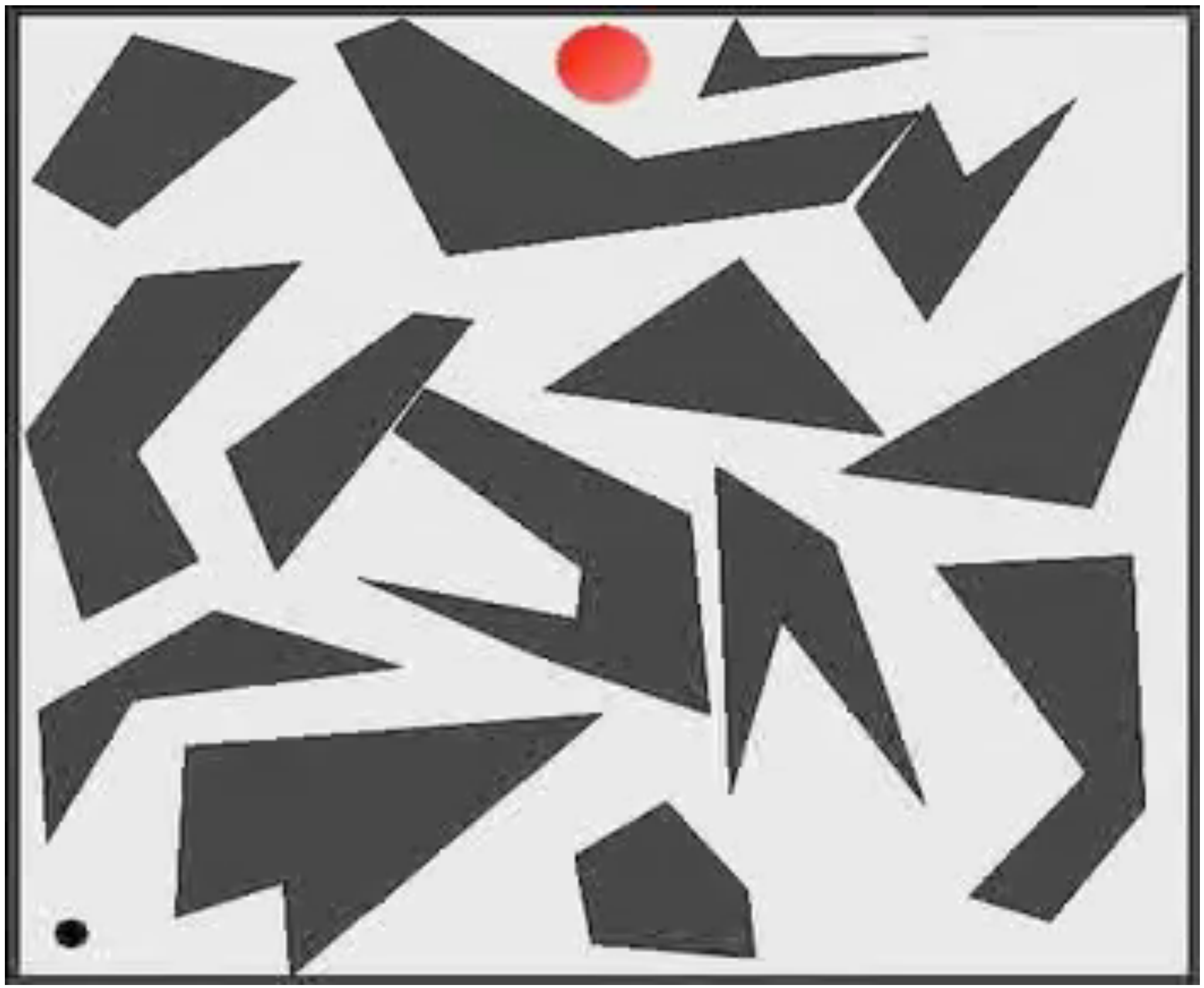
Skills should be *chainable*.

[Konidaris and Barto, NIPS 2009]

Skill Chaining

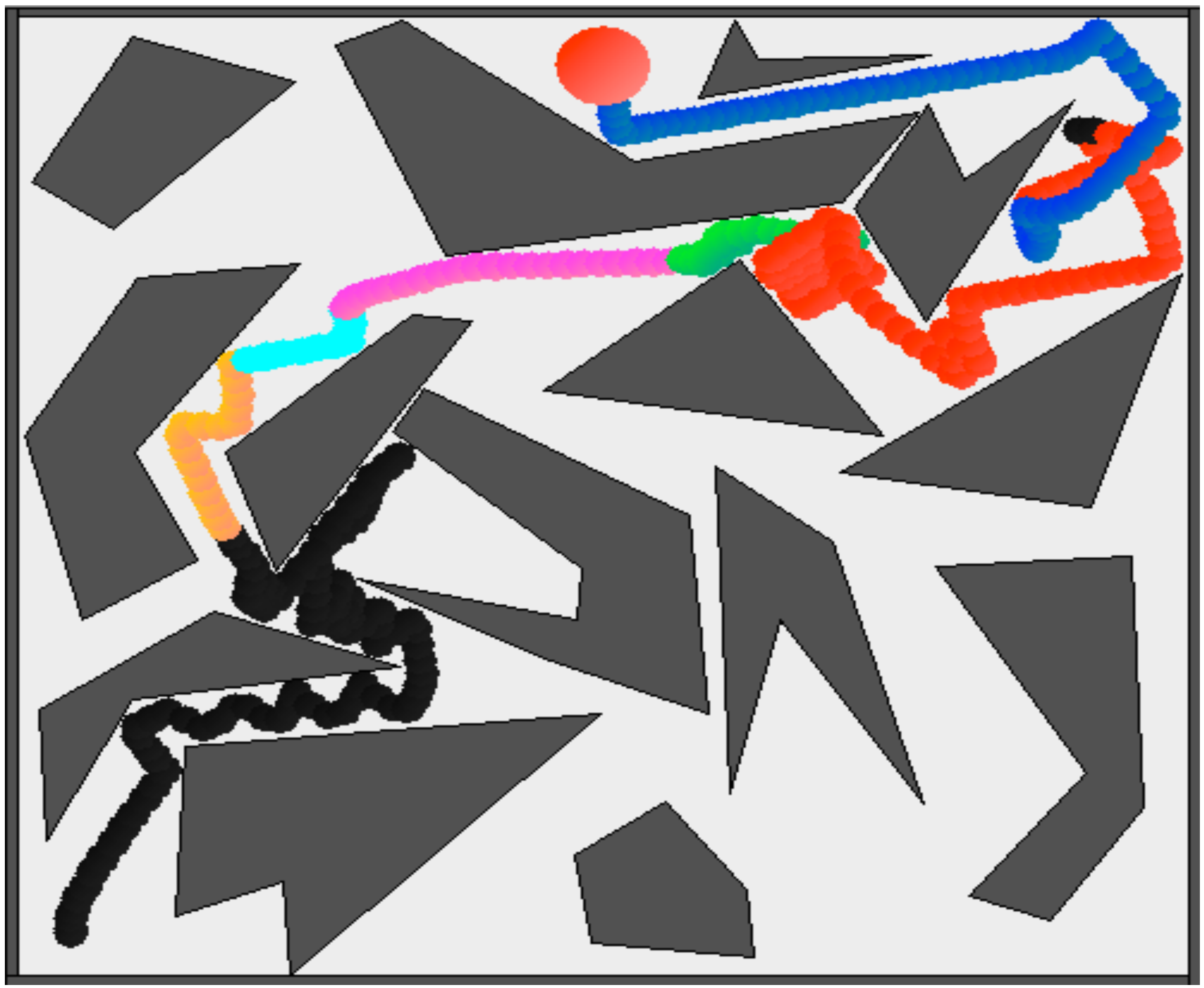


Skill Chaining: Results



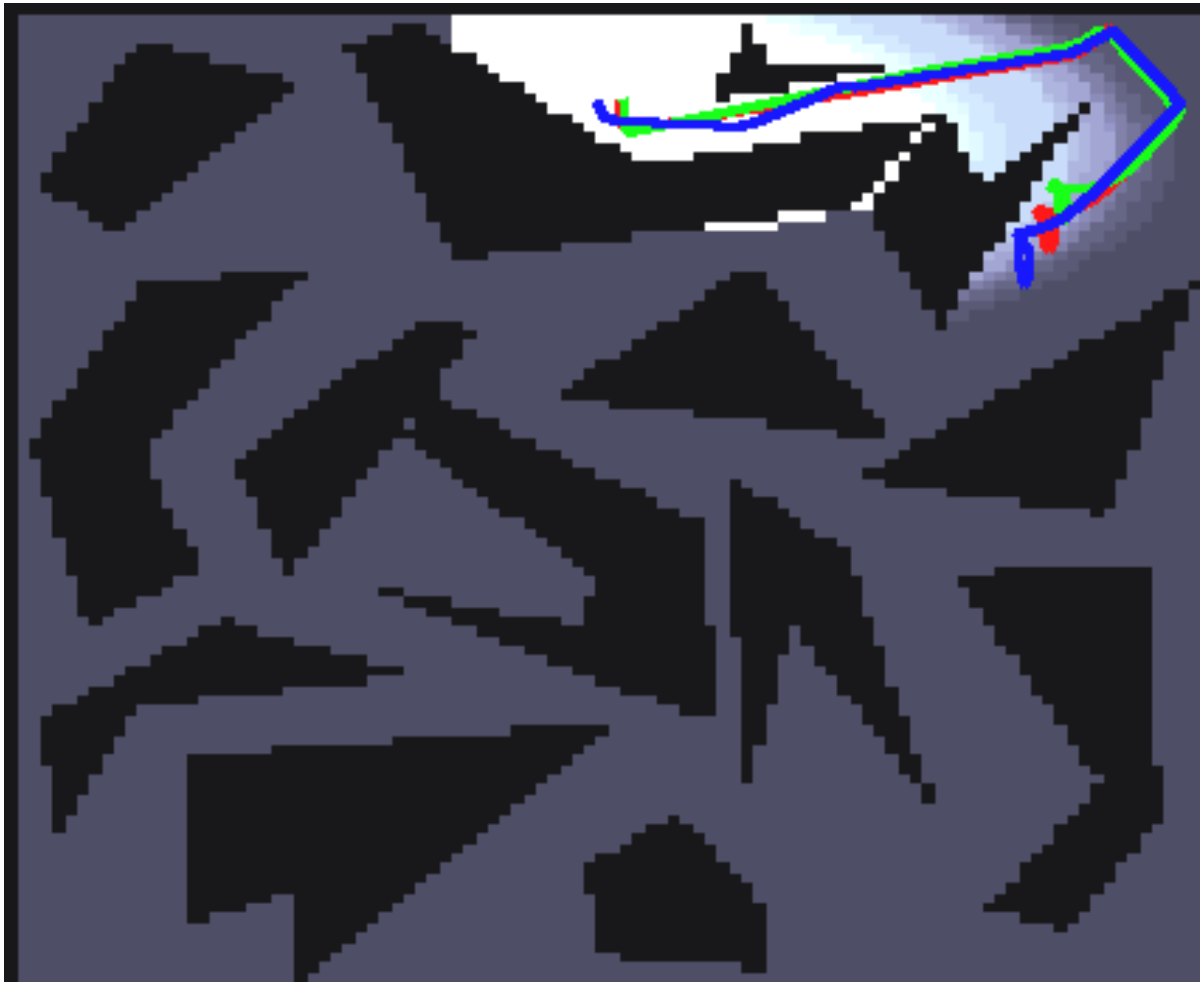
[NIPS 2009]

Skill Chaining: Results

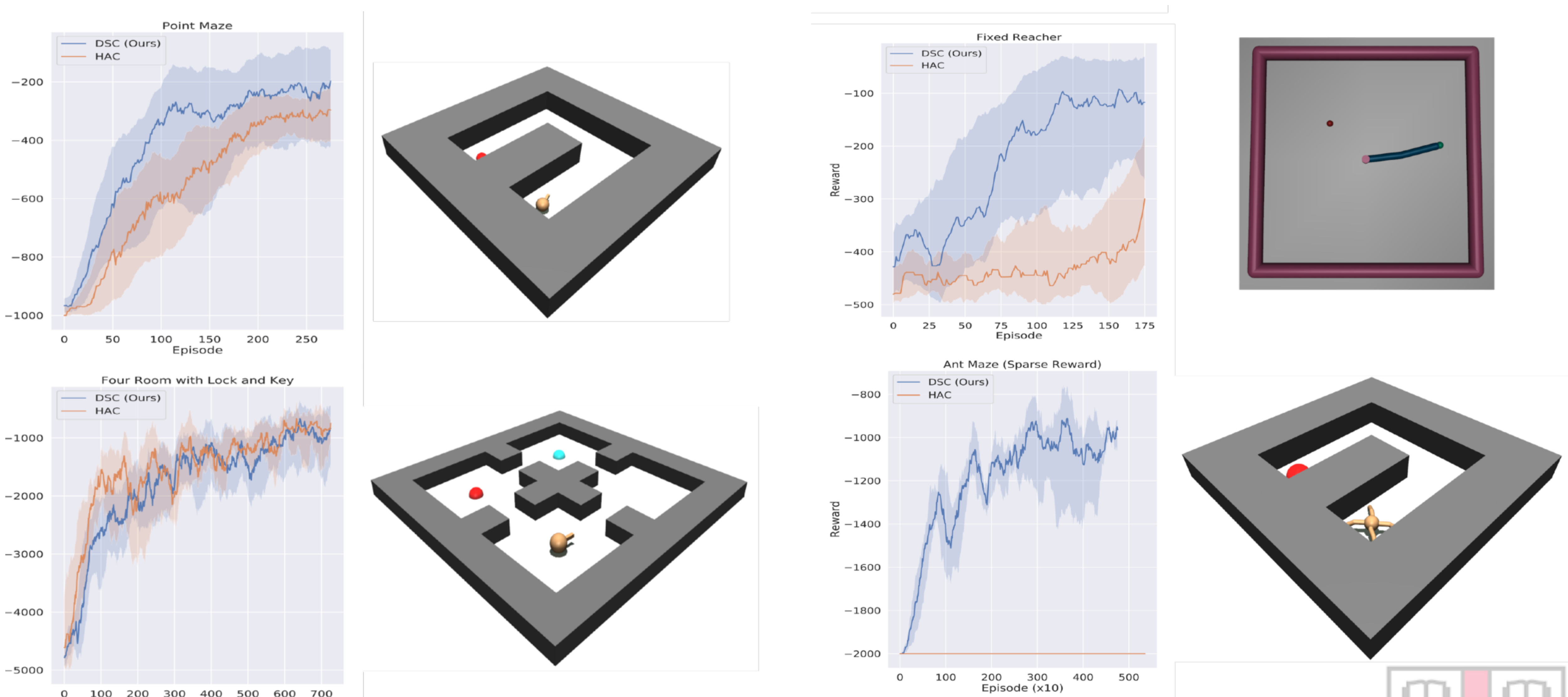


[NIPS 2009]

Skill Chaining: Results



Deep Skill Chaining



(b) (c)

[Bagaria and Konidaris, in submission]

Option-Critic (Bacon et al.)

- Use policy gradient to simultaneously learn high-level policy and option policies / termination conditions
- Assume options can be initiated anywhere
- Options are learned based directly on performance, rather than a heuristic!

Feudal Nets (Vezhnevits et. al)

- Manager module sets goals for the worker and receives environmental reward
- Worker module is rewarded for completing goals set by manager
- Learns to set and accomplish goals that are best for optimizing expected return — no heuristics