State Abstraction Discovery from Irrelevant State Variables

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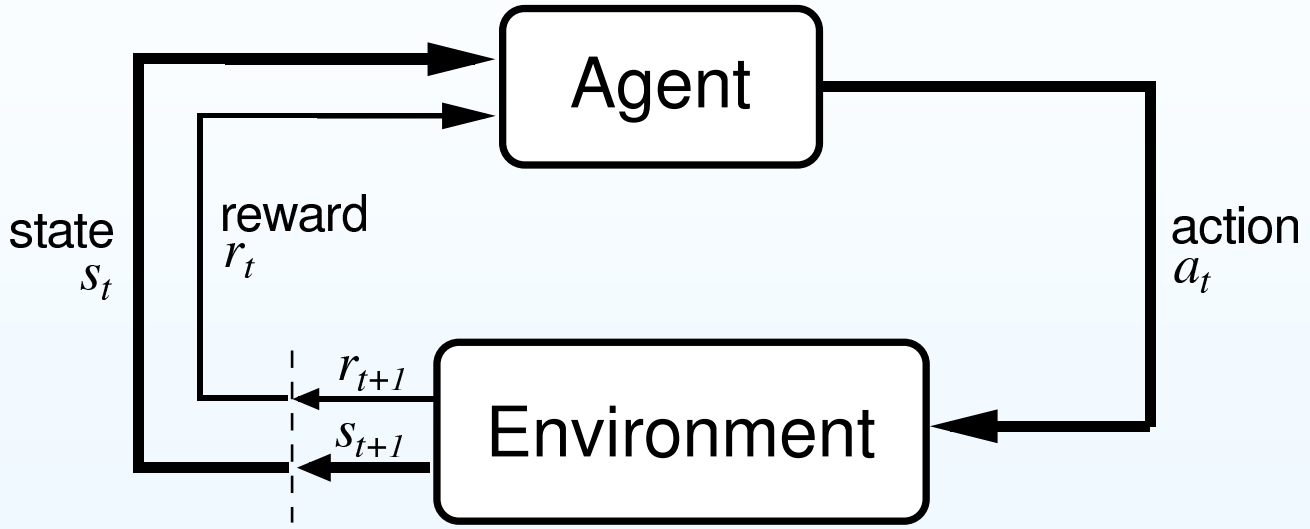
Reinforcement Learning



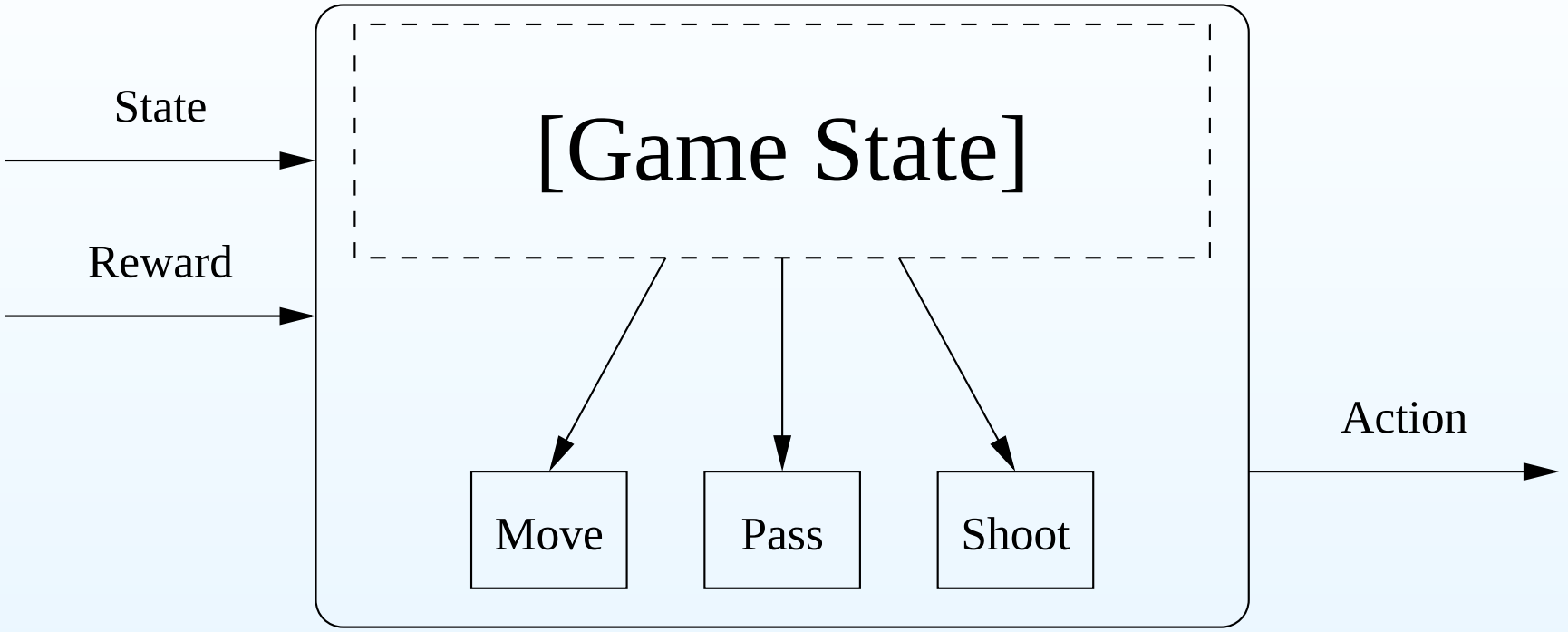
- Task: Maximize rewards in an unknown environment
- Only given: the state-action interface
- Much research: learn policies given an arbitrary interfaces
- Our research: discover interfaces that are easier to learn

Value-Based RL

Learn: a control policy

“What action should I choose in each state?”

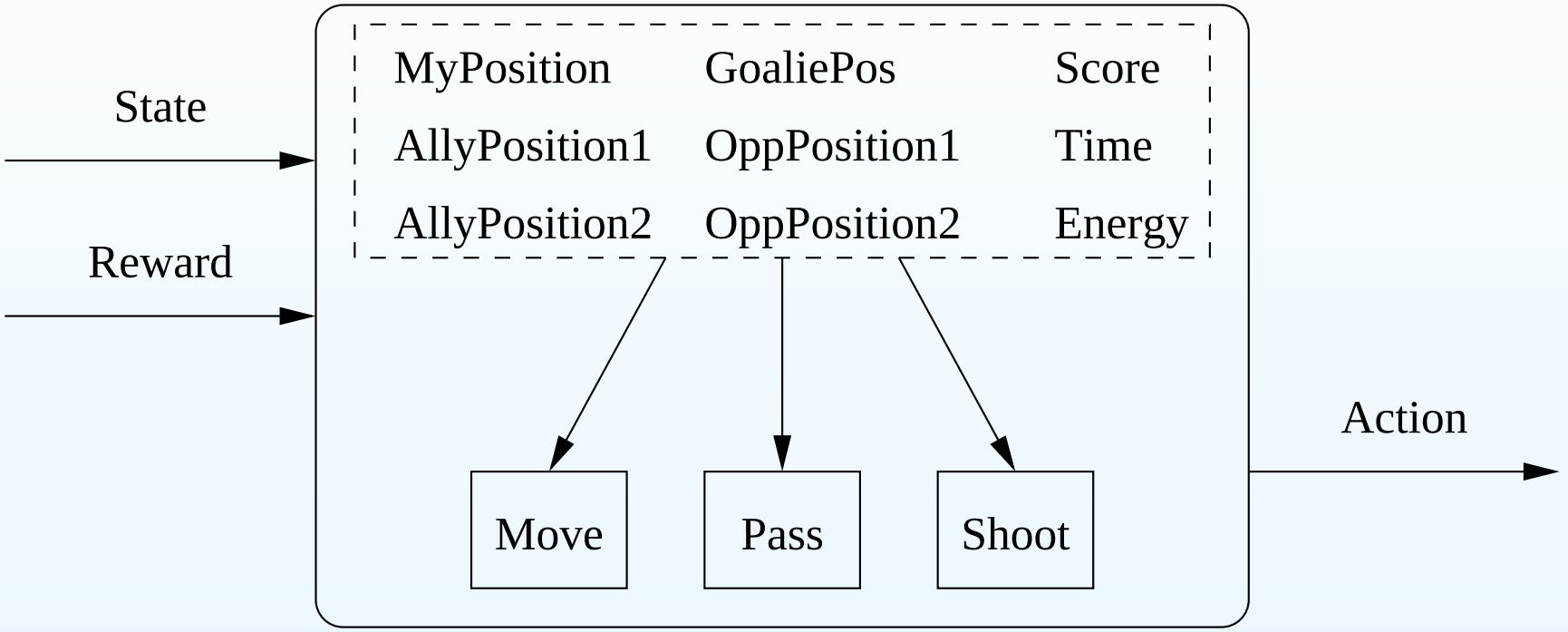
Value-Based RL



Learn: $Q : S \times A \rightarrow \mathbb{R}$

"How much reward can I earn starting at s by choosing a?"

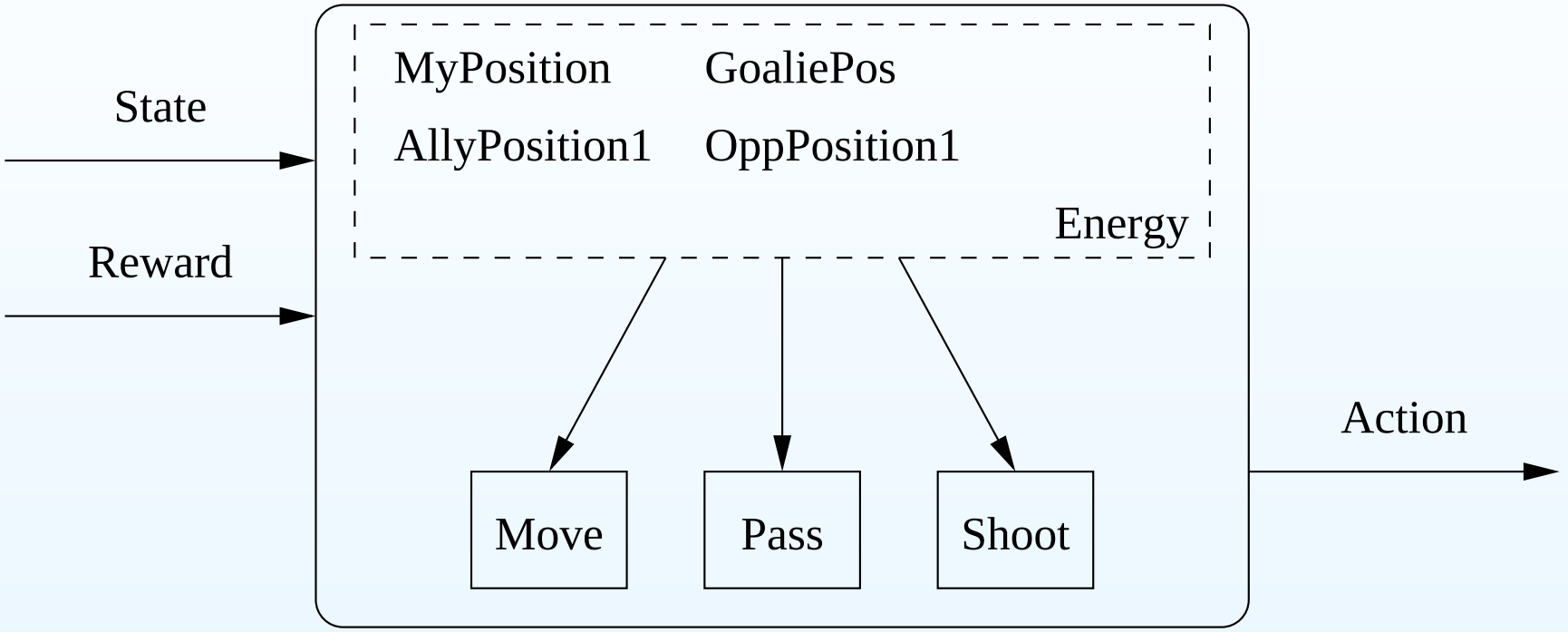
Value-Based RL



Learn: $Q : F_1 \times F_2 \times F_3 \times F_4 \times F_5 \times F_6 \times F_7 \times F_8 \times F_9 \times A \rightarrow \mathbb{R}$

In practice: high-dimensional state spaces

Value-Based RL



Learn: $Q : F_1 \times F_2 \times F_4 \times F_5 \times F_9 \times A \rightarrow \mathbb{R}$

State abstraction: ignore the irrelevant dimensions

State abstraction as qualitative knowledge

- Traditional sources of abstraction
  - Prior knowledge from a human
  - Computation from a given model

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  - Prior knowledge from a human
  - Computation from a given model
- Automatic discovery?
  - But discovering structure is harder than learning policies
  - Our approach: knowledge transfer

1. Discover abstractions in easy domains
2. Transfer abstractions to hard domains

Policy irrelevance: A new basis for state abstraction

When should we ignore a feature?

- Prior work
  - *... if the states share the same abstract one-step model.*
  - Requires the true model of the environment
  - Depends on the global abstraction

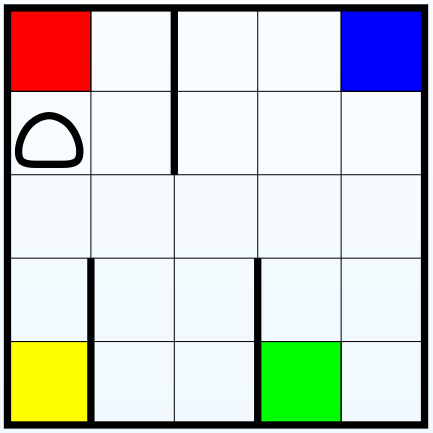
Policy irrelevance: A new basis for state abstraction

When should we ignore a feature?

- Prior work
  - *... if the states share the same abstract one-step model.*
  - Requires the true model of the environment
  - Depends on the global abstraction
- Our work
  - *... if the states share the same optimal action.*
  - Requires a learned policy for the environment
  - Independent of abstraction at other states

The Taxi domain

- Four features
  - Taxi x coordinate
  - Taxi y coordinate
  - Current passenger location
  - Passenger destination



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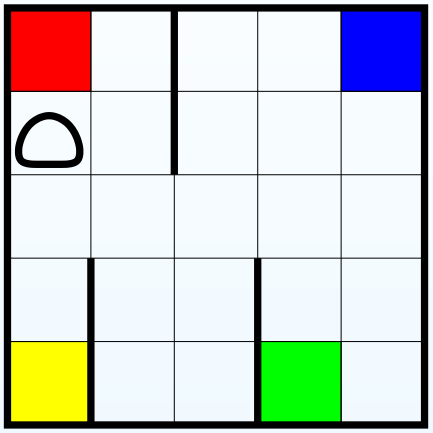

- Six actions: North, South, East, West, Pick Up, Put Down

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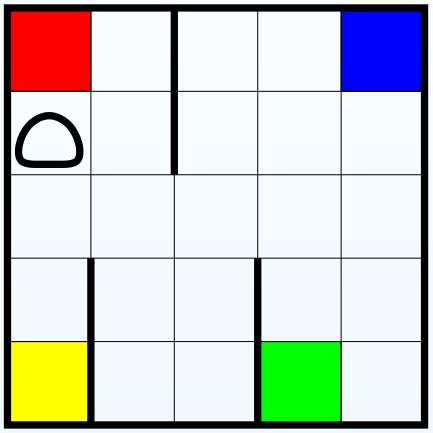

- Four features
  - Taxi x coordinate
  - Taxi y coordinate
  - Current passenger location
  - Passenger destination
- Six actions: North, South, East, West, Pick Up, Put Down
- Optimal policy:
  - Navigate to the passenger's location
  - Pick up the passenger
  - Navigate to the passenger's destination
  - Put down the passenger

Policy irrelevance in the Taxi domain

Relevance of the passenger destination. . .

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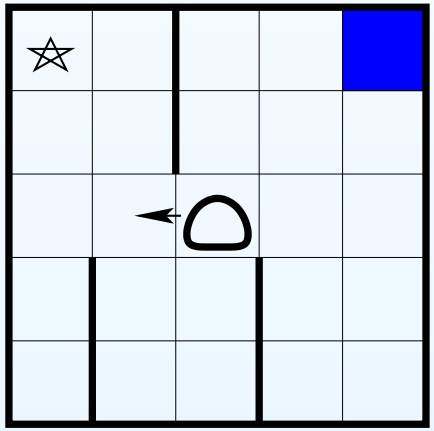
- When the passenger is not inside the taxi



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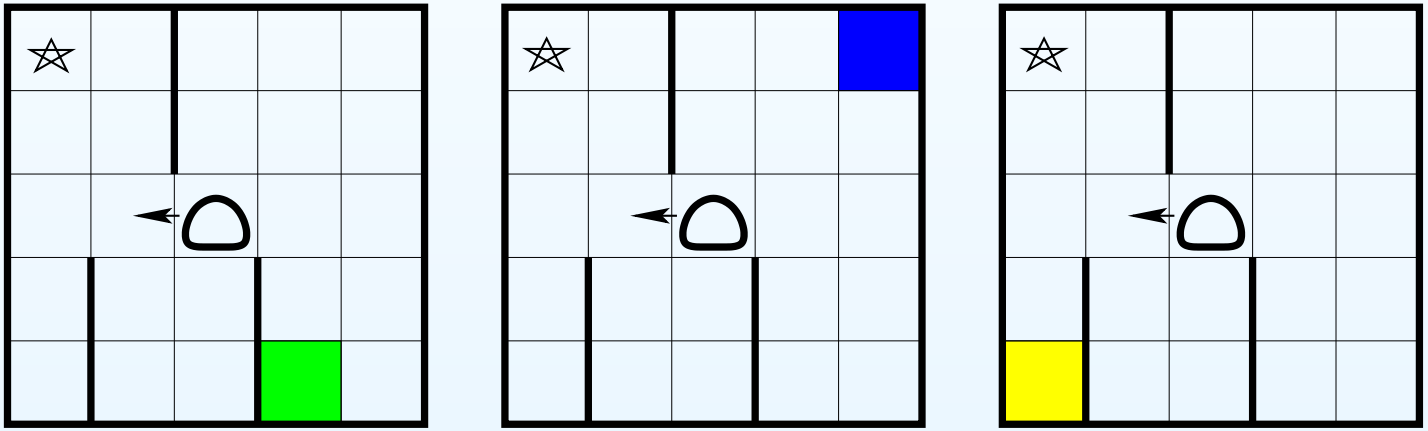

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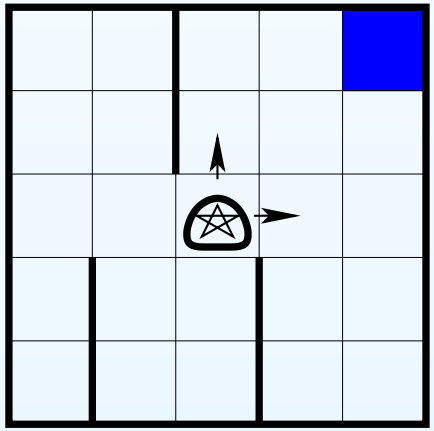

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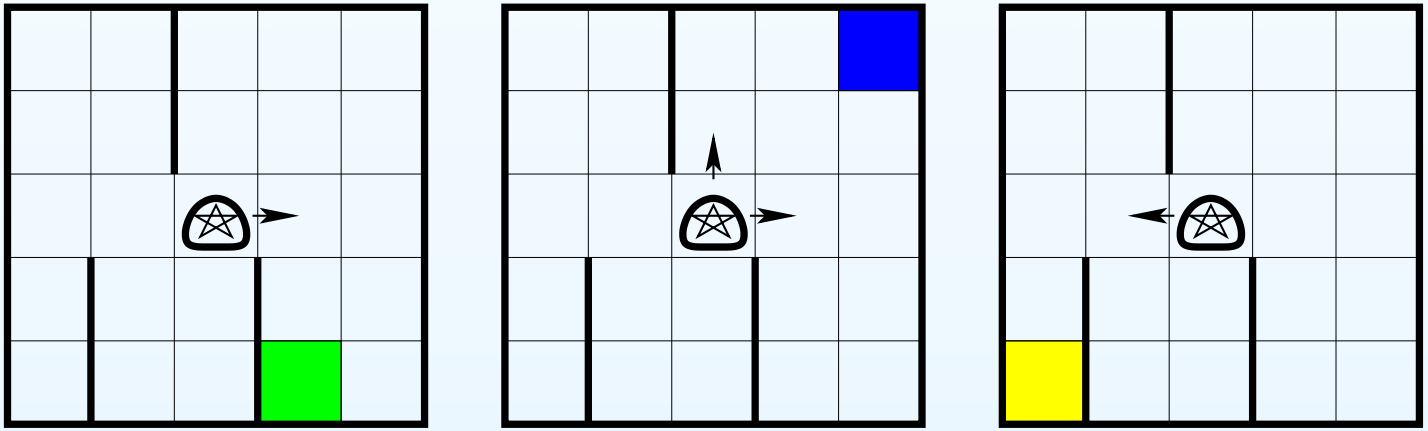

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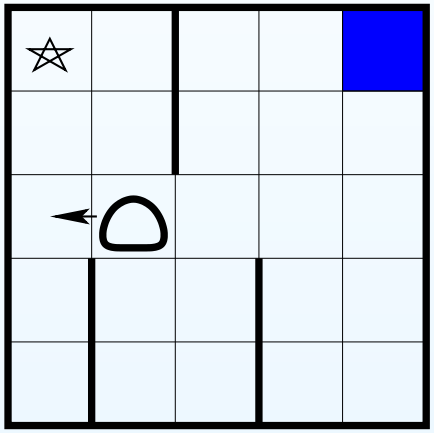

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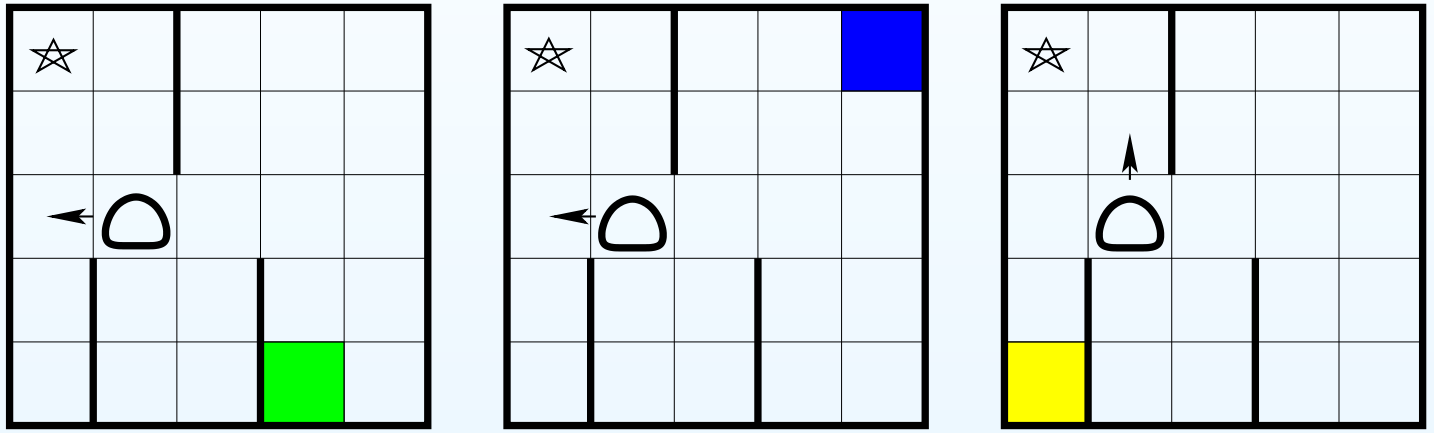

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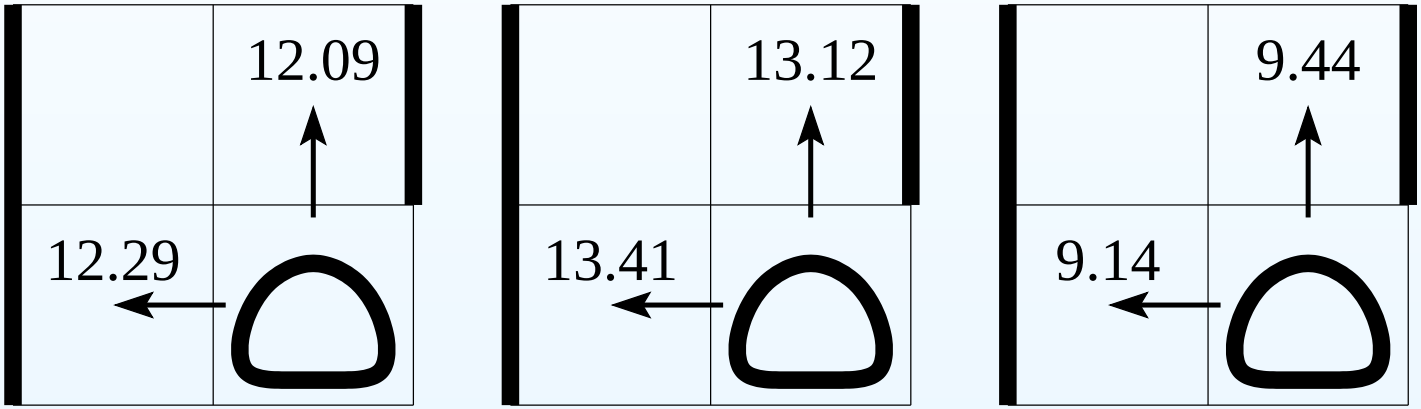

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Robust action-value comparison via sampling

$$Q(s', a) \stackrel{?}{\geq} Q(s', a')$$

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- Method 1: Statistical hypothesis testing
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  - Low computational but high sample complexity
- Method 2: Monte Carlo simulation
  - Construct a Bayesian model from an experience trace
  - Low sample but high computational complexity

Partial state abstractions

Are features F relevant at state s?

At what states is each set of features relevant?

- Train a binary classifier for certain sets of features
- Learn when each set of features is irrelevant
- Naive application: ignore F at classified states

Transferring abstractions to novel domains

- Sources of error for straightforward state aggregation
  - Statistical testing error
  - Generalization error of the learned classifiers
  - Novelty in the transfer domain
  - Disruption of value-function semantics!

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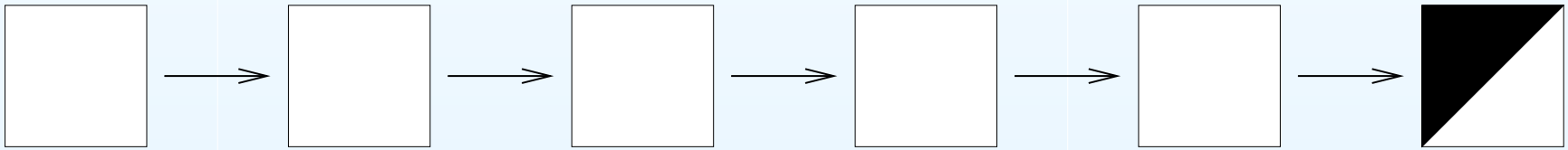

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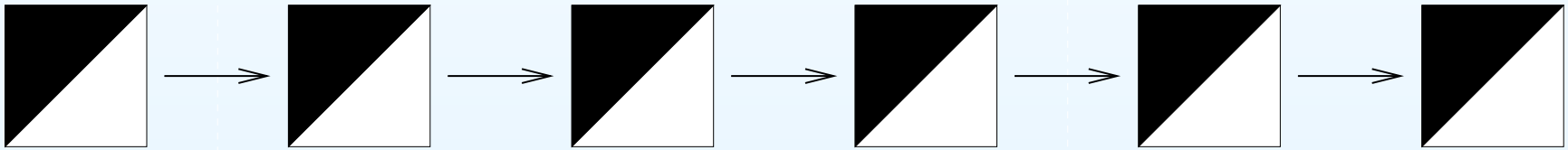

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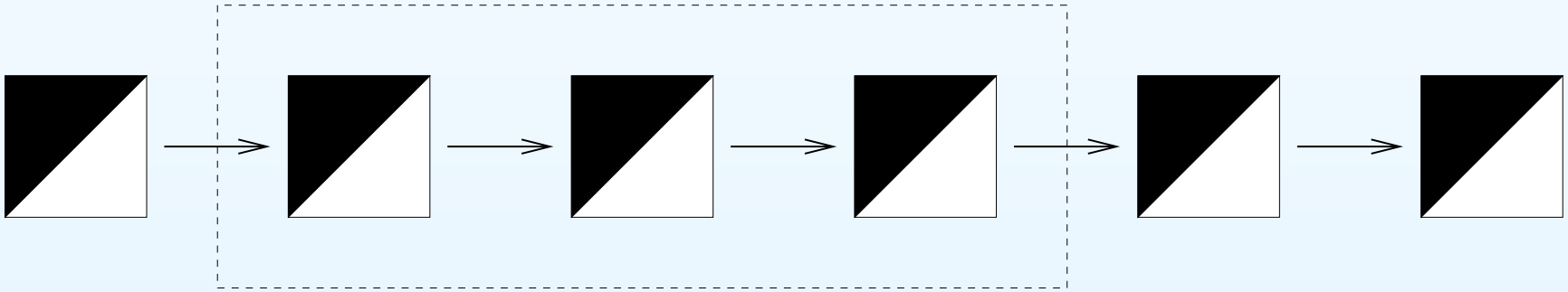

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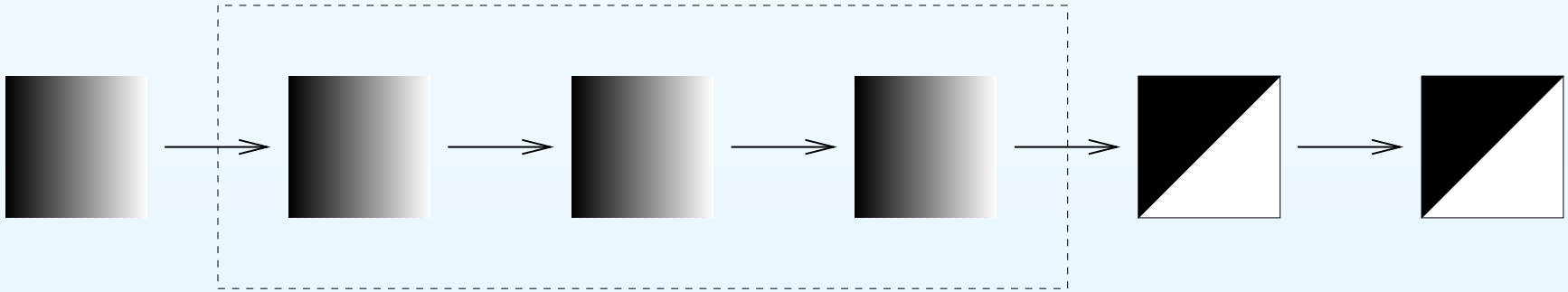

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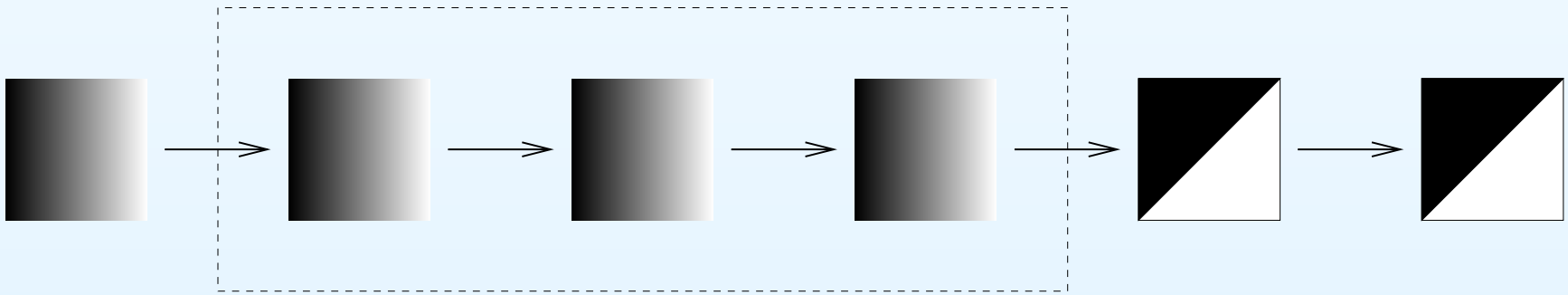

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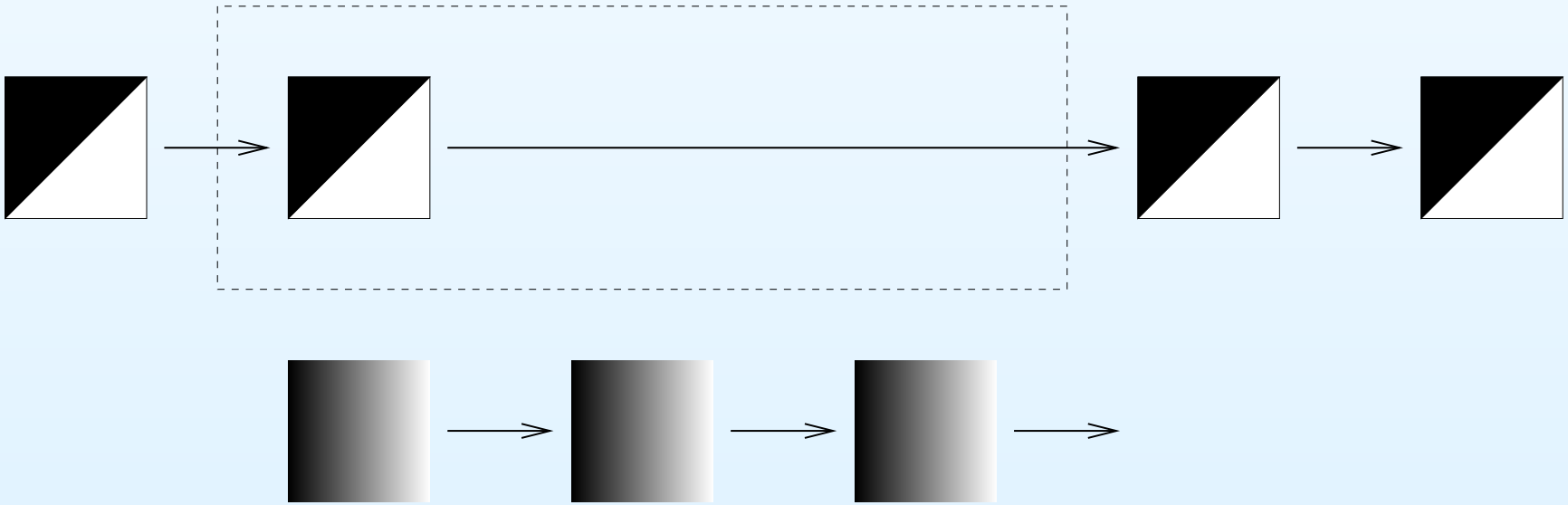

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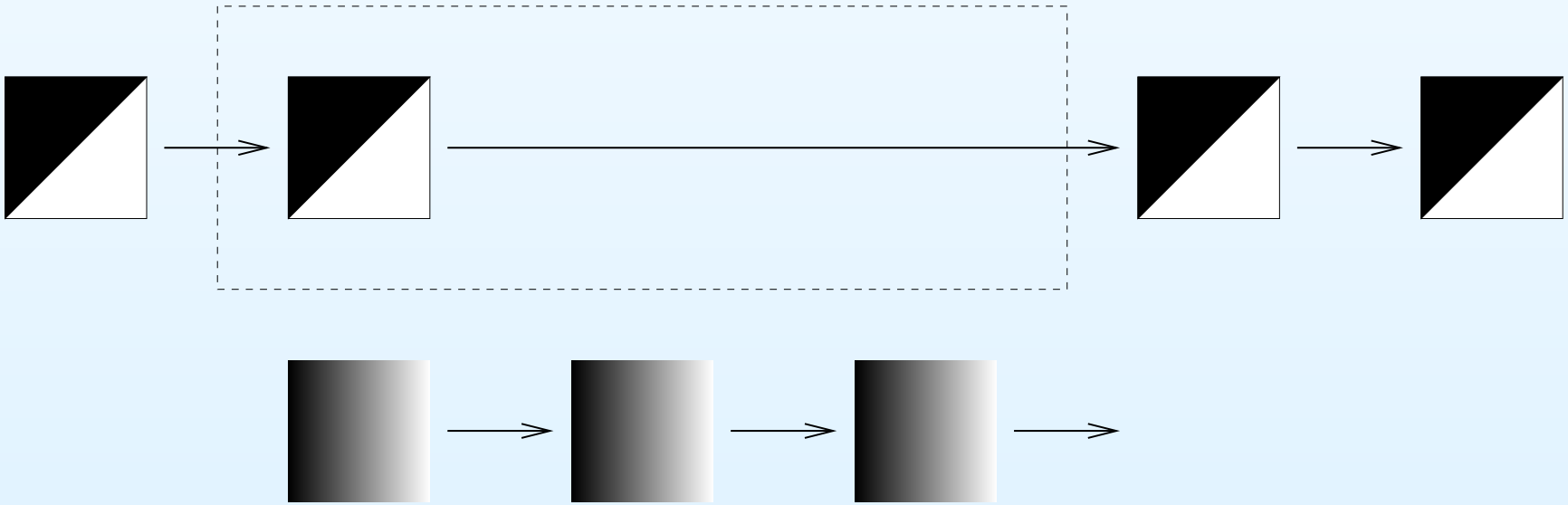
- Our research: “Ignore these features”



Temporal abstraction

- One abstract action comprises a sequence of actions
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- Prior research: “Achieve this subgoal state”
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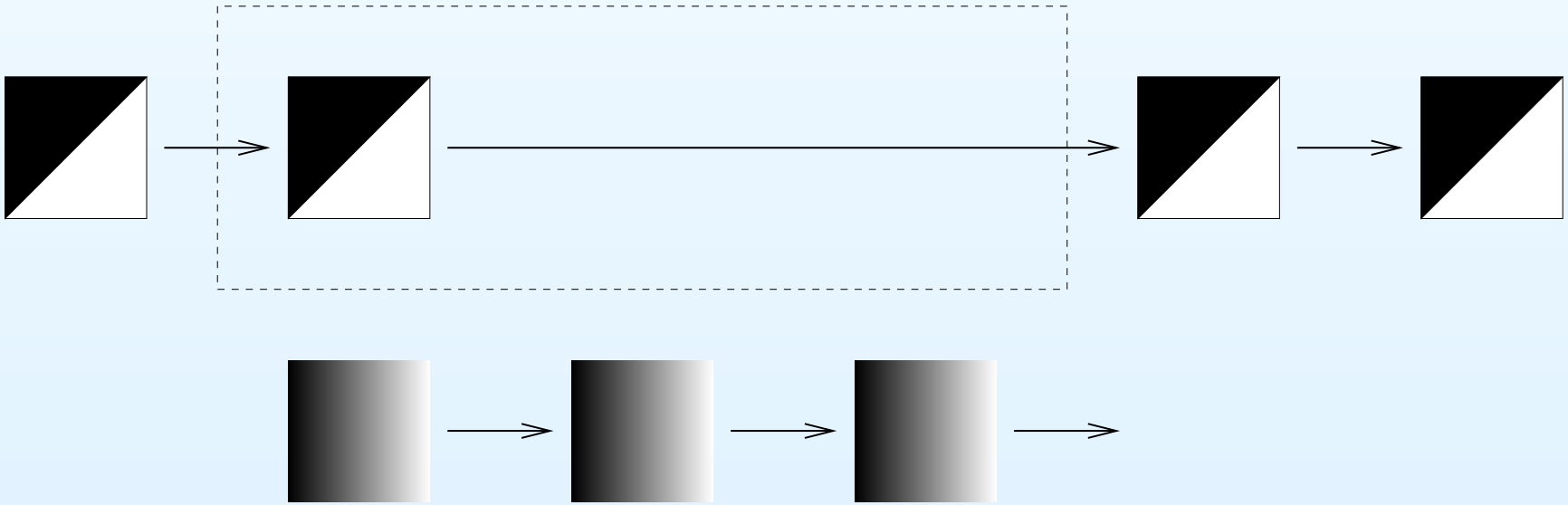
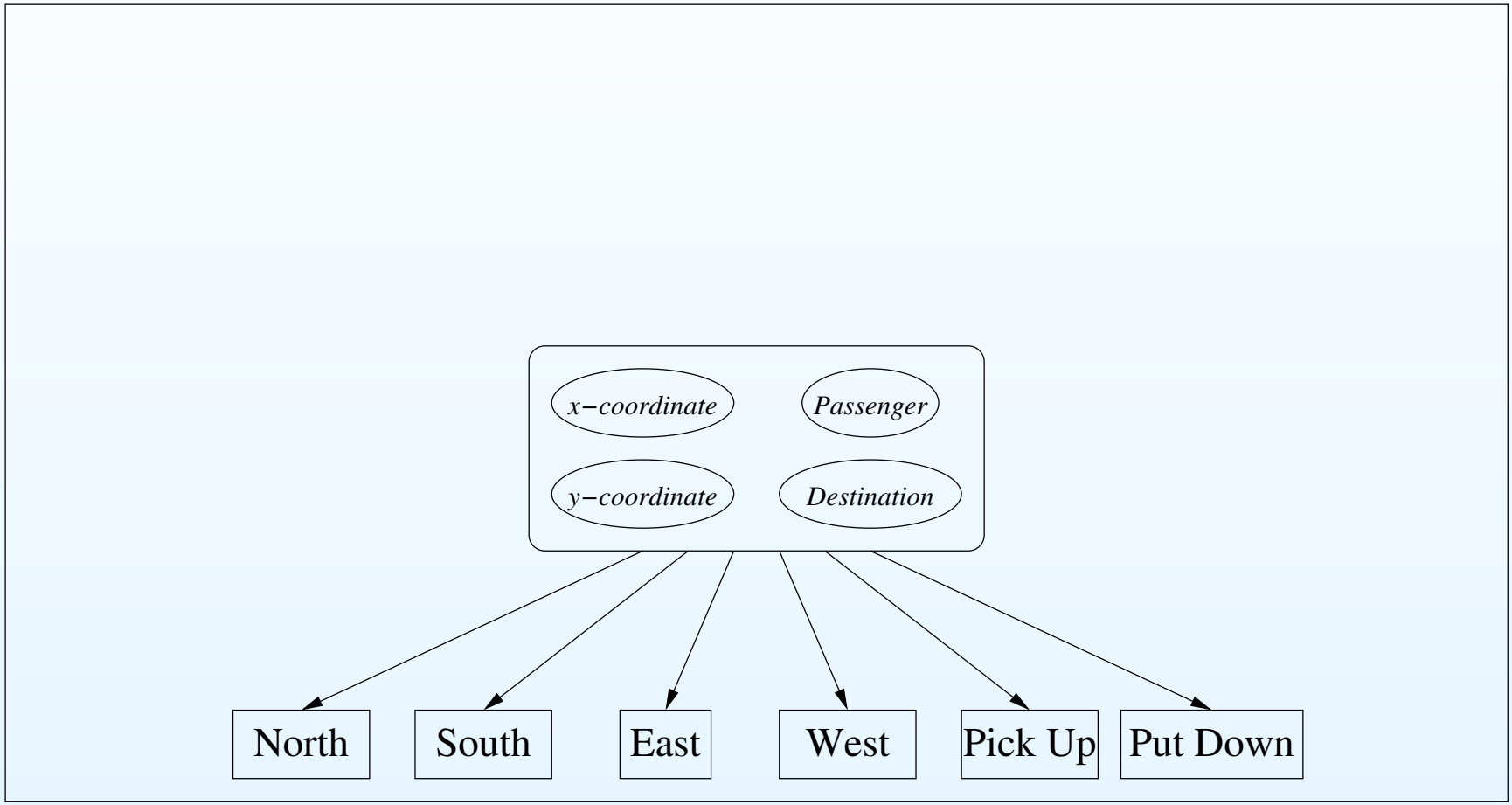
- Safe encapsulation of state abstractions into actions
- Learn when to apply discovered state abstractions!



Hierarchies of state and temporal abstractions



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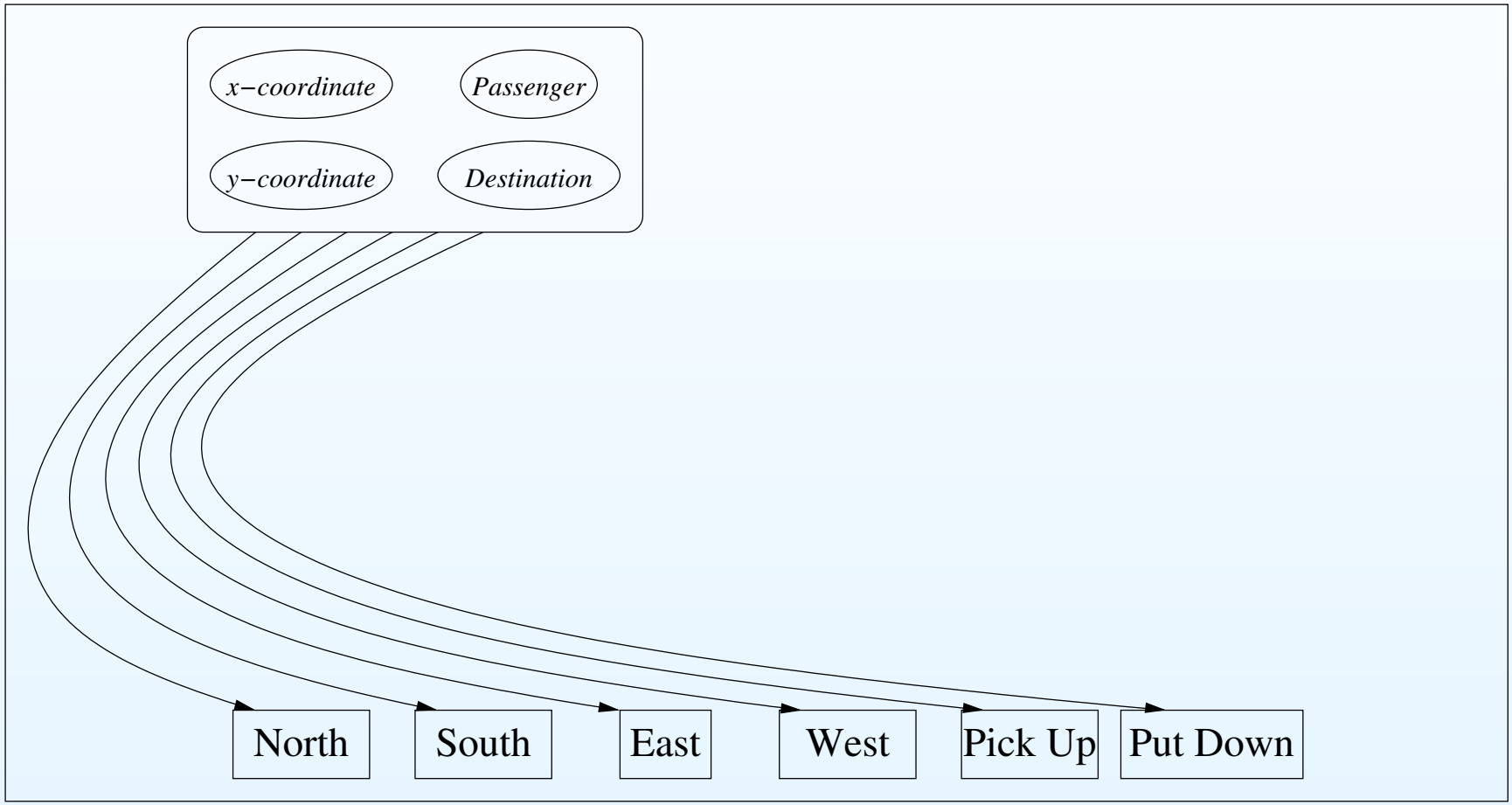

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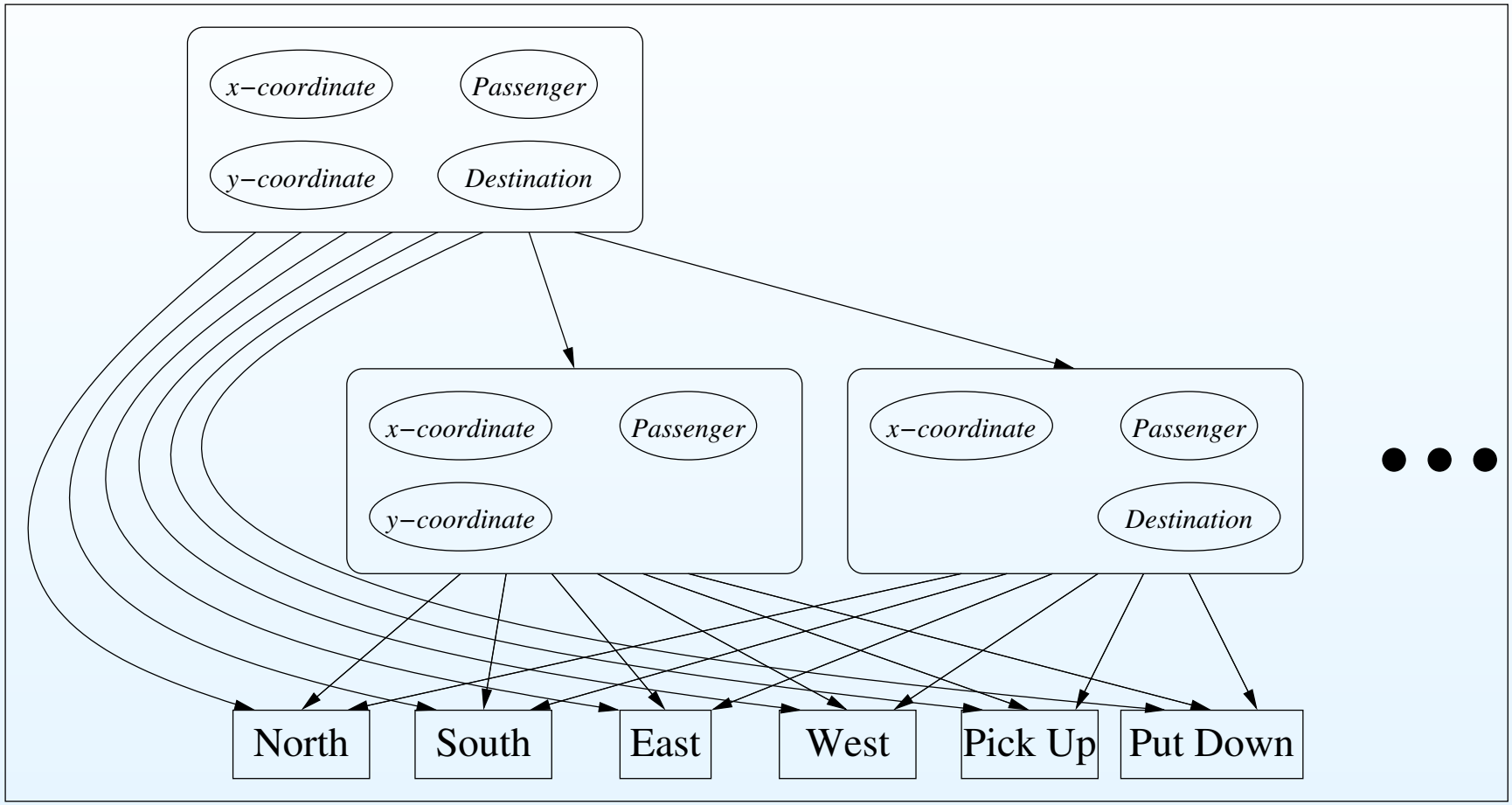

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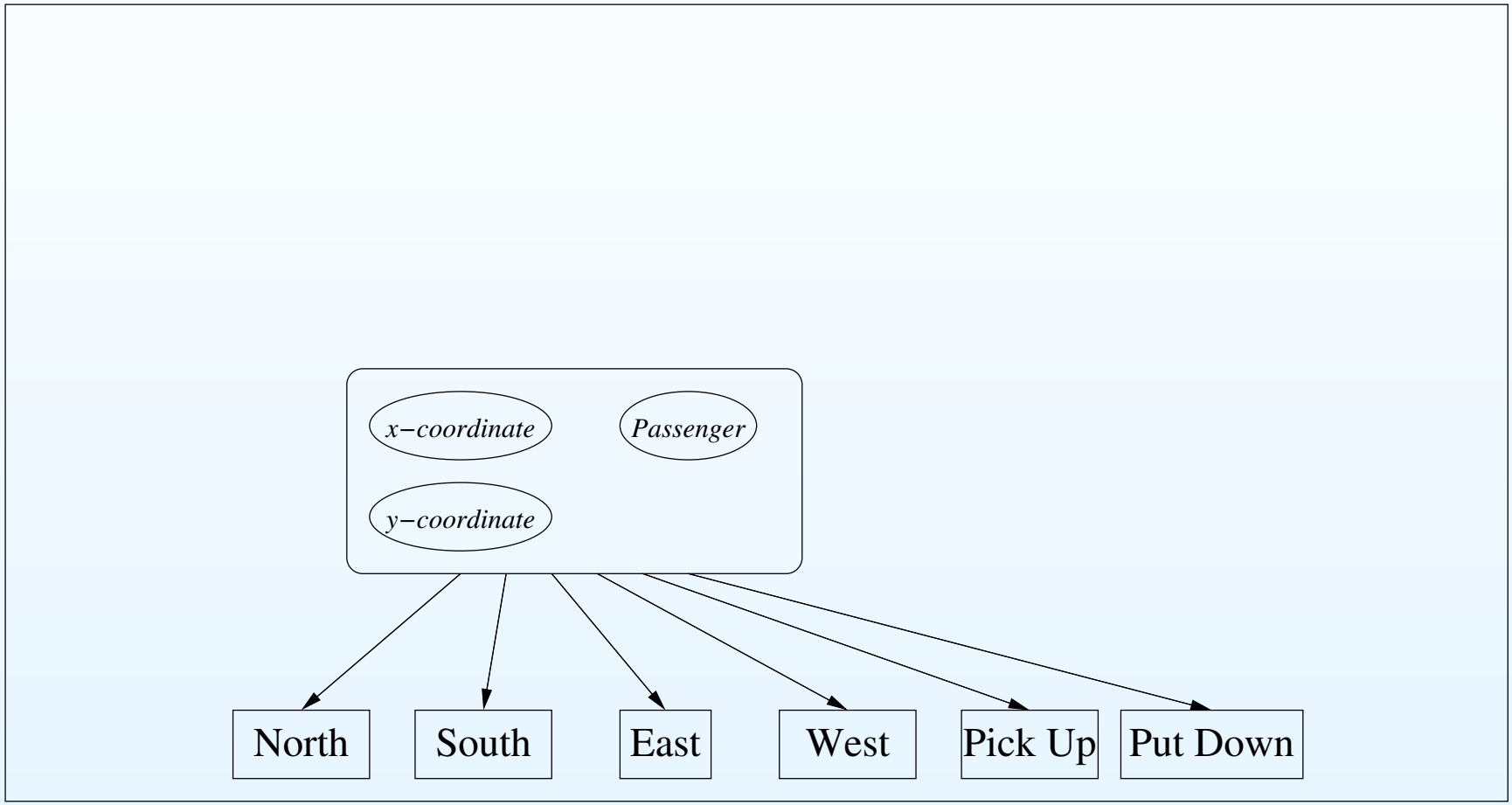
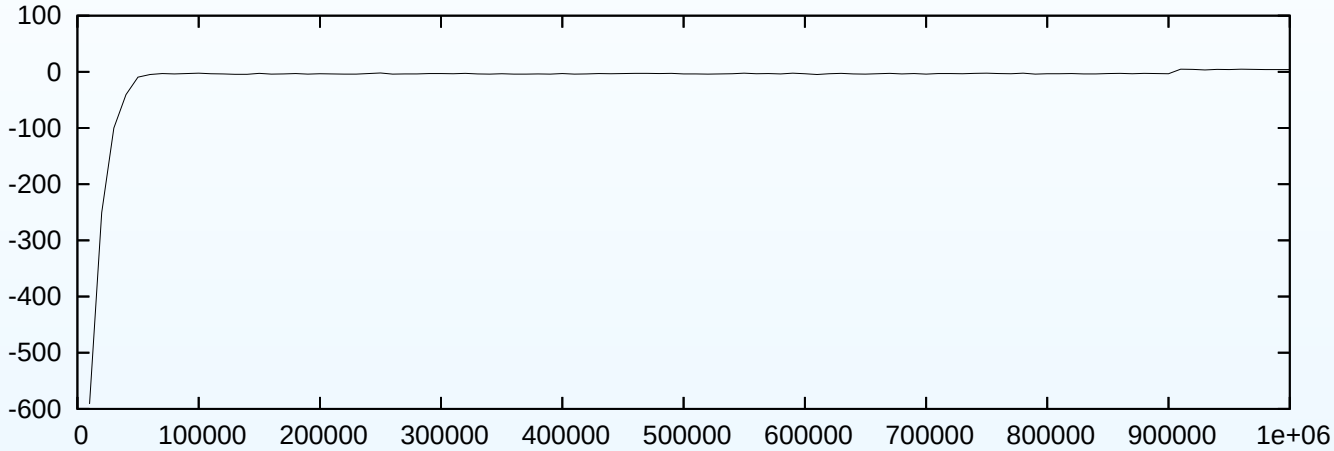


Results in the Taxi domain

- Original 5×5 domain



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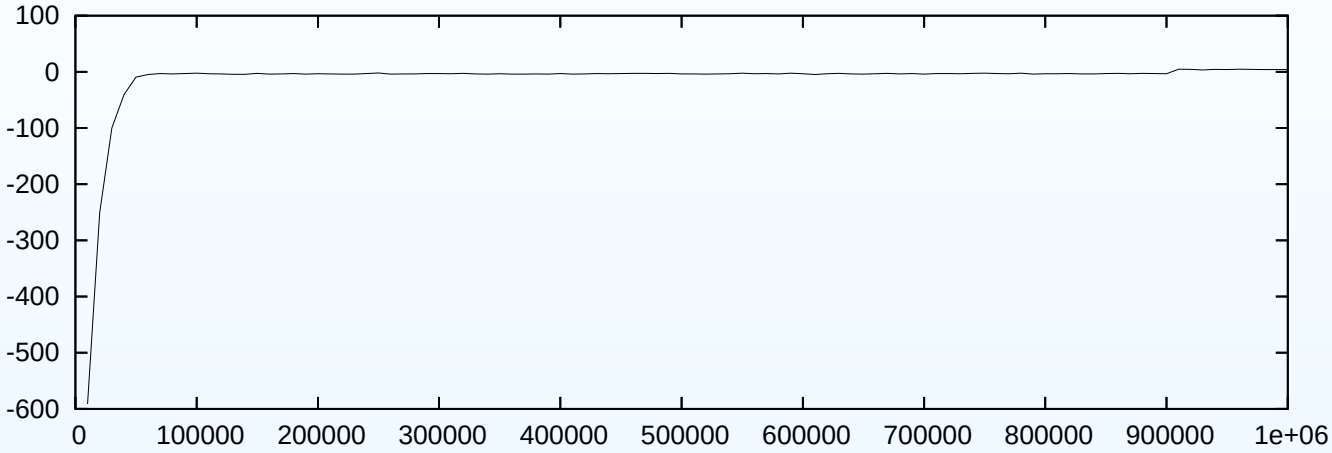
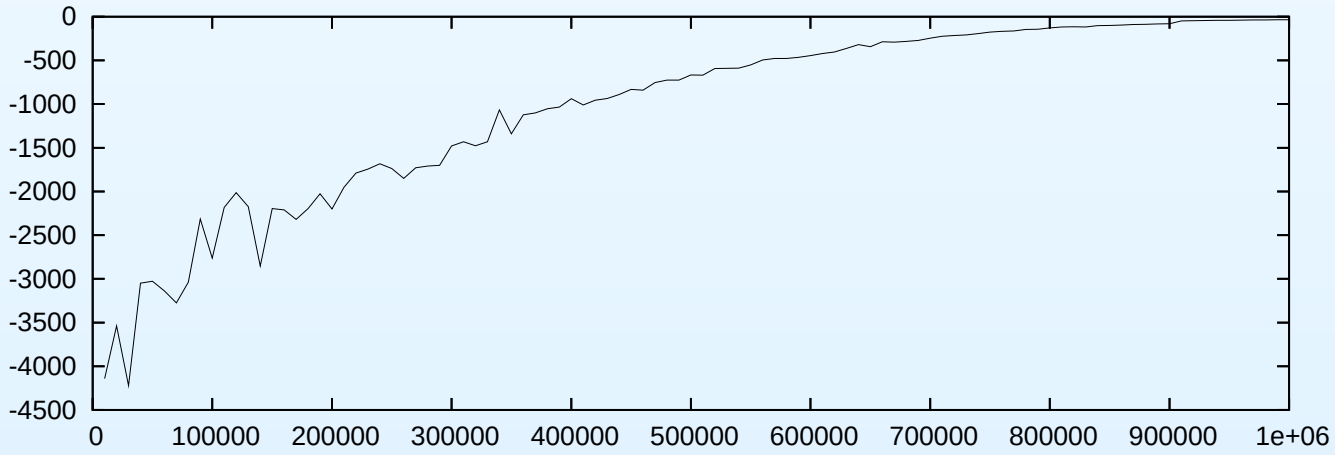


- Randomly generated 10×10 domain



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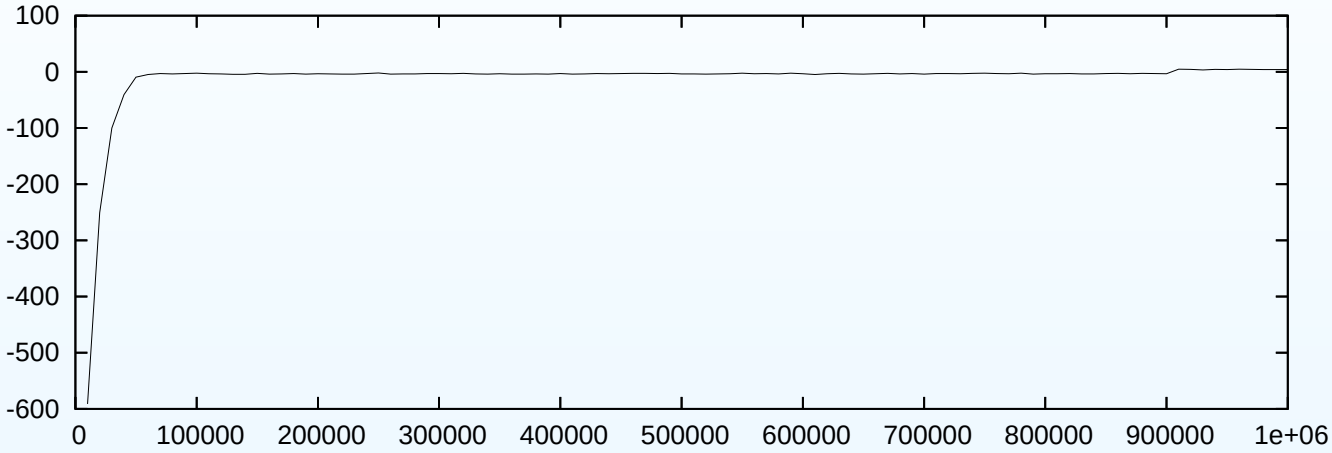

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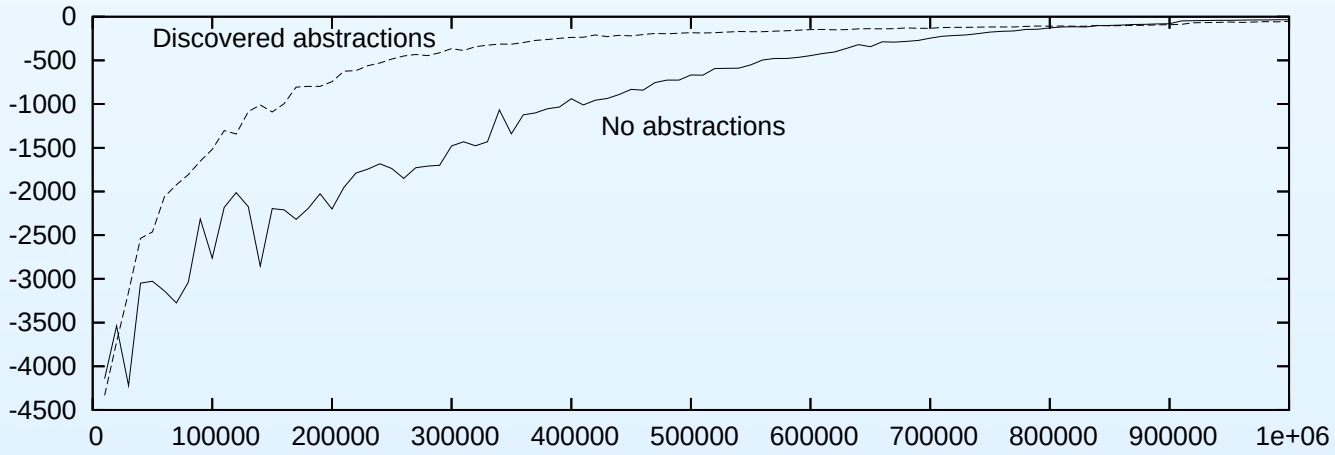


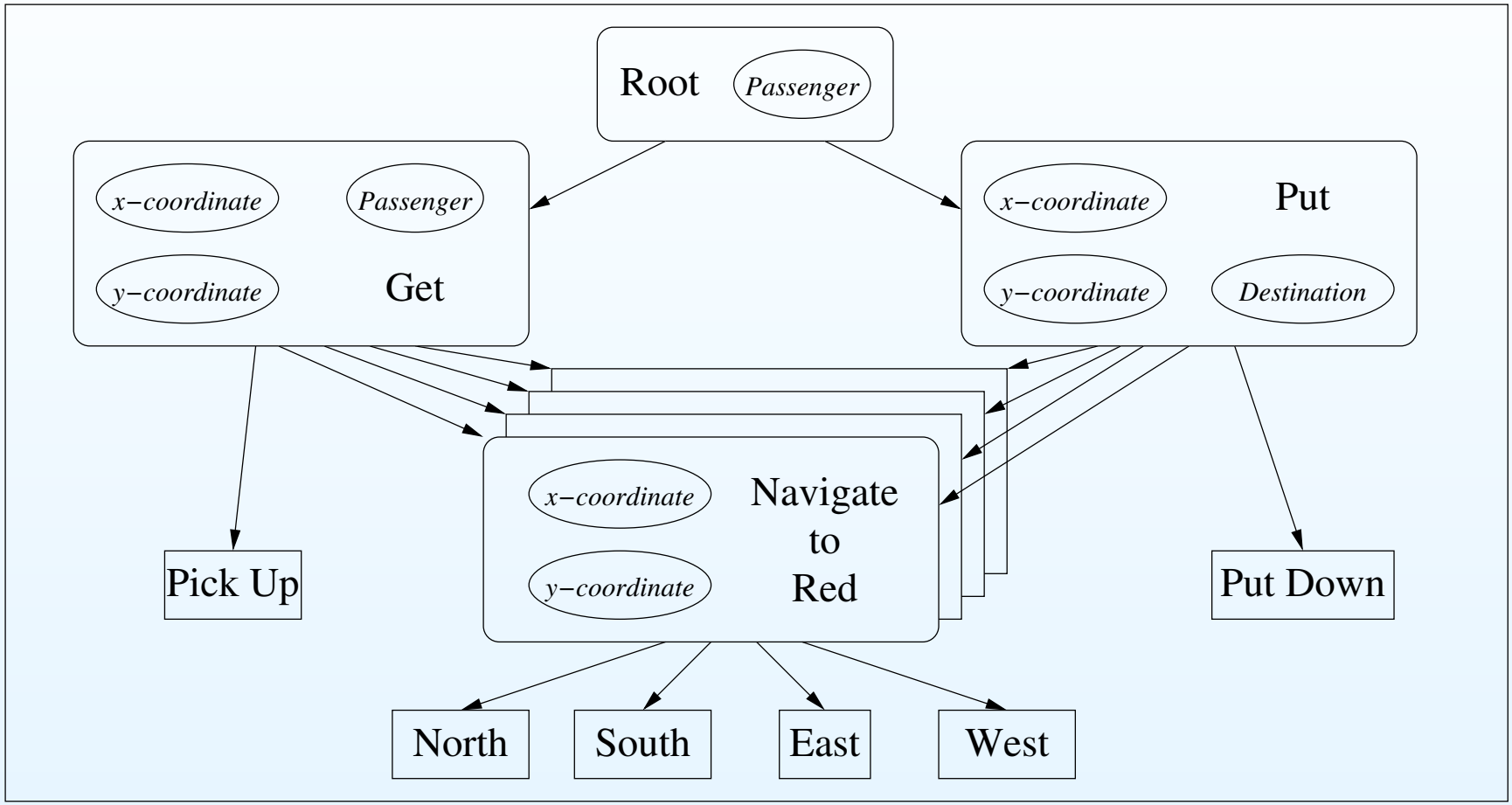
Conclusions

- Abstraction discovery as problem reformulation
- A new basis for state abstraction: policy irrelevance
  - Statistical testing methods
  - Trajectory-based discovery algorithm
- Safe transfer of state abstractions to novel domains
  - Encapsulation inside temporal abstractions
  - Synergy of temporal and state abstractions

Future work

- Adjusting abstraction-termination conditions
- Detection of dynamic domains
- Application to larger domains
  - Function approximation
  - Model-based RL algorithms
- Recursive abstraction discovery
  - Discovery of hierarchy
  - Dynamic state abstraction

Future work: discovery of hierarchy



The discovery algorithm

Which feature sets F to test at what states s?

The discovery algorithm

Which feature sets F to test at what states s?

- For given state s, test small feature sets F first and prune

s

F_1

F_2

F_3

$F_{1,2}$

$F_{1,3}$

$F_{2,3}$

$F_{1,2,3}$

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Which feature sets F to test at what states s?

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	s
F_1	yes
F_2	no
F_3	yes
$F_{1,2}$	
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The discovery algorithm

Which feature sets F to test at what states s?

- For given state s, test small feature sets F first and prune
- Sample states s from solution trajectories

	s_1	s_2	s_3	s_4	s_5	s_6	$\cdots$
F_1	yes						
F_2	no						
F_3	yes						
$F_{1,2}$	no						
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F_2	no	no	no	yes	no	yes	$\cdots$
F_3	yes	no	no	yes	yes	yes	$\cdots$
$F_{1,2}$	no	no	no	no	no	no	$\cdots$
$F_{1,3}$	no	no	no	yes	no	no	$\cdots$
$F_{2,3}$	no	no	no	no	no	no	$\cdots$
$F_{1,2,3}$	no	no	no	no	no	no	$\cdots$

The discovery algorithm

Which feature sets F to test at what states s?

- For given state s, test small feature sets F first and prune
- Sample states s from solution trajectories
- Construct a binary classification problem for each F

	s_1	s_2	s_3	s_4	s_5	s_6	$\cdots$
F_1	yes	yes	yes	yes	no	no	$\cdots$
F_2	no	no	no	yes	no	yes	$\cdots$
F_3	yes	no	no	yes	yes	yes	$\cdots$
$F_{1,2}$	no	no	no	no	no	no	$\cdots$
$F_{1,3}$	no	no	no	yes	no	no	$\cdots$
$F_{2,3}$	no	no	no	no	no	no	$\cdots$
$F_{1,2,3}$	no	no	no	no	no	no	$\cdots$

Some abstractions discovered in the Taxi domain

1. Taxi's x-coordinate:
   (a) $y = 1 \wedge$ passenger in taxi $\wedge$ destination Red $\Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*
2. Taxi's y-coordinate:
   (a) $x = 4 \wedge$ passenger in taxi $\Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*
3. Passenger's destination:
   (a) passenger in taxi $\Rightarrow$ *relevant*
   (b) otherwise, *irrelevant*
4. Passenger's location *and* destination:
   (a) $(x = 1 \wedge y = 2) \vee (x = 1 \wedge y = 1) \Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*

Some abstractions discovered in the Taxi domain

1. Taxi's x-coordinate:
   (a) $y = 1 \wedge$ passenger in taxi $\wedge$ destination Red $\Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*
2. Taxi's y-coordinate:
   (a) $x = 4 \wedge$ passenger in taxi $\Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*
3. Passenger's destination: GOOD
   (a) passenger in taxi $\Rightarrow$ *relevant*
   (b) otherwise, *irrelevant*
4. Passenger's location *and* destination:
   (a) $(x = 1 \wedge y = 2) \vee (x = 1 \wedge y = 1) \Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*

Some abstractions discovered in the Taxi domain

1. Taxi's x-coordinate: BAD: testing or classification error!
   (a) $y = 1 \wedge$ passenger in taxi $\wedge$ destination Red $\Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*
2. Taxi's y-coordinate: BAD: testing or classification error!
   (a) $x = 4 \wedge$ passenger in taxi $\Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*
3. Passenger's destination: GOOD
   (a) passenger in taxi $\Rightarrow$ *relevant*
   (b) otherwise, *irrelevant*
4. Passenger's location *and* destination: BAD: task-specific!
   (a) $(x = 1 \wedge y = 2) \vee (x = 1 \wedge y = 1) \Rightarrow$ *irrelevant*
   (b) otherwise, *relevant*