Trust-Region Policy Optimization

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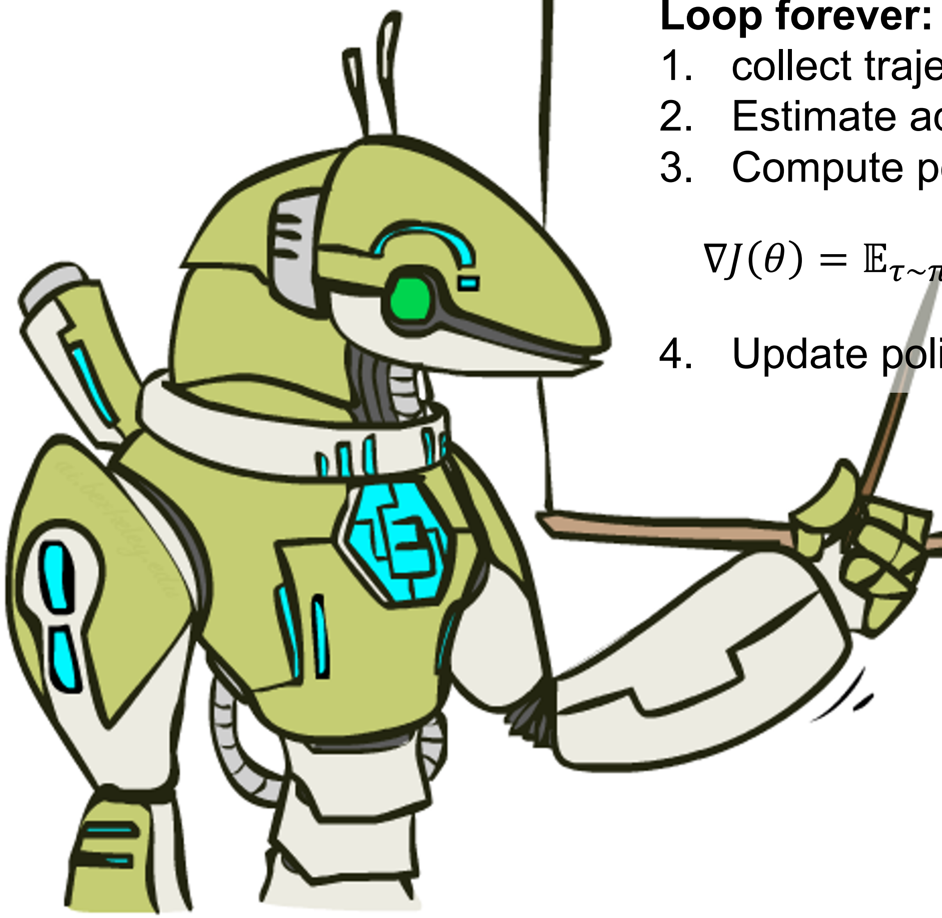
Policy Gradients so far



Loop forever:

1. collect trajectories via policy π_θ
2. Estimate advantage function $A^{\pi_\theta}(a_t|s_t)$
3. Compute policy gradient:

$$\nabla J(\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left(\sum_t \nabla_\theta \log \pi_\theta(a_t|s_t)\, A^{\pi_\theta}(a_t|s_t)\right)$$

4. Update policy parameters $\theta_{new} \leftarrow \theta + \alpha\nabla_\theta J(\theta)$

http://ai.berkeley.edu/lecture_slides.html

Policy Gradients so far

Problem

- Run gradient descent/ascent on one batch of collected experience

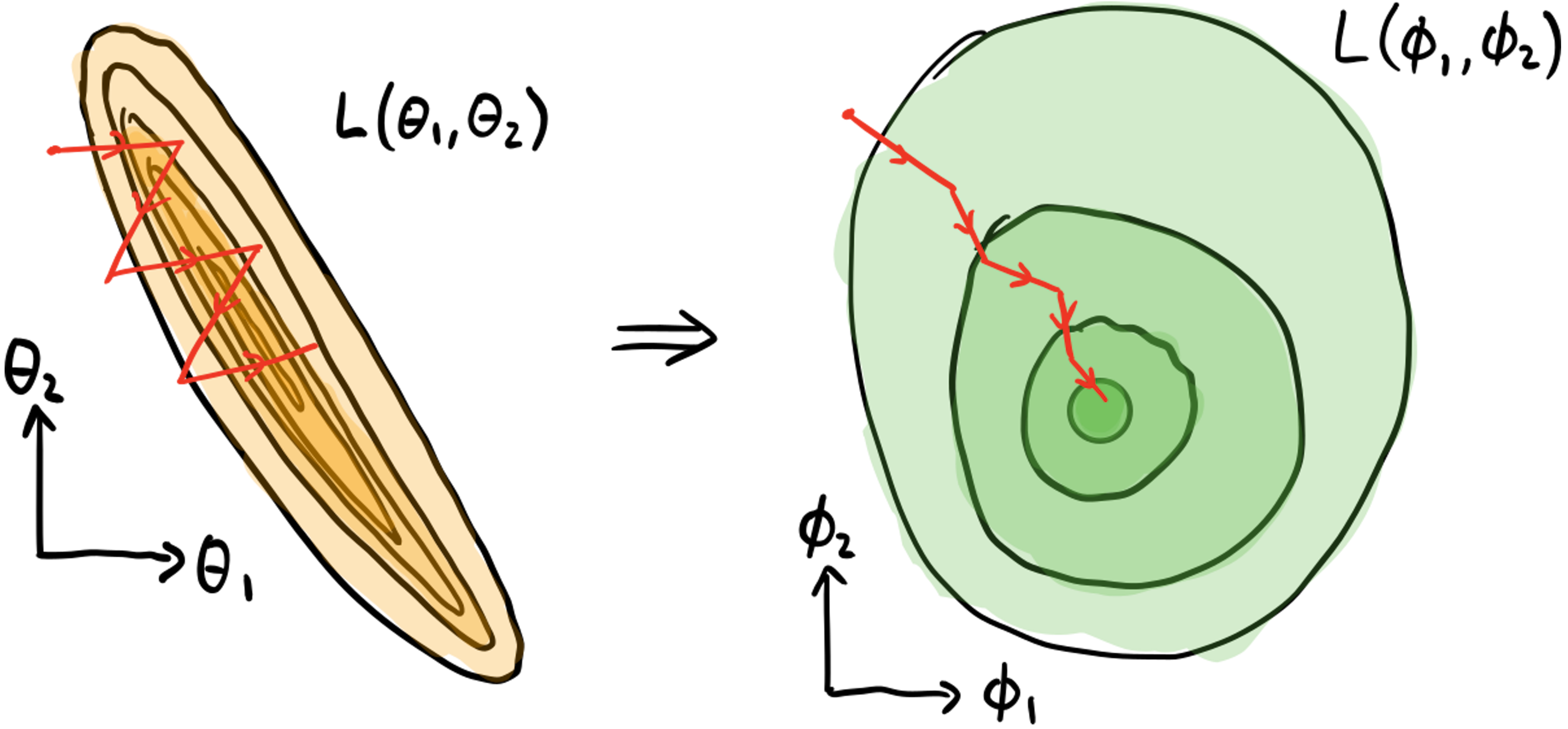
- Note: the advantage function (which is a noisy estimate) may not be accurate
  - Too large steps may lead to a disaster (even *if* the gradient is *correct*)
  - Too small steps are also bad

- Definition and scheduling of learning rates in RL is tricky as the underlying data distribution changes with updates to the policy

- Mathematical formulization:
  - First-order derivatives approximate the (parameter) surface to be flat
  - But if the surface exhibits high curvature it gets dangerous
  - Projection: small changes in parameter space might lead to large changes in policy space!

- Parameters θ get updated to areas too far out of the range from where previous data was collected *(note: a bad policy leads to bad data)*

Images taken from https://medium.com/@jonathan_hui/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9 and http://www.taiwanoffthebeatentrack.com/2012/08/23/mount-hua-华山-the-most-dangerous-hike-in-the-world/


$L(\theta_1, \theta_2)$
θ_2
θ_1
$\Rightarrow$
$L(\phi_1, \phi_2)$
ϕ_2
ϕ_1

Trust-Region Policy Optimization (TRPO)

<u>"Simple" Idea</u>

Regularize updates to the policy parameters,
such that the policy does not change too much.

Motivation: Why trust region optimization?

Line search
(like gradient ascent)

Trust region

Image credit: https://medium.com/@jonathan_hui/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9

Primer: Trust-Region Methods

Optimization in Machine Learning: two classes

1. Line Search, e.g., gradient descent
   - find a (some) direction of improvement
   - (cleverly) select a step length

2. Trust-Region Methods
   - select a trust region (analog to max step length)
   - find a point of improvement in that region

Primer: Trust-Region Methods

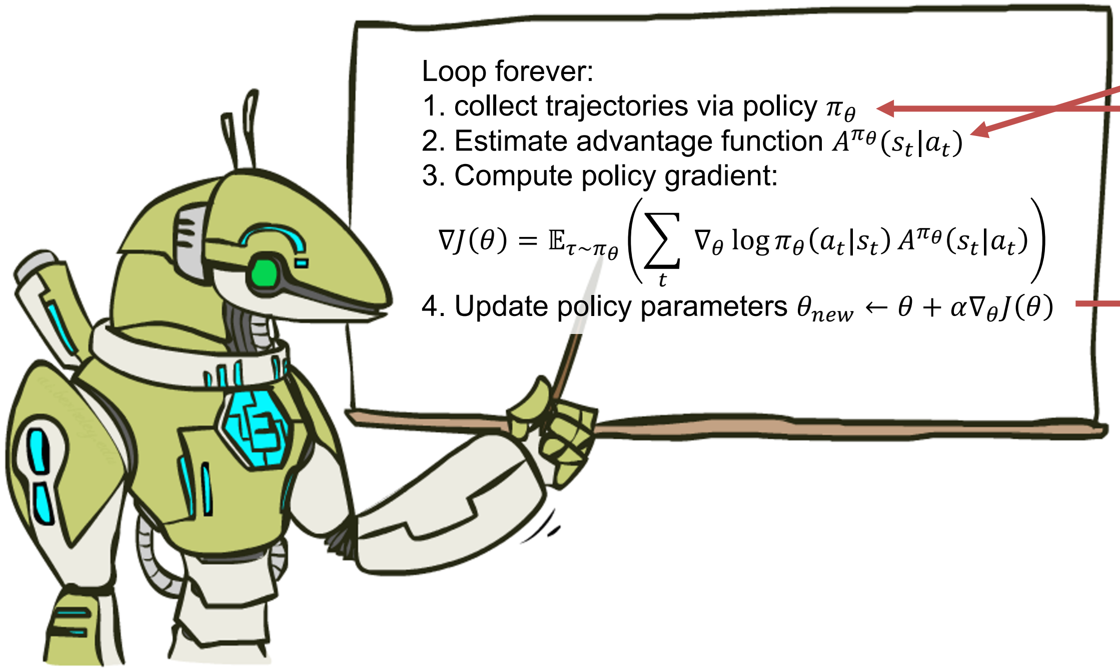
- **Idea:**
  - Approximate the real objective f with something simpler, i.e., $\tilde{f}$
  - Solve $\tilde{x}^* = \arg\min_x \tilde{f}(x)$
- **Problem:**
  - The optimum $\tilde{x}^*$ might be in a region where $\tilde{f}$ poorly approximates f
  - $\tilde{x}^*$ might be far from optimal
- **Solution:**
  - Restrict the search to a region tr where we trust $\tilde{f}$ to approximate f well
  - Solve $\tilde{x}^* = \arg\min_{x \in tr} \tilde{f}(x)$

Trust-Region Policy Optimization (TRPO)

So back to what we actually do…

The problem(s) of the Policy Gradient (PG) is that

- PG keeps old and new policy close in parameter space, while
- small changes can lead to large differences in performance, and
- “large” step-sizes hurt performance (whatever “large” means…)



random at beginning

Non-stationary input data due to changing policy and reward distribution change

Update carefully

→ We want improvement and not degradation

Trust-Region Policy Optimization (TRPO)

- We want to optimize $\eta(\pi)$, i.e., the expected return of policy π:

$$\eta(\pi) = \mathbb{E}_{s_0 \sim \rho_0, a^t \sim \pi_{old}(\cdot|s_t)}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

- We collect data with π_{old} and optimize to get a new policy π_{new}
- Let's express $\eta(\pi_{new})$ in terms of advantage over the original policy[1]:

$$\eta(\pi_{new}) = \eta(\pi_{old}) + \mathbb{E}_{\tau \sim \pi_{new}}\left[\sum_{t=0}^{\infty} \gamma^t A_{\pi_{old}}(s_t, a_t)\right]$$

Expected return of the new policy

Expected return of the old policy

Sample from new policy

[1] *Kakade et al.: Approximately Optimal Approximate Reinforcement Learning. ICML 2002.*

Trust-Region Policy Optimization (TRPO)

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$$= \eta(\pi_{old}) + \sum_s \rho_{\pi_{new}}(s) \sum_a \pi_{new}(a|s) A_{\pi_{old}}(s, a)$$

Discounted visitation frequency according to **new** policy:
$\rho_{\pi_{new}}(s) = P(s_0 = s) + \gamma P(s_1 = s) + \gamma^2 P(s_2 = s) + \cdots$

[1] *Schulman et al.: Trust-Region Policy Optimization. ICML 2015.*

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$$= \eta(\pi_{old}) + \sum_s \rho_{\pi_{new}}(s) \sum_a \underbrace{\pi_{new}(a|s) A_{\pi_{old}}(s, a)}_{> 0}$$

new expected return > old expected return

If we can guarantee this…

→ New objective guarantees improvement from $\pi_{old} \rightarrow \pi_{new}$

Trust-Region Policy Optimization (TRPO)

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$$\eta(\pi_{new}) = \eta(\pi_{old}) + \mathbb{E}_{\tau \sim \pi_{new}}\left[\sum_{t=0}^{\infty} \gamma^t A_{\pi_{old}}(s_t, a_t)\right]$$

$$= \eta(\pi_{old}) + \boxed{\sum_s \rho_{\pi_{new}}(s)} \sum_a \pi_{new}(a|s) A_{\pi_{old}}(s, a)$$

However, this cannot be easily estimated. The state visitations that we sampled so far are coming from the old policy!

→ **we cannot optimize this in the current form!**

Trust-Region Policy Optimization (TRPO)

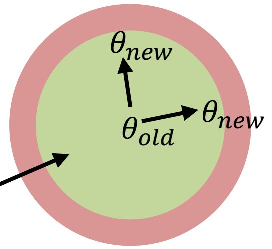
$$\eta(\pi_{new}) = \eta(\pi_{old}) + \sum_s \rho_{\pi_{new}}(s) \sum_a \pi_{new}(a|s) A_{\pi_{old}}(s,a)$$

approximate locally $\approx$

$$L(\pi_{new}) = \eta(\pi_{old}) + \sum_s \rho_{\pi_{old}}(s) \sum_a \pi_{new}(a|s) A_{\pi_{old}}(s,a)$$

← This we already sampled → We already have this!

- The approximation is accurate within step size δ (trust region)
  - δ needs to be chosen based on a lower-bound approximation error
- Monotonic improvement guaranteed
  - (within the green region!)



Trust region
→ $\pi_{\theta_{new}}(s,a)$ does not change too much

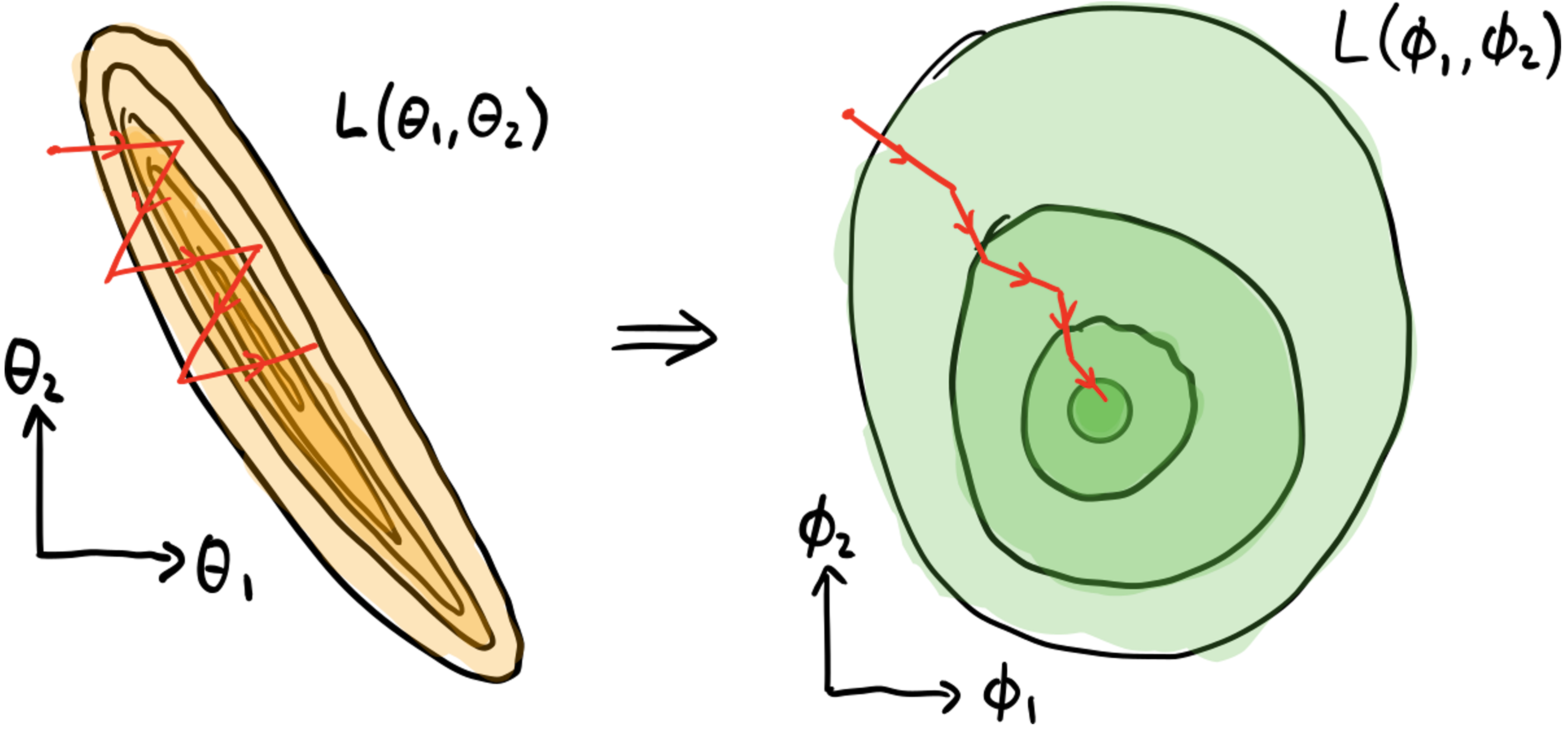
Trust-Region Policy Optimization (TRPO)

- If we want to optimize $L(\theta_{new})$ instead of $\eta(\theta_{new})$ …
  with a guarantee of monotonic improvement on $\eta(\theta_{new})$, …
  … we need a bound on $L(\theta_{new})$.

- It can be proven that there exists the following bound[1,2]:

$$\eta(\pi_{new}) \geq L(\pi_{new}) - C \cdot D_{KL}^{max}(\pi_{old}, \pi_{new}), \text{ where } C = \frac{4\varepsilon\gamma}{(1-\gamma)^2}$$

[1] Schulman et al.: Trust-Region Policy Optimization. ICML 2015.
[2] Kakade et al.: Approximately Optimal Approximate Reinforcement Learning. ICML 2002.


L(θ1,θ2)
θ2
θ1
⇒
L(φ1,φ2)
φ2
φ1

Trust-Region Policy Optimization (TRPO)

- A monotonically increasing policy can be defined by (minorization-maximization algorithm):

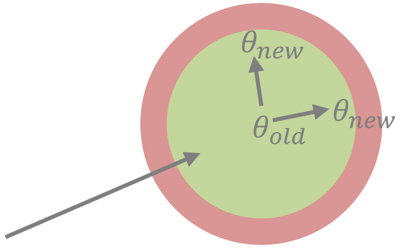
$$\pi = \arg\max_{\pi}[L(\pi_{new}) - C \cdot D_{KL}^{max}(\pi_{old}, \pi_{new})], \text{ where } C = \frac{4\varepsilon\gamma}{(1-\gamma)^2}$$

Side-note:

- A constraint on the KL-divergence between new and old policy (i.e., a trust region constraint) allows larger step sizes while being mathematically equivalent:

$$\pi = \arg\max_{\pi} L_{\pi_{old}}, \text{ such that } D_{KL}^{max}(\pi_{old}, \pi) \leq \delta$$

- Approximation with L is accurate within δ
  → here, monotonic improvement guaranteed



Trust region
→ $\pi_{\theta_{new}}(s, a)$ does not change too much

PPO (clipping version)

$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s,a), \;\; g(\epsilon, A^{\pi_{\theta_k}}(s,a)) \right),$$

where

$$g(\epsilon, A) = \begin{cases} (1+\epsilon)A & A \geq 0 \\ (1-\epsilon)A & A < 0. \end{cases}$$