REINFORCEMENT LEARNING: THEORY AND PRACTICE Modern Landscape: Part II

Profs. Amy Zhang and Peter Stone

Logistics Questions?

Agenda for today

Natural progression of policy gradient and actor-critic deep RL methods: Trust Region Policy Optimization Proximal Policy Optimization Deep Deterministic Policy Gradients Twin Delayed DDPG

Maximum Entropy RL Framework

Soft Actor-Critic

Christopher Mutschler

Policy Gradients so far

Policy Gradients so far

Problem

- Run gradient descent/ascent on one batch of collected experience
- Note: the advantage function (which is a noisy estimate) may not be accurate
	- Too large steps may lead to a disaster (even *if* the gradient is *correct*)
	- Too small steps are also bad
- Definition and scheduling of learning rates in RL is tricky as the underlying data distribution changes with updates to the policy
	- Mathematical formulization:
		- First-order derivatives approximate the (parameter) surface to be flat
		- But if the surface exhibits high curvature it gets dangerous
		- Projection: small changes in parameter space might lead to large changes in policy space!
- Parameters θ get updated to areas too far out of the range from where previous data was collected *(note: a bad policy leads to bad data)*

*Images taken from https://medium.com/@jonathan_hui/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9 and http://www.taiwanoffthebeatentrack.com/2012/08/23/mount-hua-*华山*-the-most-dangerous-hike-in-the-world/*

"Simple" Idea

Regularize updates to the policy parameters, such that the policy does not change too much.

Motivation: Why trust region optimization?

Trust region

Image credit: https://medium.com/@jonathan_hui/rl-trust-region-policy-optimization-trpo-explained-a6ee04eeeee9

Primer: Trust-Region Methods

Optimization in Machine Learning: two classes

- 1. Line Search, e.g., gradient descent
	- find a (some) direction of improvement
	- (cleverly) select a step length

- 2. Trust-Region Methods
	- select a trust region (analog to max step length)
	- find a point of improvement in that region

Primer: Trust-Region Methods

- **Idea:**
	- Approximate the real objective f with something simpler, i.e., \tilde{f}
	- Solve $\tilde{x}^* = \arg \min$ χ $\tilde{f}(x)$
- **Problem:**
	- The optimum \tilde{x}^* might be in a region where \tilde{f} poorly approximates f
	- \tilde{x}^* might be far from optimal
- **Solution:**
	- Restrict the search to a region tr where we trust \tilde{f} to approximate f well
	- Solve $\tilde{x}^* = \arg\min_{x \in tr} \tilde{f}(x)$

So back to what we actually do…

The problem(s) of the Policy Gradient (PG) is that

- PG keeps old and new policy close in parameter space, while
- small changes can lead to large differences in performance, and
- "large" step-sizes hurt performance (whatever "large" means…)

• We want to optimize $\eta(\pi)$, i.e., the expected return of policy π :

$$
\eta(\pi) = \mathbb{E}_{s_0 \sim \rho_0, a^t \sim \pi_{old}(\cdot | s_t)} \left[\sum_{t=0}^{\infty} \gamma^t r_t \right]
$$

- We collect data with π_{old} and optimize to get a new policy π_{new}
- Let's express $\eta(\pi_{new})$ in terms of advantage over the original policy¹:

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- We collect data with π_{old} and optimize to get a new policy π_{new}
- Let's express $\eta(\pi_{new})$ in terms of advantage over the original policy¹:

$$
\eta(\pi_{new}) = \eta(\pi_{old}) + \mathbb{E}_{\tau \sim \pi_{new}} \left[\sum_{t=0}^{\infty} \gamma^{t} A_{\pi_{old}}(s_t, a_t) \right]
$$

$$
= \eta(\pi_{old}) + \sum_{s} \rho_{\pi_{new}}(s) \sum_{a} \pi_{new}(a|s) A_{\pi_{old}}(s, a)
$$

Discounted visitation frequency according to **new** policy:

$$
\rho_{\pi_{new}}(s) = P(s_0 = s) + \gamma P(s_1 = s) + \gamma^2 P(s_2 = s) + \cdots
$$

• We want to optimize $\eta(\pi)$, i.e., the expected return of policy π :

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$$

= $\eta(\pi_{old}) + \sum_{s} \rho_{\pi_{new}}(s) \sum_{a} \pi_{new}(a|s) A_{\pi_{old}}(s, a)$
new expected > 0
return

 \rightarrow New objective guarantees improvement from $\pi_{old} \rightarrow \pi_{new}$

• We want to optimize $\eta(\pi)$, i.e., the expected return of policy π :

$$
\eta(\pi) = \mathbb{E}_{s_0 \sim \rho_0, a^t \sim \pi_{old}(\cdot | s_t)} \left[\sum_{t=0}^{\infty} \gamma^t r_t \right]
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$$

$$
= \eta(\pi_{old}) + \sum_{s} \rho_{\pi_{new}}(s) \sum_{a} \pi_{new}(a|s) A_{\pi_{old}}(s, a)
$$

However, this cannot be easily estimated. The state visitations that we sampled so far are coming from the old policy!

\rightarrow we cannot optimize this in the current form!

$$
\eta(\pi_{new}) = \eta(\pi_{old}) + \sum_{s} \rho_{\pi_{new}}(s) \sum_{a} \pi_{new}(a|s) A_{\pi_{old}}(s, a)
$$
\n
$$
\approx L(\pi_{new}) = \eta(\pi_{old}) + \sum_{s} \rho_{\pi_{old}}(s) \sum_{a} \pi_{new}(a|s) A_{\pi_{old}}(s, a) \qquad \qquad \text{This we already sampled}
$$
\n
$$
L(\pi_{new}) = \eta(\pi_{old}) + \sum_{s} \rho_{\pi_{old}}(s) \sum_{a} \pi_{new}(a|s) A_{\pi_{old}}(s, a) \qquad \qquad \text{This we already have this!}
$$

- The approximation is accurate within step size δ (trust region)
	- \cdot δ needs to be chosen based on a lower-bound approximation error
- Monotonic improvement guaranteed
	- (within the green region!)

- If we want to optimize $L(\theta_{new})$ instead of $\eta(\theta_{new})$... with a guarantee of monotonic improvement on $\eta(\theta_{new}), \ldots$... we need a bound on $L(\theta_{new})$.
- It can be proven that there exists the following bound^{1,2}:

$$
\eta(\pi_{new}) \ge L(\pi_{new}) - C \cdot D_{KL}^{max}(\pi_{old}, \pi_{new})
$$
, where $C = \frac{4\varepsilon\gamma}{(1-\gamma)^2}$

• A monotonically increasing policy can be defined by (minorization-maximization algorithm):

$$
\pi = \arg \max_{\pi} [L(\pi_{new}) - C \cdot D_{KL}^{max}(\pi_{old}, \pi_{new})], \text{ where } C = \frac{4\epsilon\gamma}{(1-\gamma)^2}
$$

Side-note:

• A constraint on the KL-divergence between new and old policy (i.e., a trust region constraint) allows larger step sizes while being mathematically equivalent:

$$
\pi = \arg \max_{\pi} L_{\pi_{old}}
$$
 , such that $D_{KL}^{max}(\pi_{old}, \pi) \leq \delta$

Approximation with L is accurate within δ \rightarrow here, monotonic improvement guaranteed

Limitations of TRPO

- Hard to use with architectures with multiple outputs, e.g., policy and value function (need to weight different terms in distance metric)

- Empirically performs poorly on tasks requiring deep CNNs and RNNs, e.g., Atari benchmark (more suitable for locomotion)

- Conjugate gradients makes implementation more complicated than SGD

Reading Questions: TRPO

Michelle Ding

[Trust Region Policy Optimization] What is achieved by keeping old and new policies sufficiently close together? Why is this not guaranteed by the policy gradient theorem from Chapter 13, which I though ensures the convergence of normal policy gradient? i.e. If a single bad step can collapse performance, how is convergence guaranteed?

Alperen Duru

Didn't we already discuss in the policy gradient methods that one such advantage of using the policy gradient methods is to smooth out the performance changes compared to the action-value methods? Why does TRPO discuss the normal policy gradient methods that a single small step can collapse the policy performance?

PPO (clipping version)

$$
L(s, a, \theta_k, \theta) = \min \left(\frac{\pi_{\theta}(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s, a), \ g(\epsilon, A^{\pi_{\theta_k}}(s, a)) \right),
$$

where

$$
g(\epsilon, A) = \begin{cases} (1+\epsilon)A & A \ge 0 \\ (1-\epsilon)A & A < 0. \end{cases}
$$

[Schulman et al, Proximal Policy Optimization Algorithms]

Reading Questions

Young-ho

Compared to TRPO, does PPO converge faster than TRPO?

-PPO: first-order

-TRPO: complex second-order

I understand that speed can be faster than TRPO, but how can performance be better than TRPO, even with a less complex model?

Laith Altarabishi

(PPO) Suppose that the advantage is negative and the new policy value increases massively in value - ie the action becomes a lot more likely. Since the clipping is a maximization term - doesn't this mean that the objective would see a massive update that is not controlled? Are we only concerned with the case where the objective increases - if so why?

Reading Questions

Saloni Modi

Does the epsilon in PPO have to be constant? I feel like I'd be cool with the new policy initially getting really far from the original policy but then overtime decaying the acceptable distance.

Deterministic Policy Gradient (DPG)

Objective

$$
J(\mu_{\theta}) = \mathbb{E}_{s \sim \rho^{\mu}} \left[Q^{\mu}(s, \mu_{\theta}(s)) \right]
$$

 $Q^{\mu}(s,a)$ Q-function following policy μ $\mu_{\theta}(s)$ Deterministic policy function (e.g., if action is continuous)

> [Deterministic Policy Gradient Algorithms, D. Silver et al.] 125

Deterministic Policy Gradient (DPG)

Taking Derivative w.r.t θ :

No need to take gradient w.r.t μ Because of deterministic policy **gradient theorem**

$$
\nabla_{\theta} J(\mu_{\theta}) = \mathbb{E}_{s \sim \rho^{\mu}} \left[\nabla_{\theta} \mu_{\theta}(s) \nabla_{a} Q^{\mu}(s,a) \vert_{a = \mu(s)} \right]
$$

Scalar gradient at (s, a)

Sample state from the distribution, $s_i \sim \rho^{\mu}$:

$$
\nabla_{\theta} J(\mu_{\theta}) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\theta} \mu_{\theta}(s_i) \nabla_a Q^{\mu}(s_i, a)|_{a = \mu(s_i)}
$$

126 [Deterministic Policy Gradient Algorithms, D. Silver et al.]

DDPG (Deep Deterministic Policy Gradient)

- Use deep networks to represent policy / Q.
- Generate trajectories with current policy + noise
	- Since the policy is deterministic
- Save trajectories into replay buffer and sample from it (Off-policy!)
- Learn Q^{μ} via DQN using target network
- Learn μ using the slide above.

[Continuous Control With Deep Reinforcement Learning, Lillicrap et al. ICLR 2016] 127

Reading Questions

Justin Sasek

(DDPG) Why is importance sampling not needed even though DDPG is off-policy?

Distributed Distributional Deterministic Policy Gradients (D4PG)

- Distributional version of DDPG
- 1. Distributional critic
- 2. N-step returns
- 3. Multiple distributed parallel actors
- 4. Prioritized experience replay

Twin Delayed DDPG (TD3)

[Addressing Function Approximation Error in Actor-Critic Methods, S Fujimoto et al, ICML 2018]

CS885 Reinforcement Learning Module 2: June 6, 2020

Maximum Entropy Reinforcement Learning

Haarnoja, Tang et al. (2017) Reinforcement Learning with Deep Energy Based Policies, *ICML*.

Haarnoja, Zhou et al. (2018) Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor, *ICML*.

Reinforcement Learning

Reinforcement Learning

Deterministic Policies

- There always exists an optimal deterministic policy
- Search space is smaller for deterministic than stochastic policies
- Practitioners prefer deterministic policies

Stochastic Policies

- Search space is continuous for stochastic policies (helps with gradient descent)
- More robust (less likely to overfit)
- Naturally incorporate exploration
- Facilitate transfer learning
- Mitigate local optima

Encouraging Stochasticity

Standard MDP

- States: S
- Actions: A
- Reward: $R(s, a)$
- Transition: $Pr(s'|s, a)$
- Discount: γ

Soft MDP

- States: S
- Actions: A
- Reward: $R(s, a) + \lambda H(\pi(\cdot | s))$
- Transition: $Pr(s'|s, a)$
- Discount: γ

Reward: $R(s, a) + \lambda H(\pi(\cdot | s))$

$$
\mathrm{H}(X) := -\sum_{x \in \mathcal{X}} p(x) \log p(x).
$$

How do the rewards change? $\lambda = 1$

Reward: $R(s, a) + \lambda H(\pi(\cdot | s))$

$$
\text{H}(X):=-\sum_{x\in\mathcal{X}}p(x)\log p(x).
$$

How do the rewards change? $\lambda = 1$

Is this a valid reward function?

Is this a valid reward function? No! It is non stationary because it depends on the current policy.

What does the optimal policy look like that *also maximizes entropy* if $\gamma = 0.9$? Assume episodic setting where episode ends when $+10$ is reached.

What does the optimal policy look like that *also maximizes entropy* if $\gamma = 0.9$? Assume episodic setting where episode ends when $+10$ is reached.

 $V(S = 0) = 0 + 0.9 * 0 + 0.9^{2} * 1 + 0.9^{3} * 0 + 0.9^{4} * 10 = 7.371$

What does the optimal policy look like that *also maximizes entropy* if $\gamma = 0.9$? Assume episodic setting where episode ends when $+10$ is reached.

 $V(S = 0) = 0 + 0.9 * 0 + 0.9^{2} * 1 - 0.9^{3} * 3 + 0.9^{4} * 10 = 5.184$

What does the optimal policy look like that *also maximizes entropy* if $\gamma = 0.9$? Assume episodic setting where episode ends when $+10$ is reached.

 $V(S = 0) = 0 + 0.9 * 0 + 0.9^{2} * 1 - 0.9^{3} * 3 + 0.9^{4} * 2 + 0.9^{5} * 0 + 0.9^{6} * 10 = 5.25$

What does the optimal policy look like that *also maximizes entropy* if $γ = 1?$

What does the optimal policy look like that *also maximizes entropy* if $γ = 1?$

Is that the same optimal policy for this problem? $\gamma=1$

Reward: $R(s, a) + \lambda H(\pi(\cdot | s))$

Optimal Policy

• Standard MDP

$$
\pi^* = \underset{\pi}{\operatorname{argmax}} \sum_{n=0}^{N} \gamma^n E_{s_n, a_n | \pi} [R(s_n, a_n)]
$$

• Soft MDP

$$
\pi_{soft}^{*} = \underset{\pi}{\operatorname{argmax}} \sum_{n=0}^{N} \gamma^{n} E_{s_{n}, a_{n} | \pi} \big[R(s_{n}, a_{n}) + \lambda H(\pi(\cdot | s_{n})) \big]
$$
\nMaximum entropy policy

\nEntropy regularized policy

Q-function

• Standard MDP

$$
Q^{\pi}(s_0, a_0) = R(s_0, a_0) + \sum_{n=1}^{\infty} \gamma^n E_{s_n, a_n | s_0, a_0, \pi}[R(s_n, a_n)]
$$

• Soft MDP

$$
Q_{soft}^{\pi}(s_0, a_0) = R(s_0, a_0) + \sum_{n=1}^{\infty} \gamma^n E_{s_n, a_n | s_0, a_0, \pi} [R(s_n, a_n) + \lambda H(\pi(\cdot | s_n))]
$$

NB: No entropy with first reward term since action is not chosen according to π

Greedy Policy

• Standard MDP (deterministic policy)

$$
\pi_{greedy}(s) = \underset{a}{\operatorname{argmax}} Q(s, a)
$$

• Soft MDP (stochastic policy)

 $\pi_{greedy}(\cdot \, | s) = \mathrm{argmax} \sum_a \pi(a | s) Q(s, a) + \lambda H\big(\pi(\cdot \, | s) \big)$ $\overline{\pi}$ = $\exp(Q(s, \cdot)/\lambda)$ $\sum_{\bm{a}} \exp(Q(s, \bm{a})/\lambda)$ $= softmax(Q(s,·)/\lambda)$

when $\lambda \to 0$ then softmax becomes regular max

Soft Policy Iteration

SoftPolicyIteration(MDP,
$$
\lambda
$$
)

\nInitialize π_0 to any policy

\n $i \leftarrow 0$

\nRepeat

\nPolicy evaluation:

\nRepeat until convergence

\n
$$
Q_{soft}^{\pi_i}(s, a) \leftarrow R(s, a)
$$
\n
$$
+ \gamma \sum_{s'} \Pr(s'|s, a) \left[\sum_{a'} \pi_i(a'|s') Q_{soft}^{\pi_i}(s', a') + \lambda H(\pi_i(\cdot | s')) \right] \forall s, a
$$
\nPolicy improvement:

\n
$$
\pi_{i+1}(a|s) \leftarrow softmax \left(Q_{soft}^{\pi_i}(s, a) / \lambda \right) = \frac{exp\left(Q_{soft}^{\pi_i}(s, a) / \lambda \right)}{\sum_{a'} exp\left(Q_{soft}^{\pi_i}(s, a') / \lambda \right)} \forall s, a
$$
\n
$$
i \leftarrow i + 1
$$
\nUntil $\left| \left| Q_{soft}^{\pi_i}(s, a) - Q_{soft}^{\pi_{i-1}}(s, a) \right| \right|_{\infty} \le \epsilon$

University of Waterloo

Soft Actor-Critic

- RL version of soft policy iteration
- Use neural networks to represent policy and value function
- At each policy improvement step, project new policy in the space of parameterized neural nets

Soft Actor Critic (SAC)

Final Logistics

Next lecture: Abstractions and Options Reading assignments due **2PM Monday**

Final project literature review due at **11:59pm on Thursday, 4/11**

Complete Programming Assignment for Chapters 12+13 on edx by Sunday 11:59 PM CST