Fall 2023-24

PHYSICS OF MATERIALS, TYPES OF MECHANISMS

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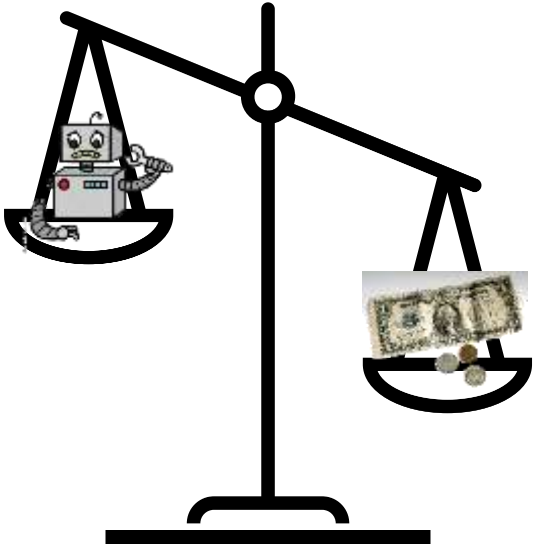
Recapping last class

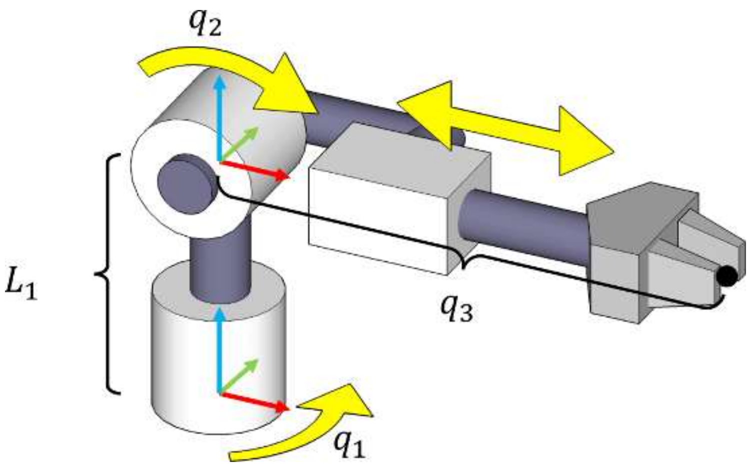
- Three Newton's Laws
  - $\sum F = 0$
  - $\sum F = m \cdot a$
  - Action←→Reaction
- Forces + Torques = Wrenches
- Friction
  - Coulomb (Stiction + Kinetic Friction)
  - Viscous
  - Rolling
- Grasping

What will you learn today?

- “Rigid” bodies is an assumption from Physics 101 we must move away from.
  - Bodies are deformable
  - Bodies can break, stretch, or fatigues (stress/strain)
  - Bodies can be connected by joints.

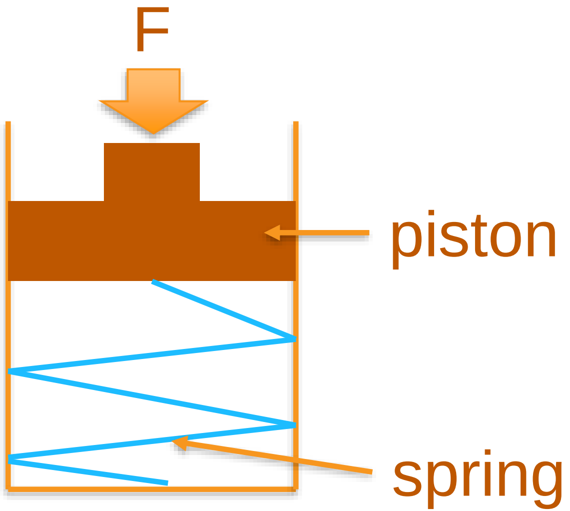
Why does this matter?



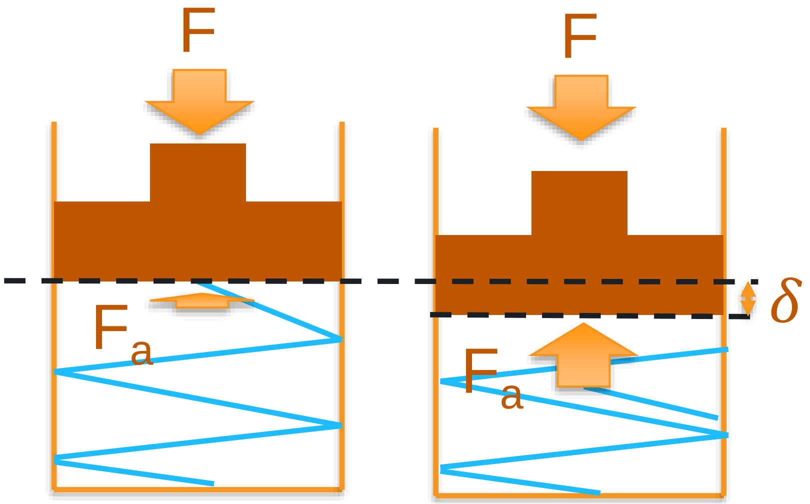


Preliminaries: Force/Displacement Relationship in Deformable Bodies

- Until now we assumed perfectly rigid bodies
  - Not true! All bodies deform
- Physics of deformation
  - Linear (Hooke's Law)
  - Non-linear (d=f(F))
  - Elastic vs non-elastic
  - Fatigue



Analysis of Forces in the Piston (Hooke's Law)



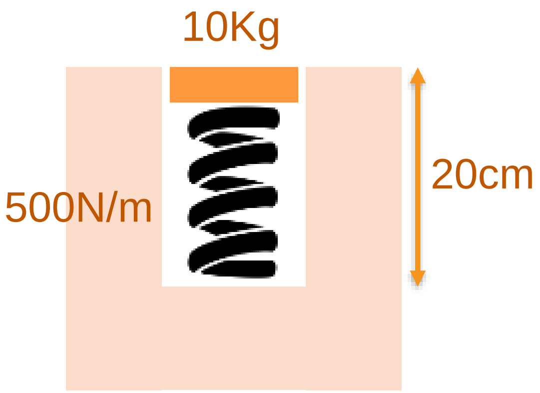
$$F_a = k\delta$$

Free body diagram (equilibrium)

$$\sum F = 0$$

$$-mg + k\delta = 0$$

Exercise



- Initial position of the piston: 20cm
- Mass of piston: 10 Kg
- k = 500 N/m
- What is the final position of the piston?

Connecting to Deformable Bodies

- Deformable bodies are like pistons
  - When supporting a mass, they deform until the “reactive force” equals the weight of the object
  - Harder objects have large K, soft objects lower K
- Until the deformation required to counteract the mass is too large → break!

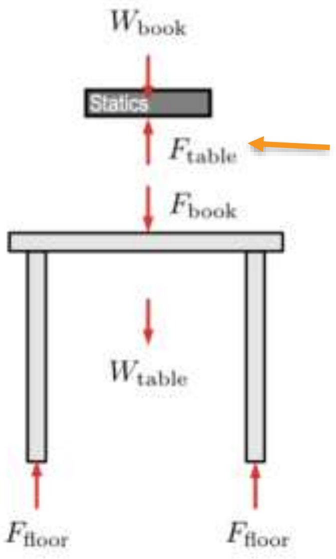


table deforms until book weight is compensated (or table breaks)

Stress-Strain Relationship

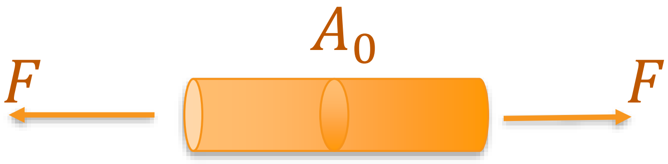
- Why?
  - Choosing materials for robot components
  - Choosing gripper material
  - Task dependent: go from A to B, apply force F with sufficient positional tolerance.
  - We want lightweight robots
    - Low energy
    - Cheap
    - Safe

Stress-Strain Relationship

Mechanical engineers and material scientists use **stress and strain** for defining mechanical properties of materials

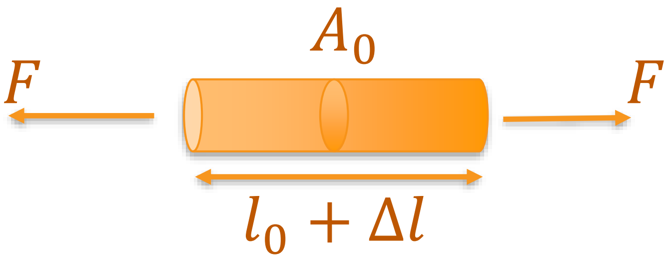
Leg material	Tree branches	Coffee stirrers	Steel rods
	• Light • Cheap • Doesn't break • Bends easily	• Light • Cheap • Breaks easily	• Doesn't break • Doesn't bend • Heavy • Expensive

Stress



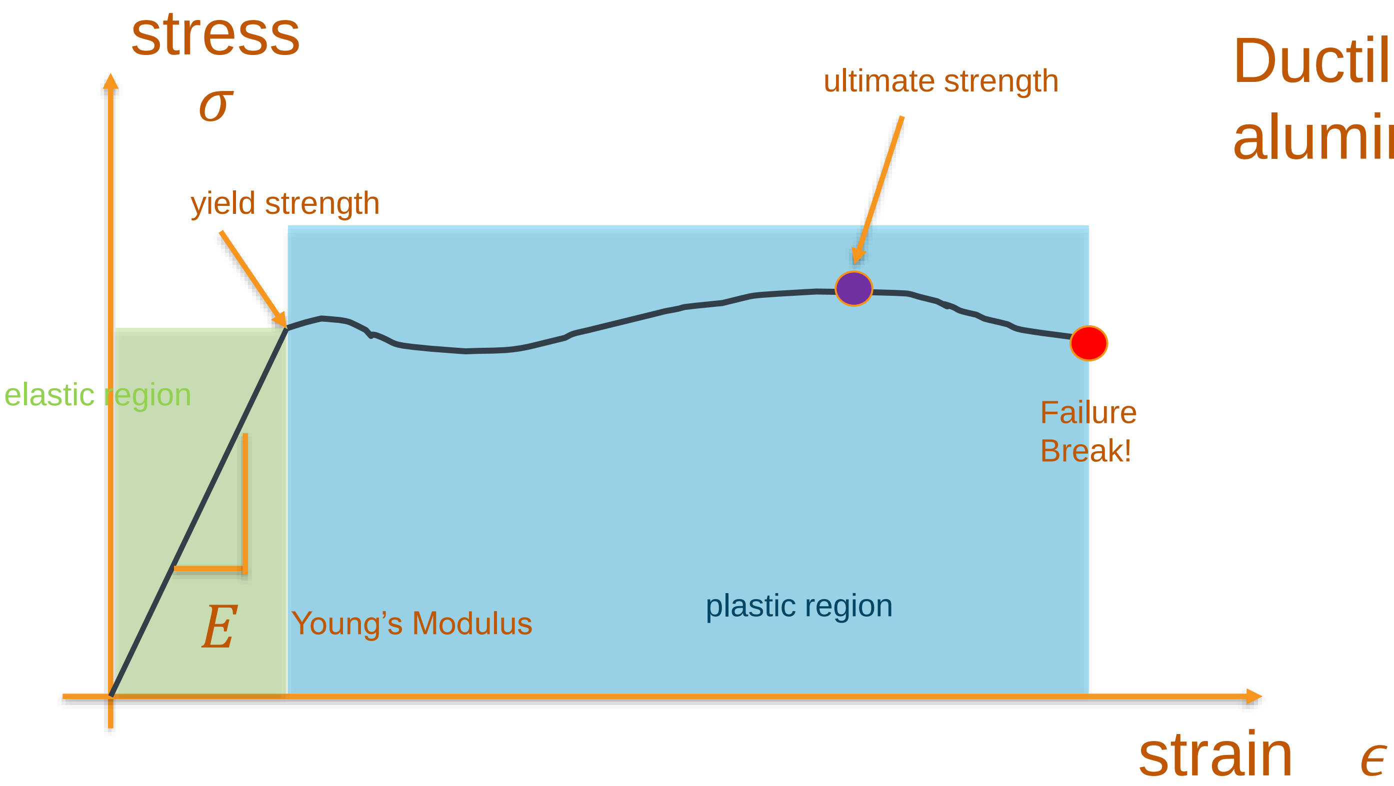
- Stress: imagine that for a rod made of certain material that we are considering for our robot pulling force (tensile) is applied from both sides
- The mechanical stress is defined as the force divided by the cross-section area
- $\sigma = F/A_0$

Strain



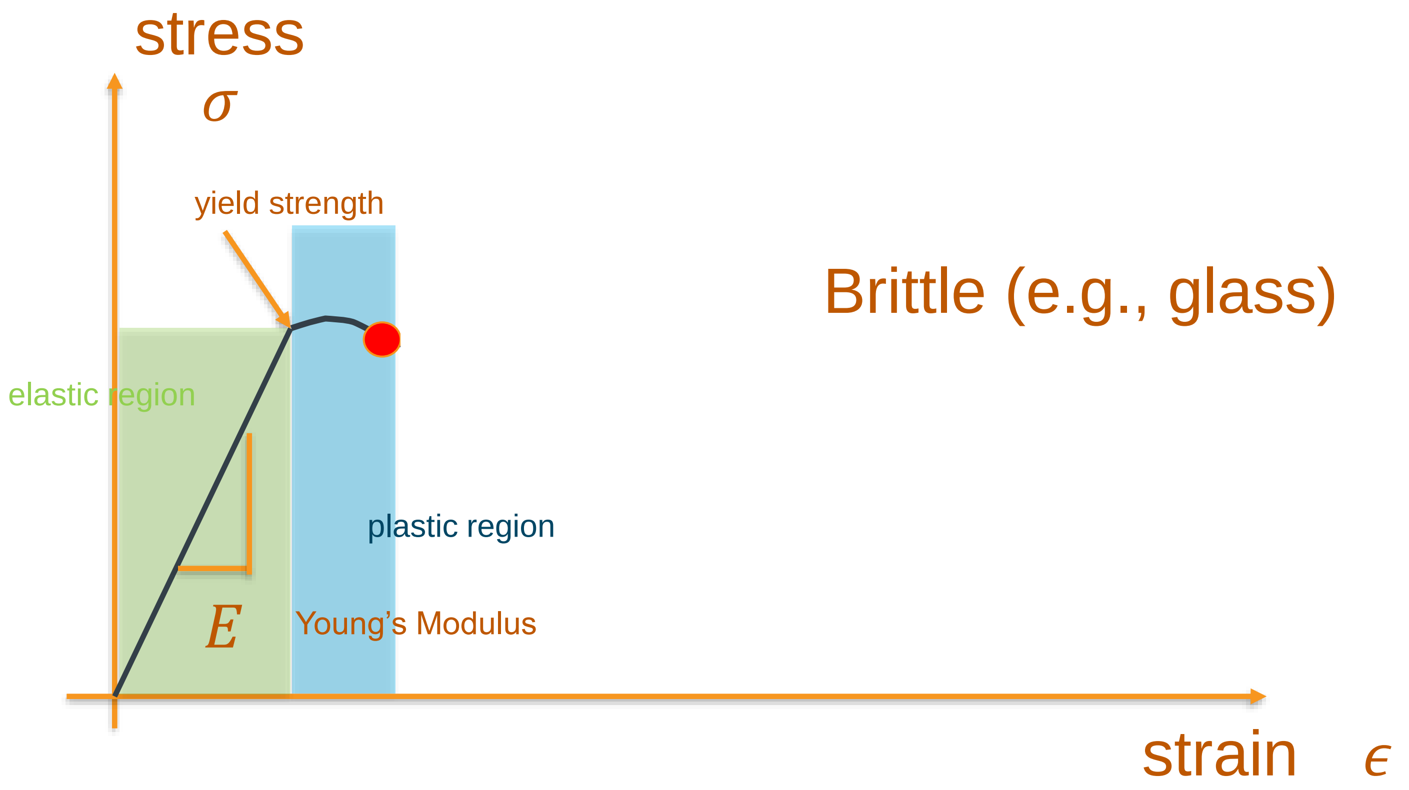
- Strain: now imagine that with this pulling force, our rod stretches a bit (Δl)
  - Strain: the ratio of change in length by the length when no force is applied
- $\epsilon = \frac{\Delta l}{l_0}$
- Understanding the relationship between strain and stress, we can decide what material is best for our robot

Stress-Strain Relationship



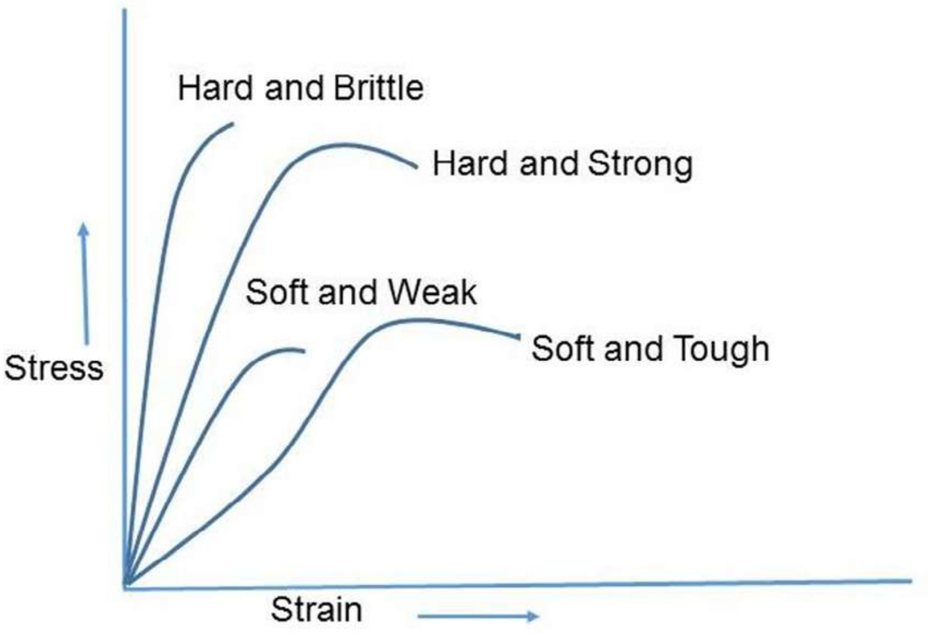
Ductile (e.g., steel, aluminium)

Stress-Strain Relationship



Brittle (e.g., glass)

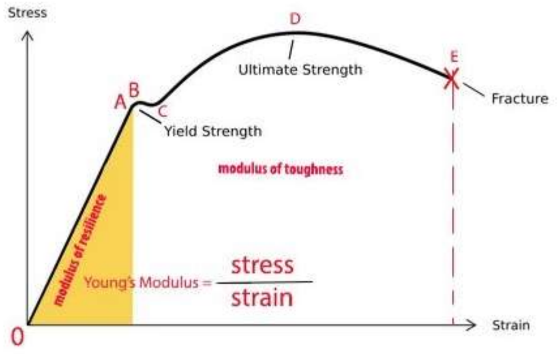
Characterizing Materials



- Strength
- Ductility
- Toughness

Stress-Strain Relationship

- Let's consider two materials:
  - One is ductile (a silicon rod)
  - One is brittle (a coffee stirrer)
- At low pulling forces, stress-strain have a linear relationship for both
- $\sigma = E\epsilon$ → Hook's Law
- The slope is called the Youngs modulus (E)
  - Units of pressure
    - Pa
    - Pounds per square inch (PSI)

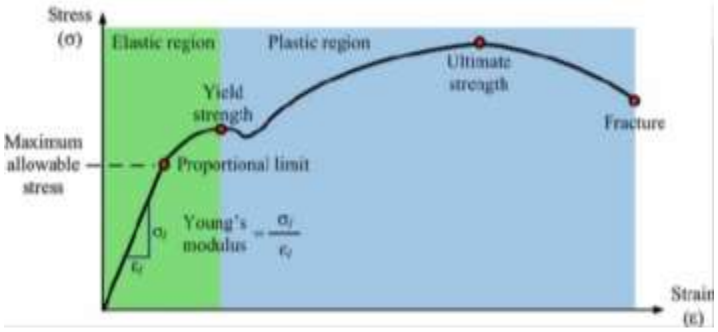


Material	E
Steel	30e6 psi
Rubber	150 psi
Acrylic	0.5e6 psi

Stress-Strain Relationship

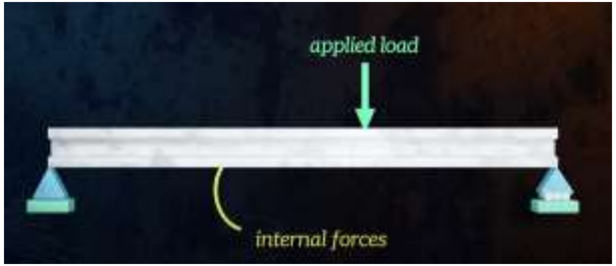
- The linear portion of the stress-strain curve is also called the elastic regime
  - The material will bounce back (recover its original shape) if the load is removed
- The second portion of the curve is the plastic regime
  - More load leads to yield permanent change of shape
- Ultimately, the end of the curve is when the part breaks

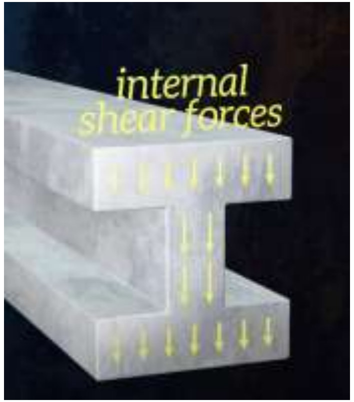
- We need to design our robots to avoid 1) plastic regime, or worse, 2) breaking point!
- We can change
  - **the material** (make parts harder and stronger) or
  - **the geometry** (same force, more area, make parts ”thicker”)

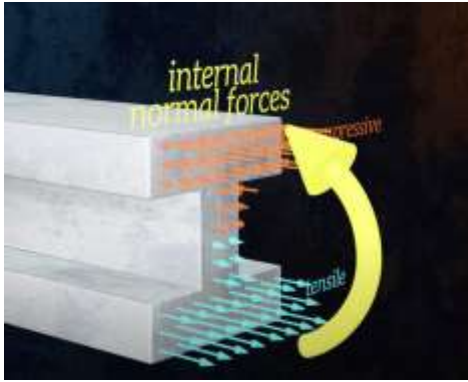


Shear Force and Bending Moment

- When a beam is loaded, there are internal forces that appear to maintain equilibrium
  - Forces parallel to the beam section
  - Forces perpendicular to the beam section (along the beam)
- Shear force → Force parallel to the cross section of the beam
- Bending moment → Caused by unequal forces along the beam creating a Torque/Moment

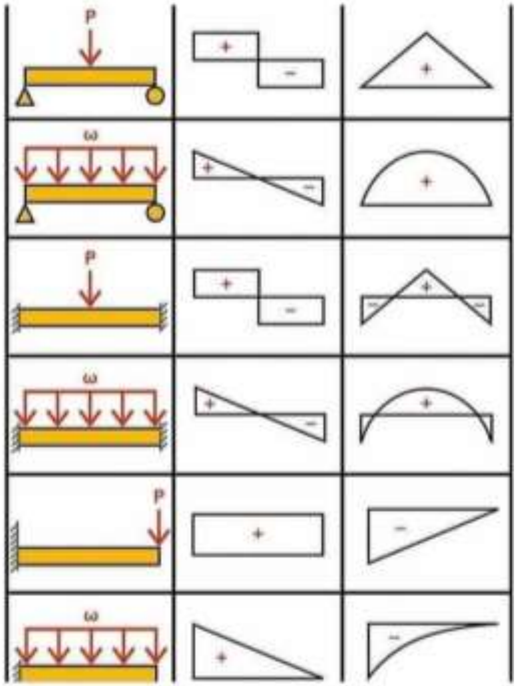






Shear Force and Bending Moment Diagram

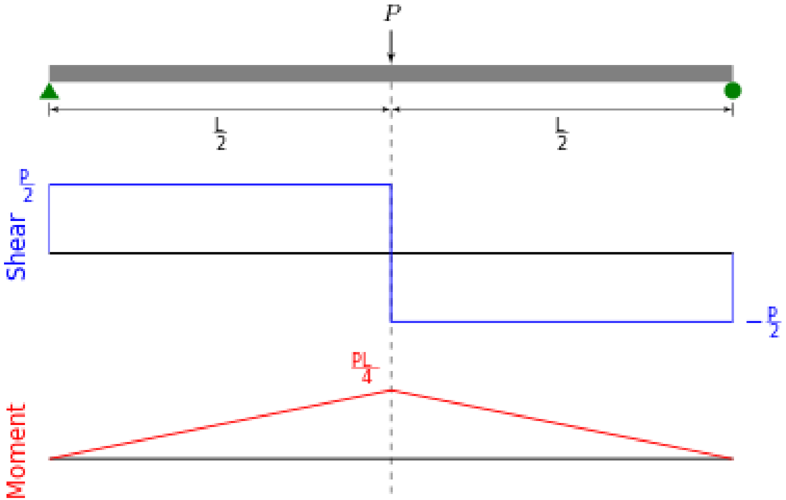
1. Analyze the loads acting on the beam
2. Localize the supports and their types:
    1. pinned support
    2. roller support
    3. fixed support
3. Draw the free body diagram
    1. Acting forces/moments
    2. Reacting forces/moments
4. Determine internal shear forces and bending moments



Example Shear Force and Bending Moment Diagram

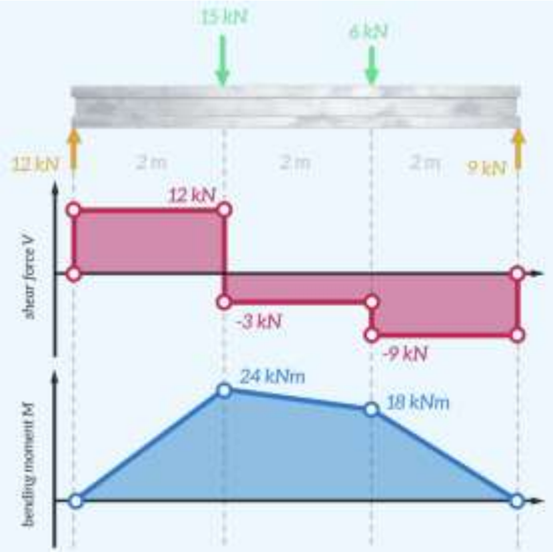
- Convention
  - Positive:

- High shear force and bending moment will cause material failure!
- How do we know if it breaks?
  - Sheer force → Sheer stress (proportional to SheerForce/Section)
  - Bending moment → Strain at rod surface



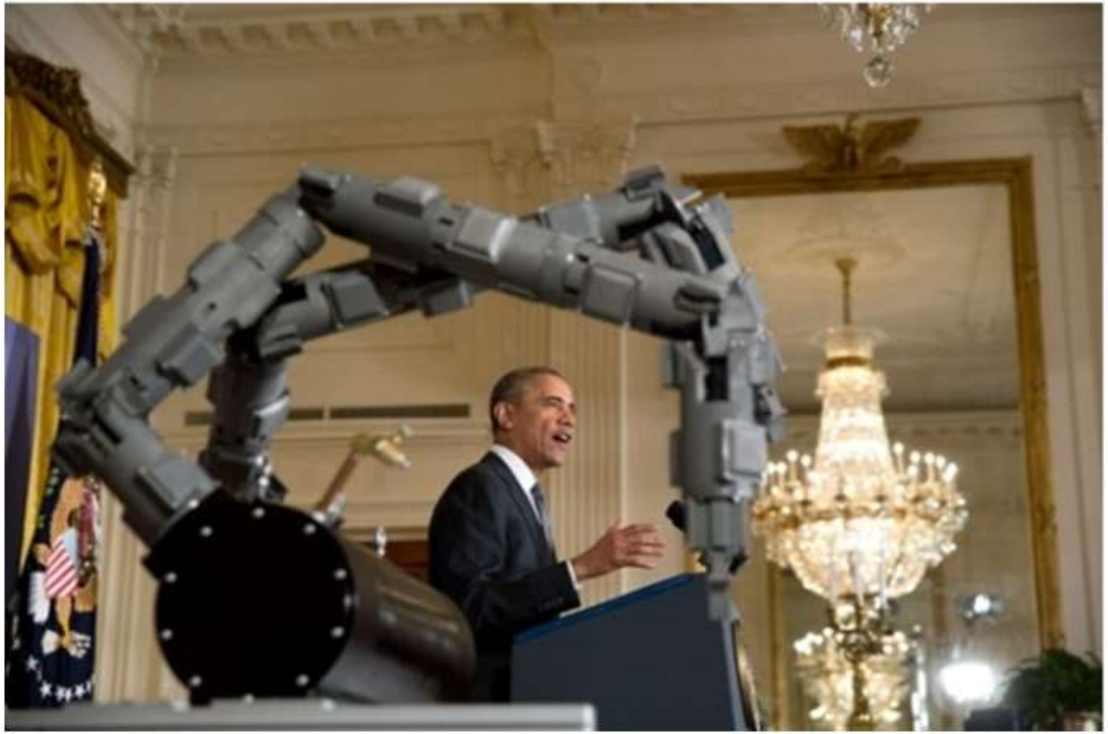
Exercise

- Industrial robot
- Sheer force and bending moment diagram given
- Q1: What is the most probable breaking point if the material does not allow large sheer force?
- Q2: What shouldn't you do to prevent failure due to sheer force?
- Q3: What is the most probable breaking point if the material does not allow large bending moment?
- Q4: What shouldn't you do to prevent failure due to bending moment?



And things can get really complicated.

- 3D Printing means complicated shapes
- But very unique benefits
  - Partial infill (lighter)
  - Connects built into links.
- Consider stresses
  - during operation
  - during fabrication



Summary

- Last Lecture:
  - How do we find the forces applied to (or applied by) a system.
    - Static, quasistatic, dynamic, friction (in all its forms)
- This Lecture:
  - How do real-world materials react to forces?
- The super crash course to understand why we must consider realistic applications and why we can't just make our robots out of cooked (or raw) spaghetti.

Joints

- Why?
  - A robot is just a structure of links (rigid parts) and joints connecting them
- In general, two free bodies can move in SIX dimensions with respect to each other (three displacement, three rotation)
- A joint will constraint the relative motion between two rigid bodies to <6 degrees of freedom

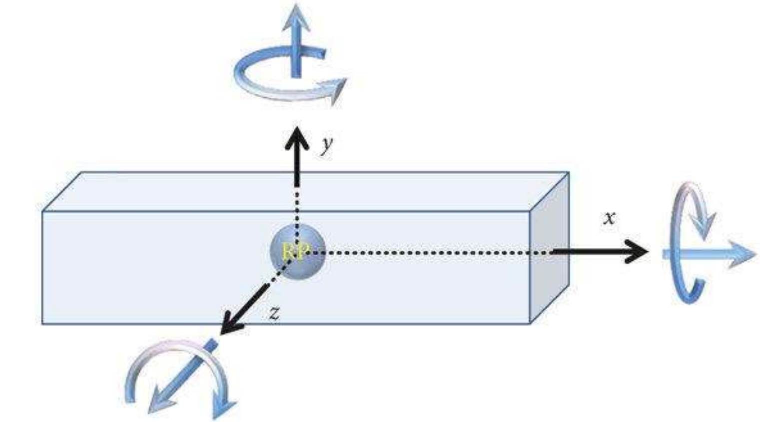
What will you learn today?

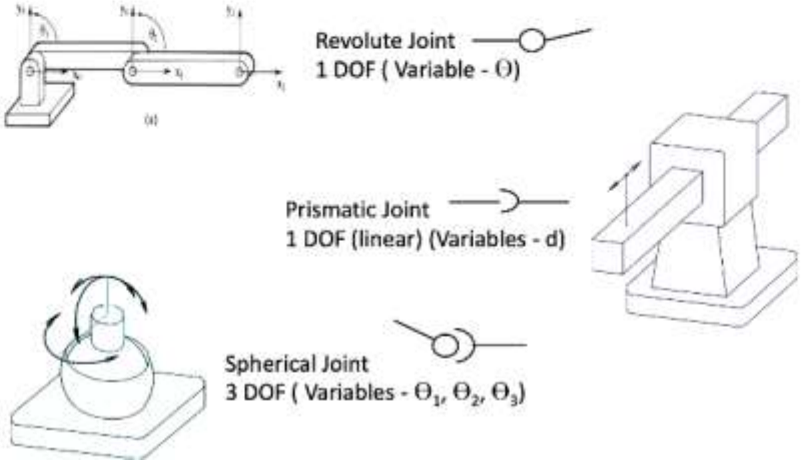
- "Rigid" bodies is an assumption from Physics 101 we must move away from.
  - Bodies can be connected by joints.
  - Joint can be actuated (active) or passive.

Help me find the active and passive joints.

Joints

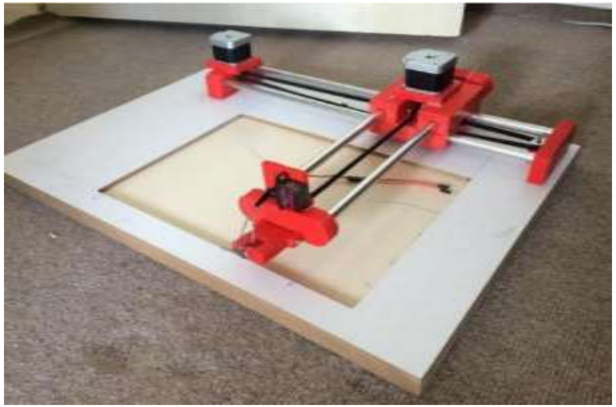
- Two ***free bodies*** can move in 6-dimensions with respect to each other.
  - x, y, z, roll, pitch, yaw
- A ***joint*** constrains the relative motion between two rigid bodies.
- The ***degrees of freedom (DOF)*** is the dimension of unconstrained free movements afforded by a joint.

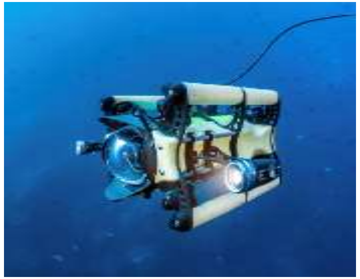




What is a Degree of Freedom?

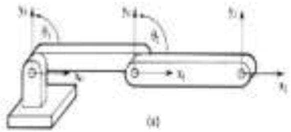
- the number of independent parameters that can fully define the Configuration Space (c) of an object.



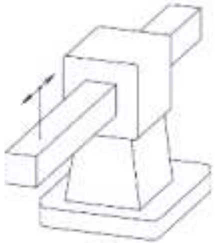


- What is the DOF of each system above?

Technically there are lots of joint types.

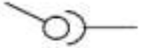


Revolute Joint
1 DOF (Variable - Θ)

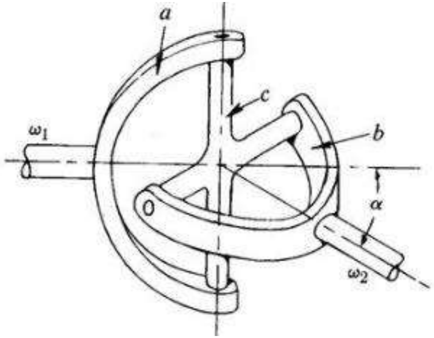


Prismatic Joint
1 DOF (linear) (Variables - d)

Spherical Joint
3 DOF (Variables - Θ_1, Θ_2, Θ_3)

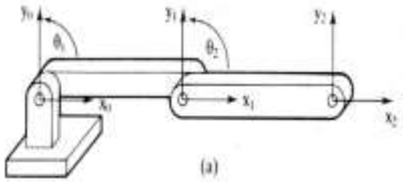


Universal Joint
2 DOF



Degrees of freedom	Free rotations	Free translations	Name	Kinematic pair	
				Form closure	Force closure
5	3	2	Sphere-plane		
4	3	1	Sphere-groove		
	2	2	Cylinder-plane		
3	3	0	Spheric		
	2	1	Sphere-slotted cylinder		
	1	2	Planar		
2	2	0	Slotted spheric		
	2	0	Toric		
	1	1	Cylindric		
	1	1	Slotted cylinder		
1	1	0	Revolute		
	0	1	Prismatic		

Types of (Lower) Joints

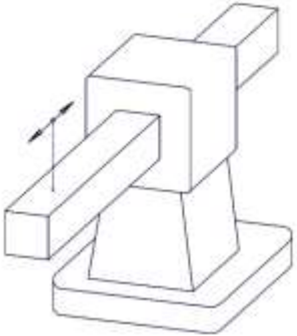


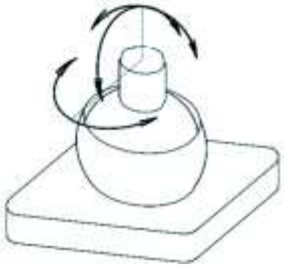
Revolute Joint 

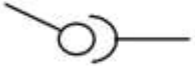
1 DOF (Variable - Θ)

Prismatic Joint 

1 DOF (linear) (Variables - d)





Spherical Joint 

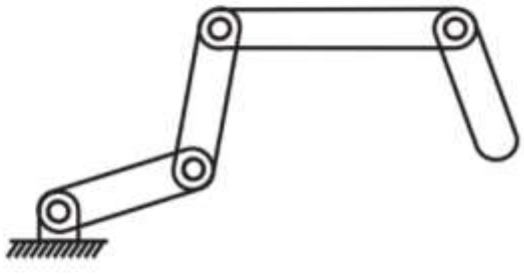
3 DOF (Variables - Θ_1, Θ_2, Θ_3)

Grübler’s Formula to “count” Degrees of Freedom

- General idea: DOF of mechanism = Link DOFs – Joint Constraints
- $M = k(n-1) - \sum_{i=1}^{j}(k - f_i) = k(n-1) - \sum_{i=1}^{j} c_i = k(n-1-j) + \sum_{i=1}^{j} f_i$
  - M: number of degrees of freedom of a mechanism
  - k: dofs of a link (3 for planar mechanisms, 6 for unconstrained)
  - n: number of links (include ground as a link!)
  - j: number of joints
  - f_i: degrees of freedom of joint i
  - c_i: degrees of freedom of joint i

What is a Degree of Freedom?

- the number of independent parameters that can fully define the Configuration Space (c) of an object.



- Atlas, 28 Joints
  - 6 x 2 arms = 12
  - 6 x 2 legs = 12
  - 3 pelvis
  - 1 neck

- What is the DOF of each system above?

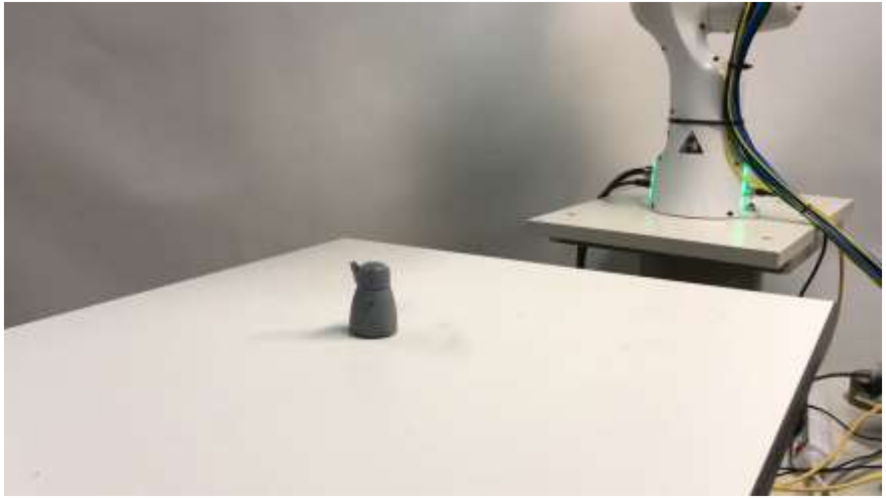
Active vs. Passive Degrees of Freedom

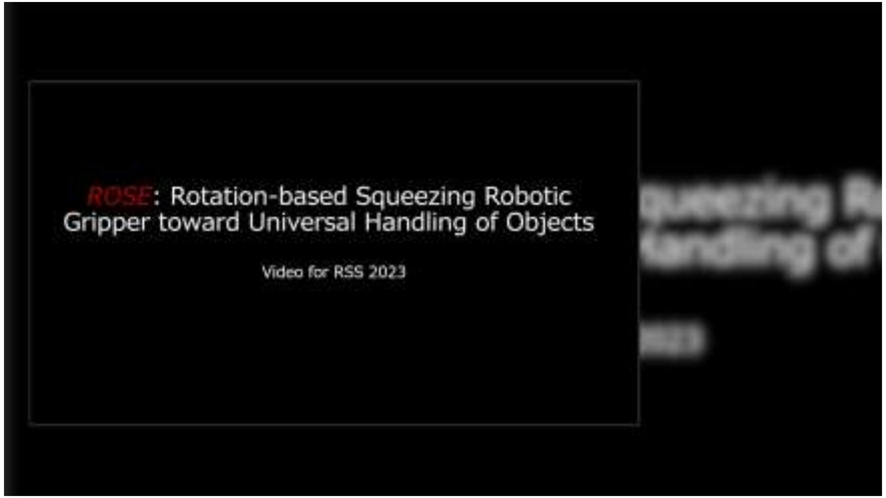
- Active DoF: its state can be directly controlled
- Passive DoF: its state is a result of active DoFs and other constraints

Fully Actuated vs. Underactuated System

- Fully actuated system: all DoF are active
- Underactuated system: there are more DoFs than active DoFs (= some DoFs are passive)

Extreme Underactuation





Morphological Computation: certain processes are performed by the body that otherwise would have to be performed by the brain.

Fully Actuated vs. Underactuated System

A Formal Definition

- State of the system: positions () and velocities ()
- Control signal to the system (we create this!):

- Dynamics of the system: $\ddot{q} = f(q, \dot{q}, u, t)$
- Fully actuated:
  - The system is fully actuated at time t and state $(q, \dot{q})$ if we are able to obtain an instantaneous acceleration $\ddot{q}$ in any direction using u
- Underactuated
  - The system is underactuated at time t and state $(q, \dot{q})$ if we cannot generate any instantaneous acceleration $\ddot{q}$ using u

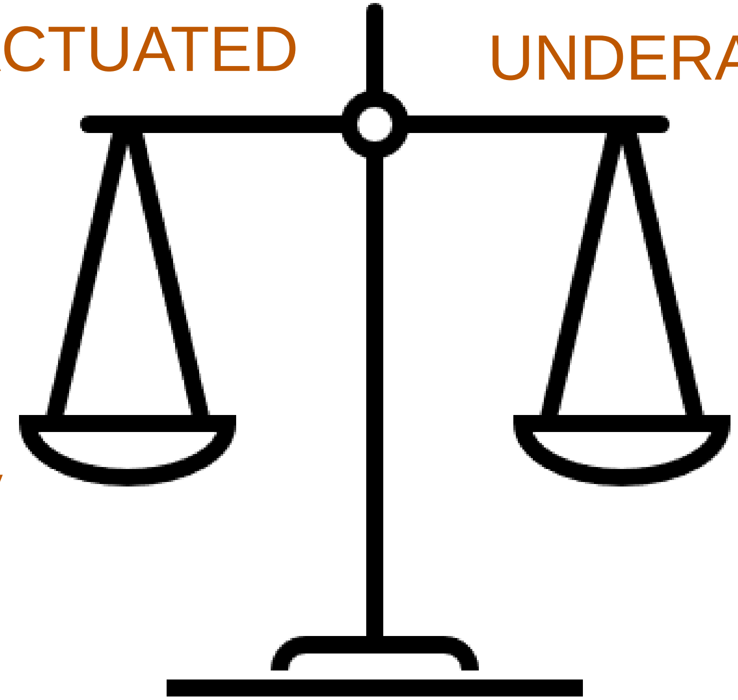
Human Motion (Walking)

- Is our walking actuated or underactuated?

Trade-offs

FULLY ACTUATED

UNDERACTUATED



control

complexity

observability

adaptation

simplicity

randomness

Summary

- In a robot rigid bodies are connected by joints.
- The number and type of joints determine the system's ***Degrees of Freedom (DoF of DOF)***
- Joints can be active or passive.
- Not every joint maps to a motion in the ***Configuration Space***
  - Some just improve the ***function*** of the robot.