Parallel Architectures
Parallel Algorithms
CUDA

Chris Rossbach

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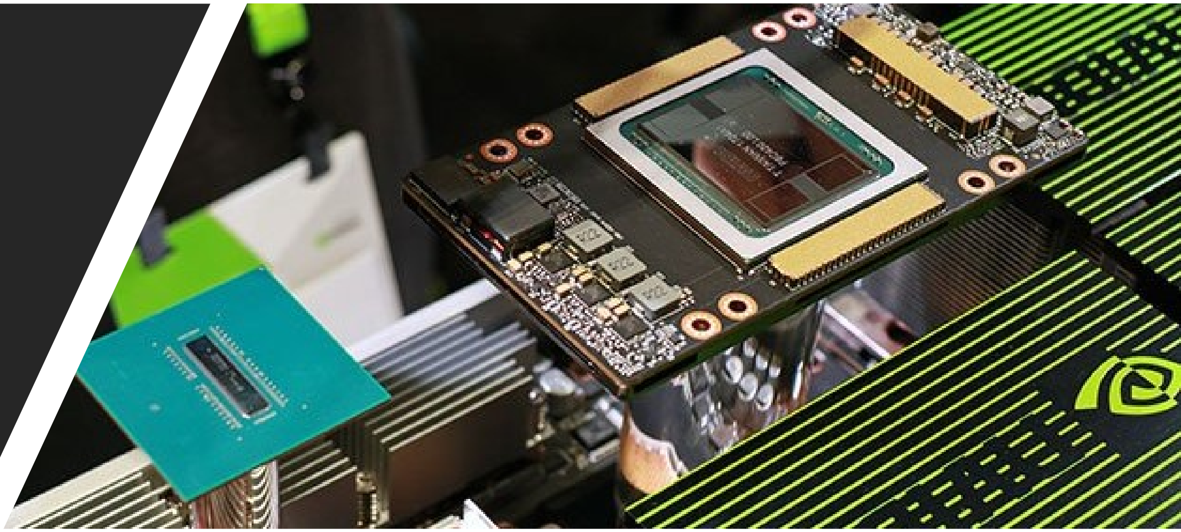
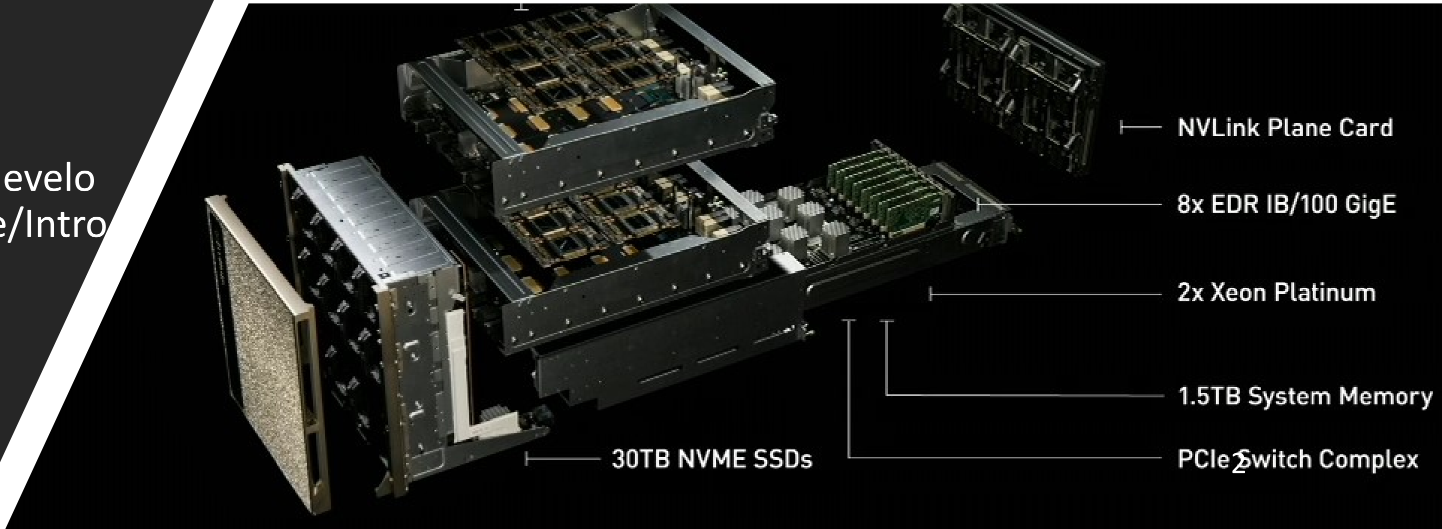
Outline for Today

- Questions?
- Administrivia
  - pedagogical-* machines should be available
- Agenda
  - Parallel Algorithms
  - CUDA

- Acknowledgements: http://developer.download.nvidia.com/compute/developertrainingmaterials/presentations/cuda_language/Introduction_to_CUDA_C.pptx

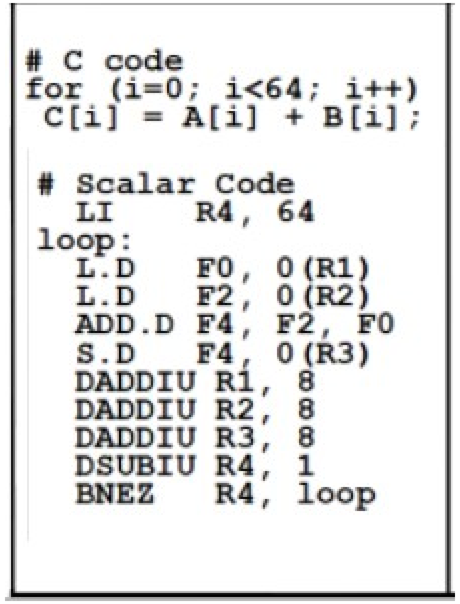




Faux Quiz Questions

- What is a reduction? A prefix sum? Why are they hard to parallelize and what basic techniques can be used to parallelize them?
- Define flow dependence, output dependence, and anti-dependence: give an example of each. Why/how do compilers use them to detect loop-independent vs loop-carried dependences?
- What is the difference between a thread-block and a warp?
- How/Why must programmers copy data back and forth to a GPU?
- What is “shared memory” in CUDA? Describe a setting in which it might be useful.
- CUDA kernels have implicit barrier synchronization. Why is __syncthreads() necessary in light of this fact?
- How might one implement locks on a GPU?
- What ordering guarantees does a GPU provide across different hardware threads’ access to a single memory location? To two disjoint locations?
- When is it safe for one GPU thread to wait (e.g. by spinning) for another?

Review: what is a vector processor?



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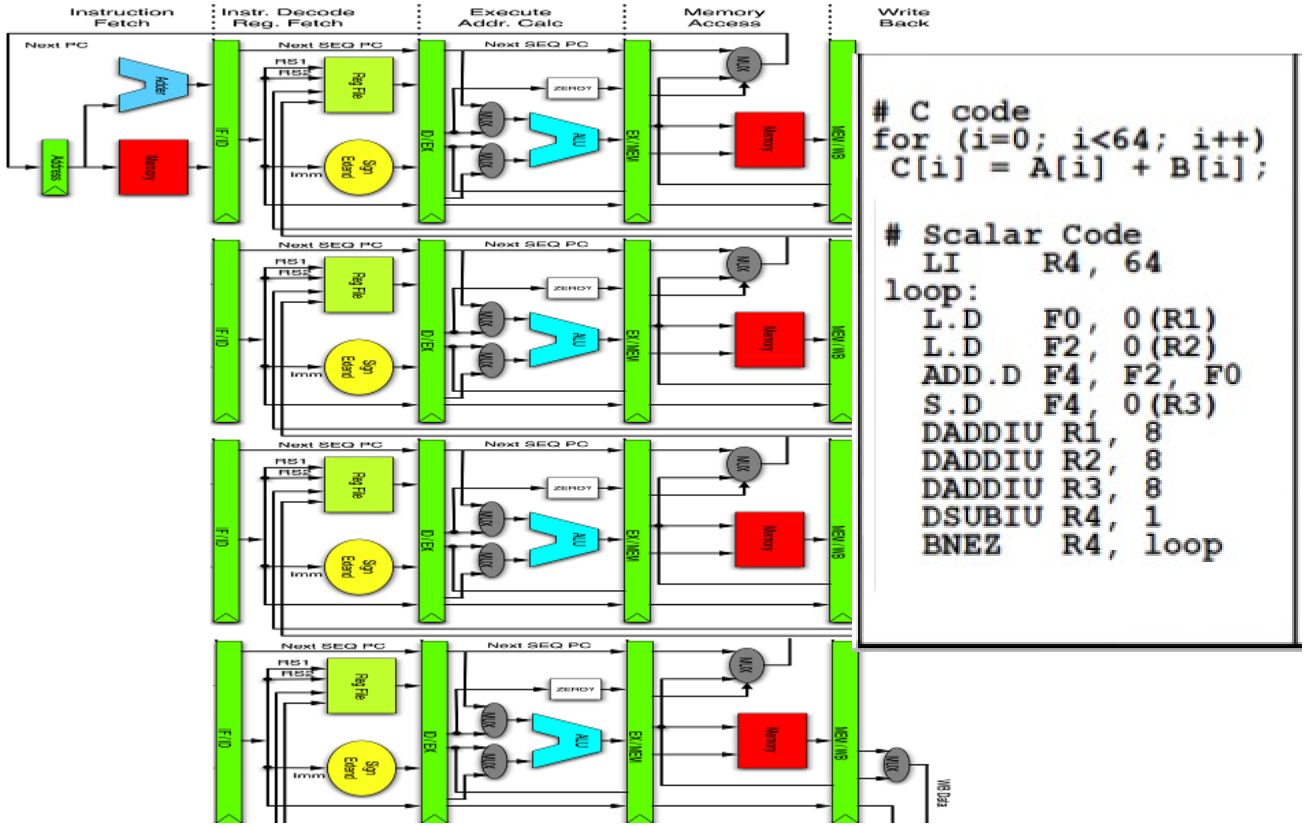

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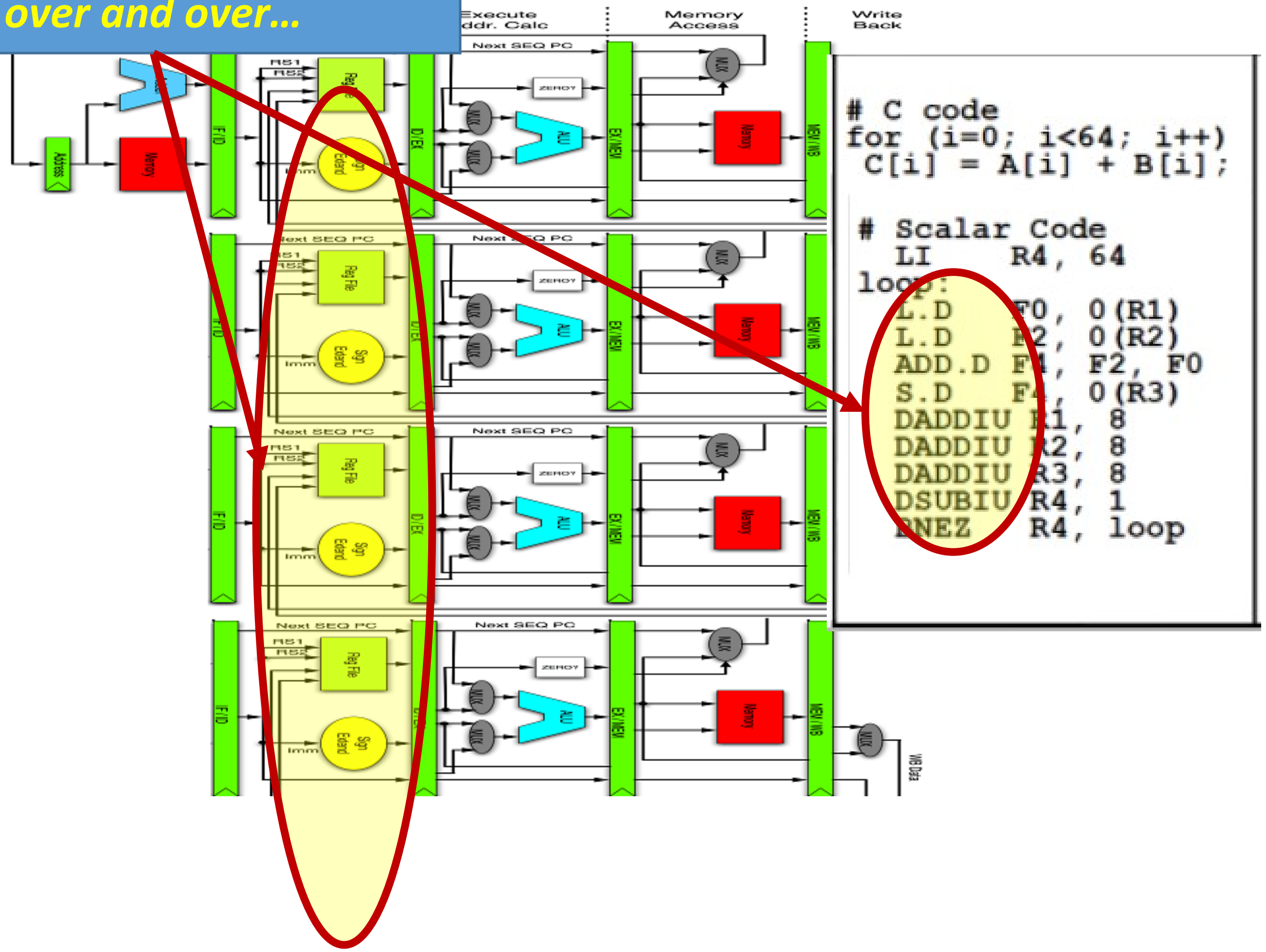
Dont decode same instruction over and over…



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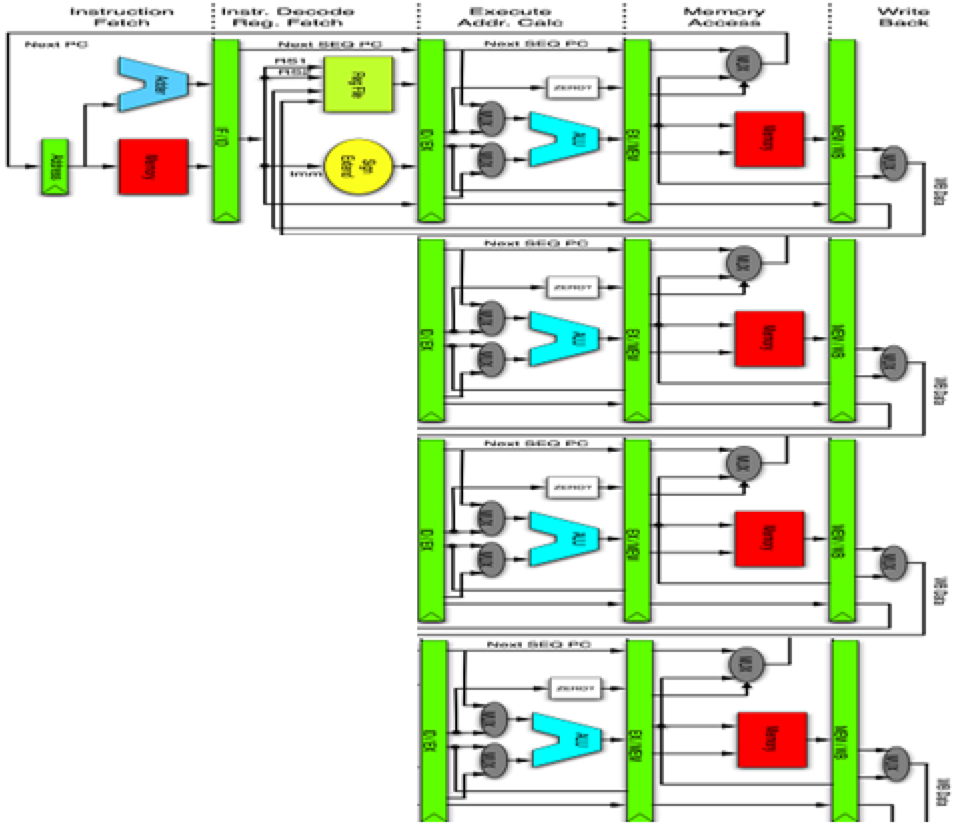

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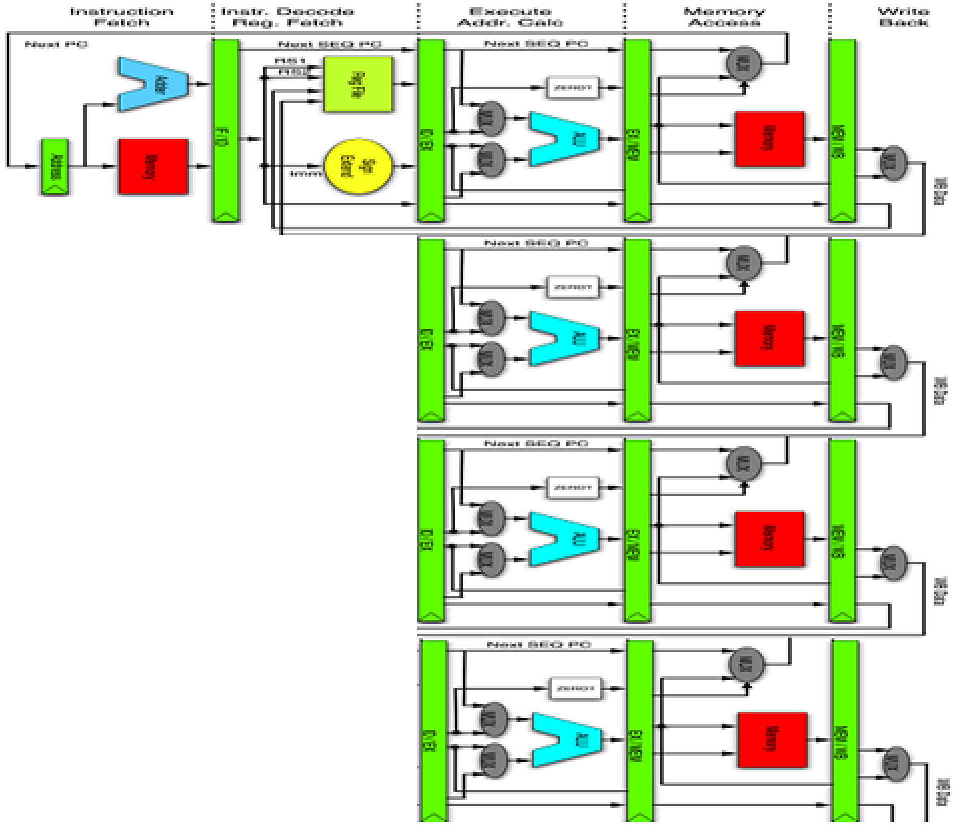

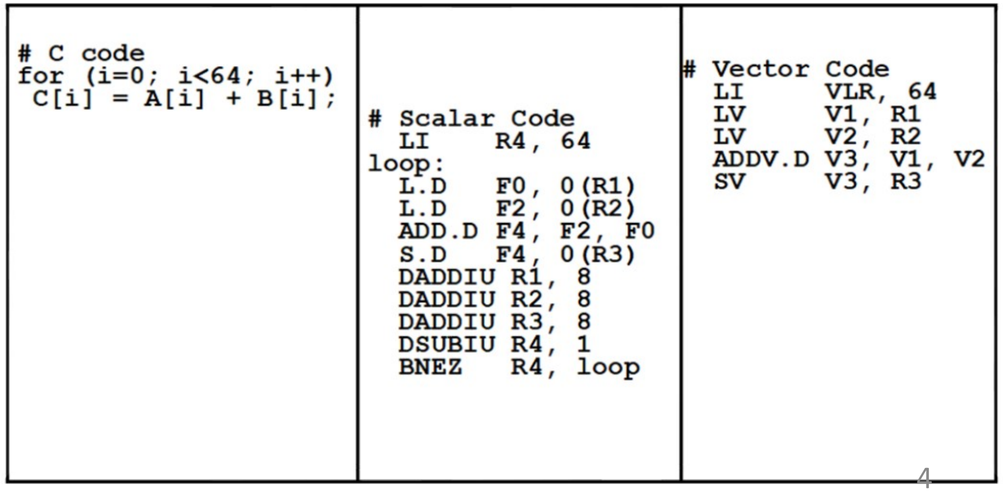

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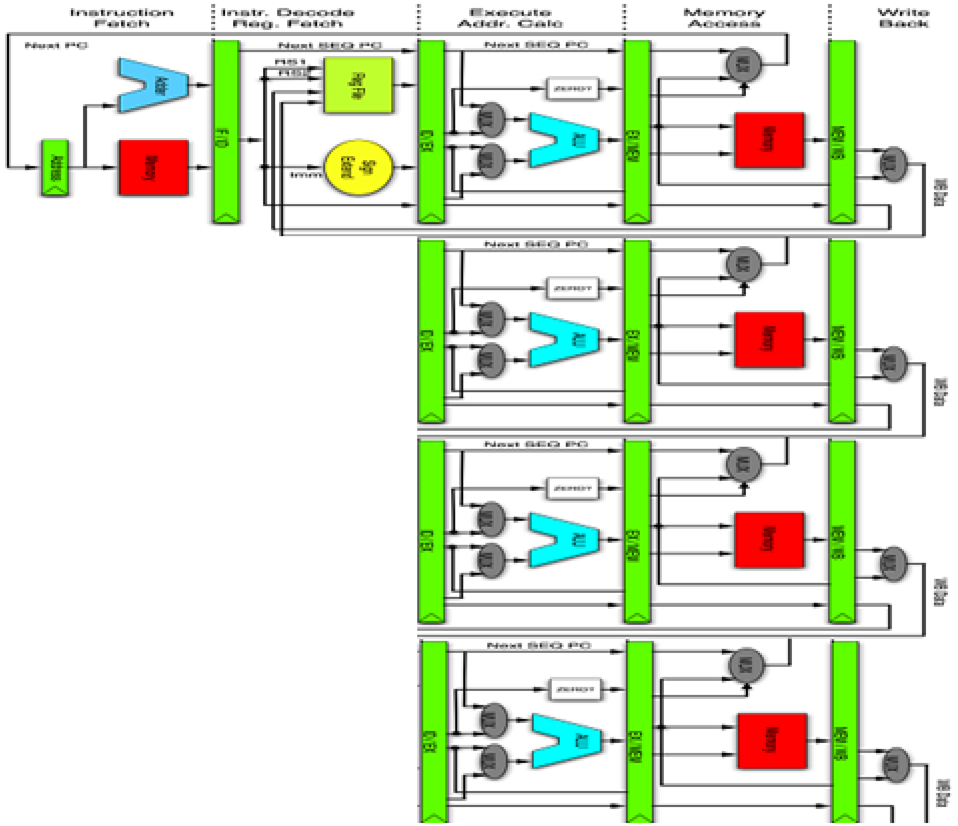
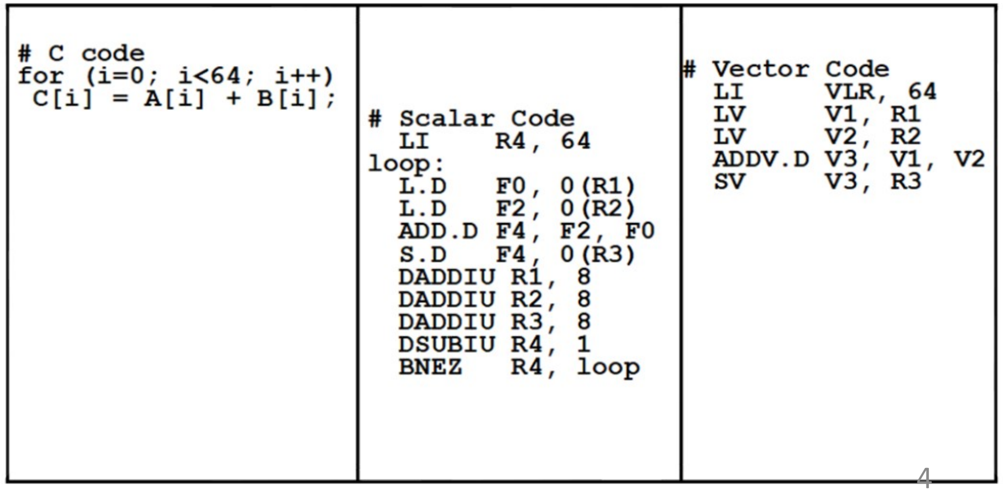


Implementation:

- Instruction fetch control logic shared
- Same instruction stream executed on
- Multiple pipelines
- Multiple different operands in parallel



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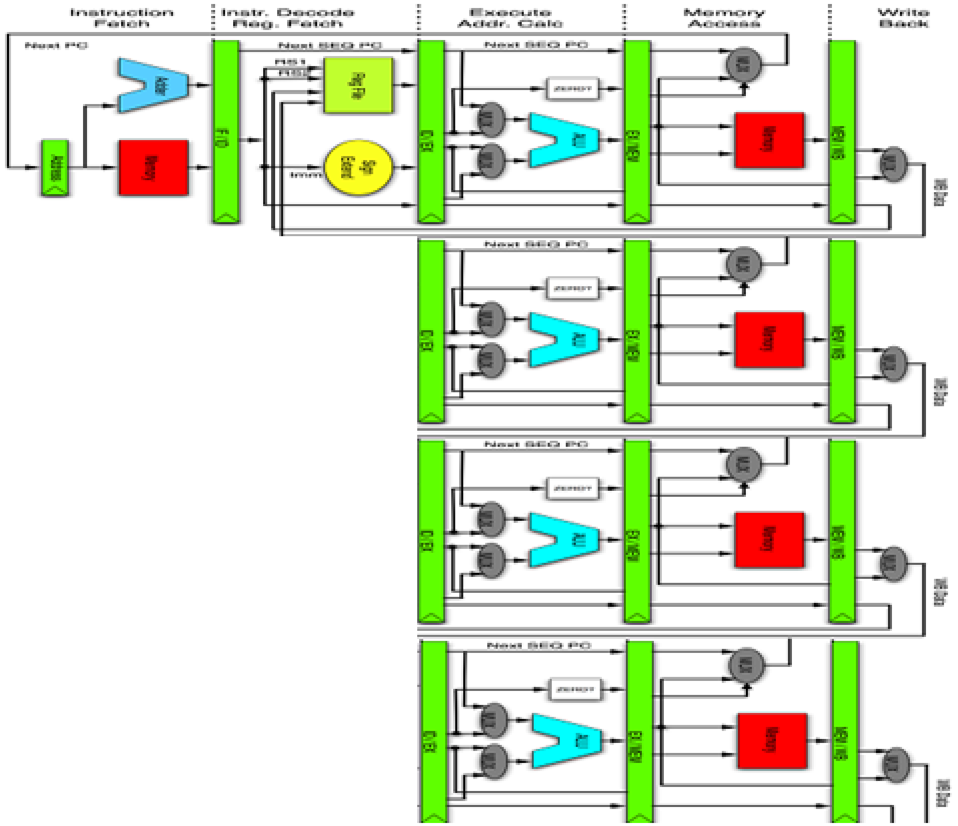

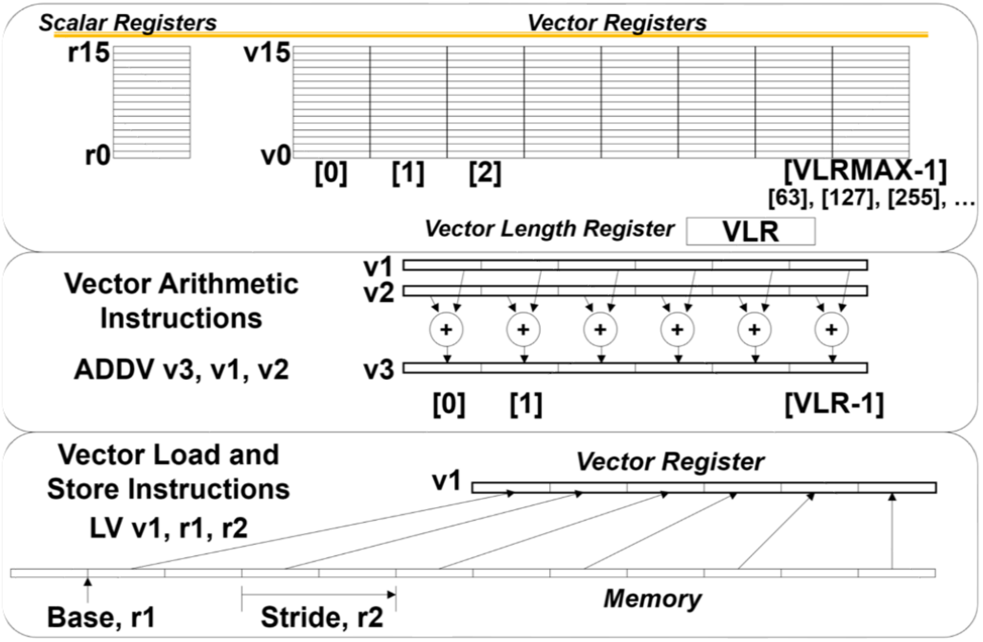

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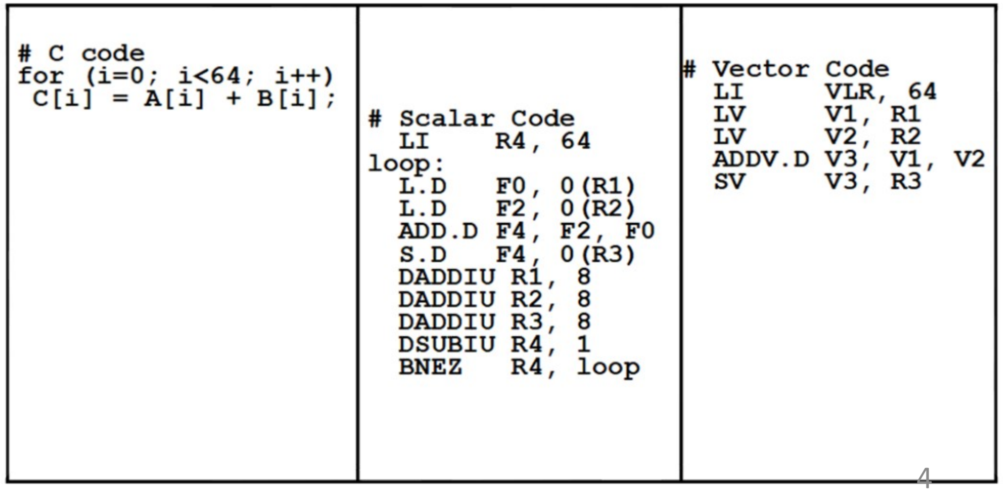

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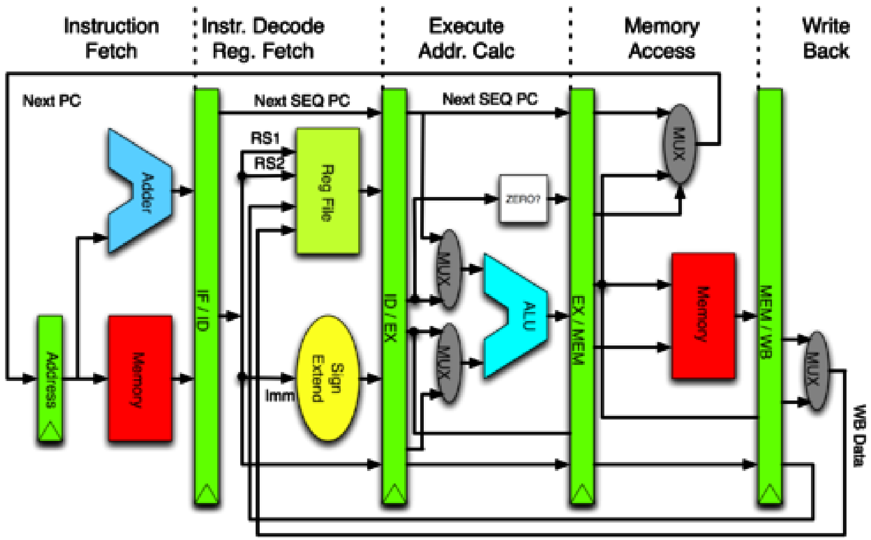

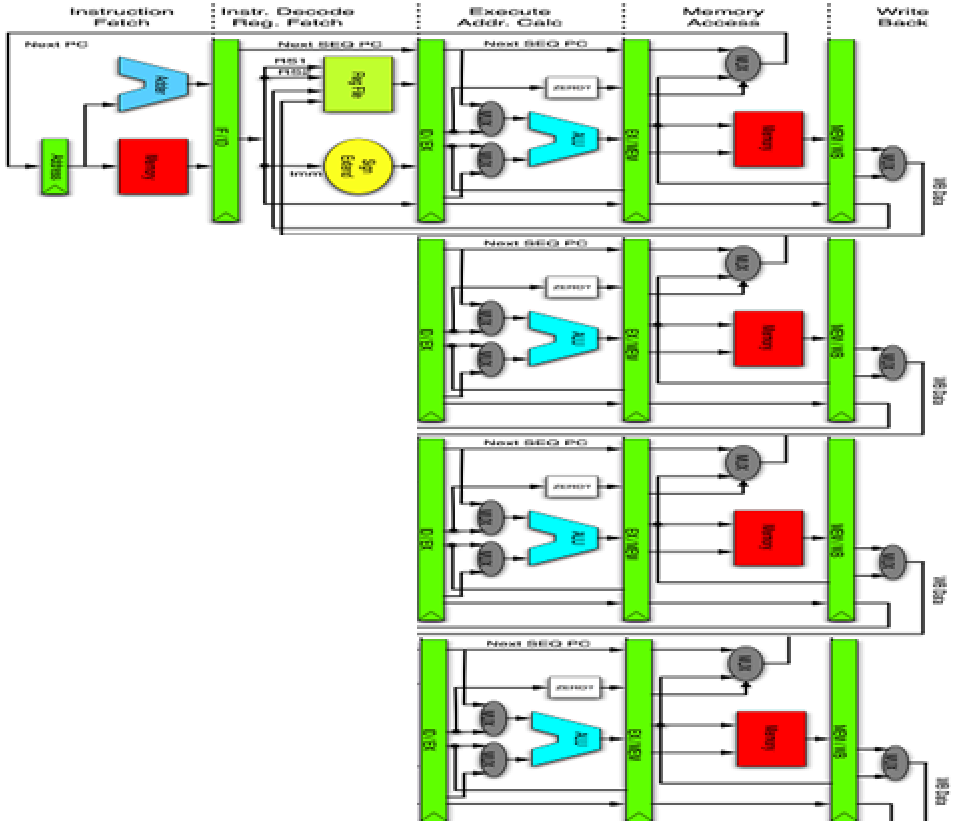

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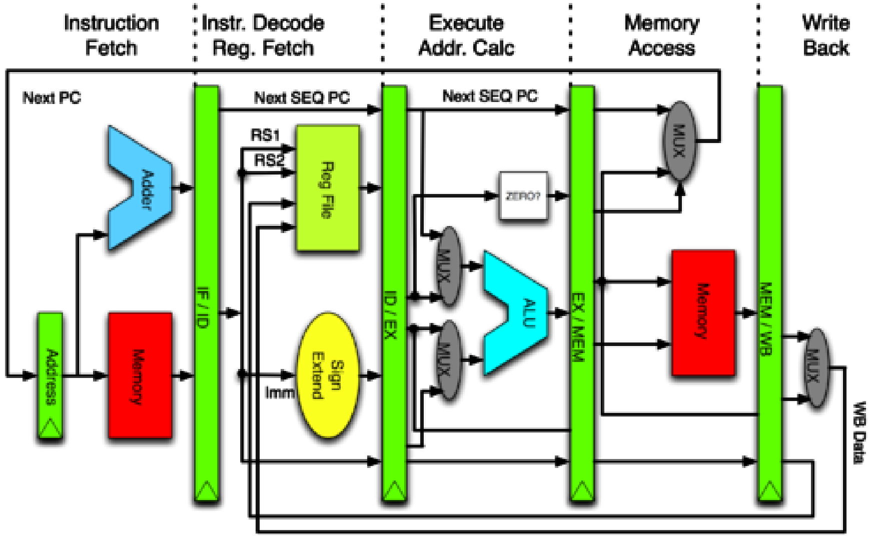
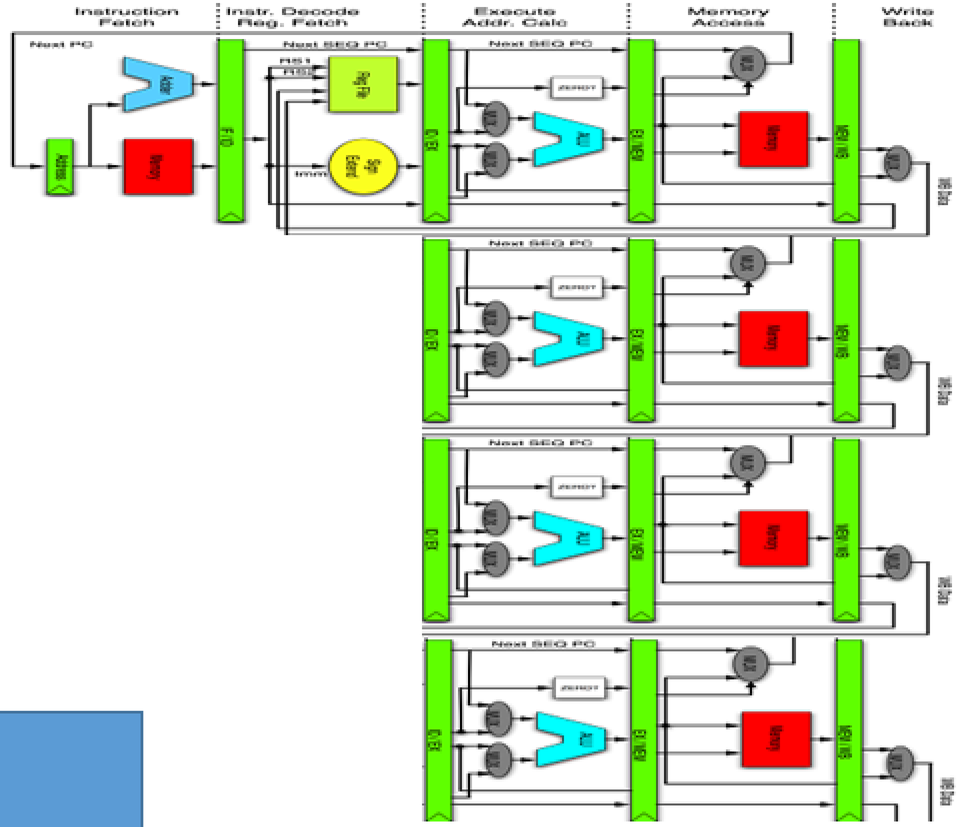
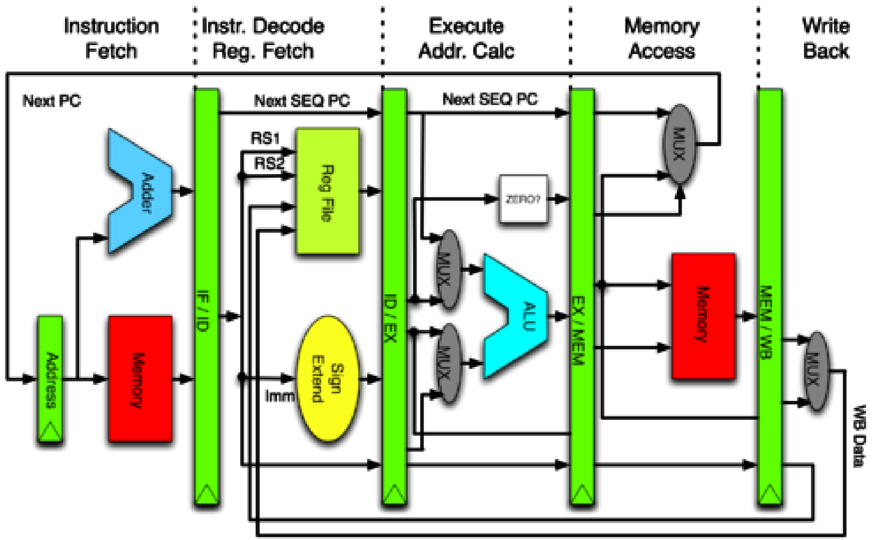
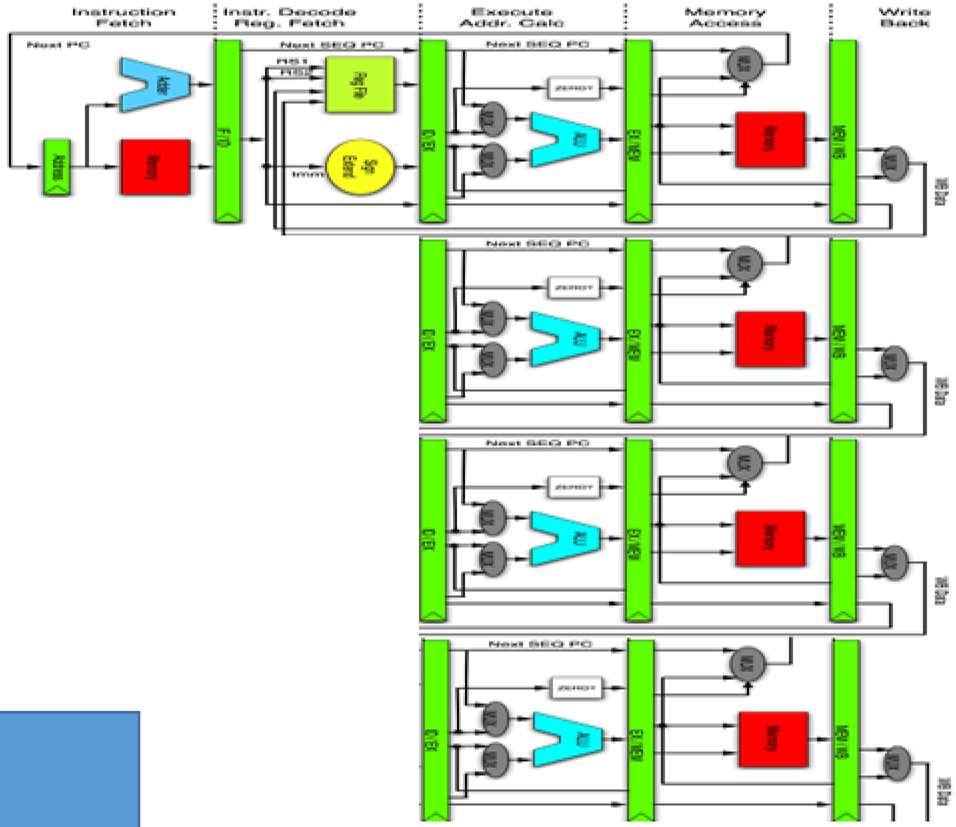




What are the potential bottlenecks here?
When can it improve throughput?

When does vector processing help?





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When can it improve throughput?

Only helps if memory can keep the pipeline busy!

Hardware multi-threading

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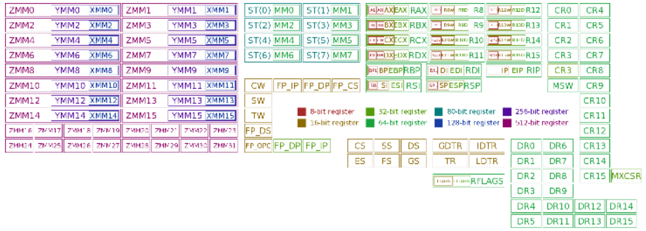

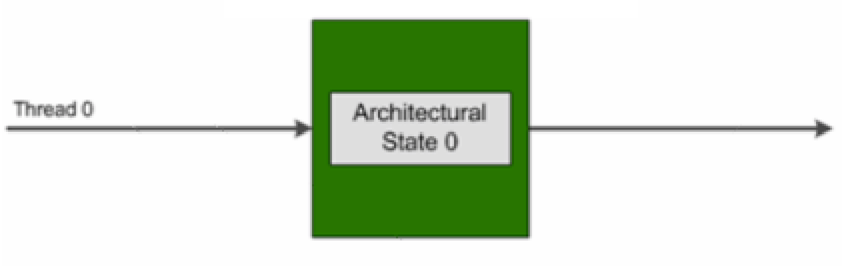

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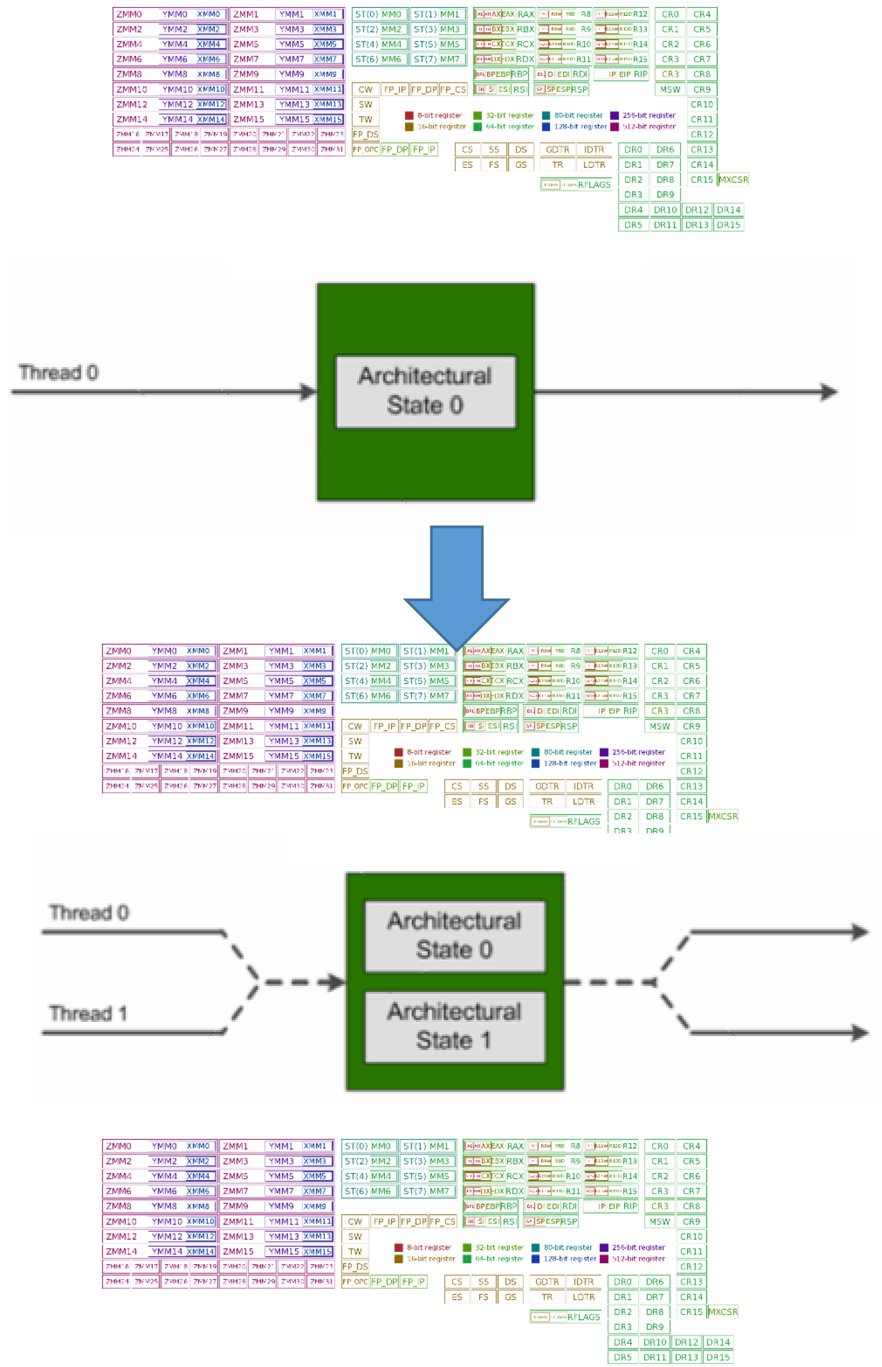

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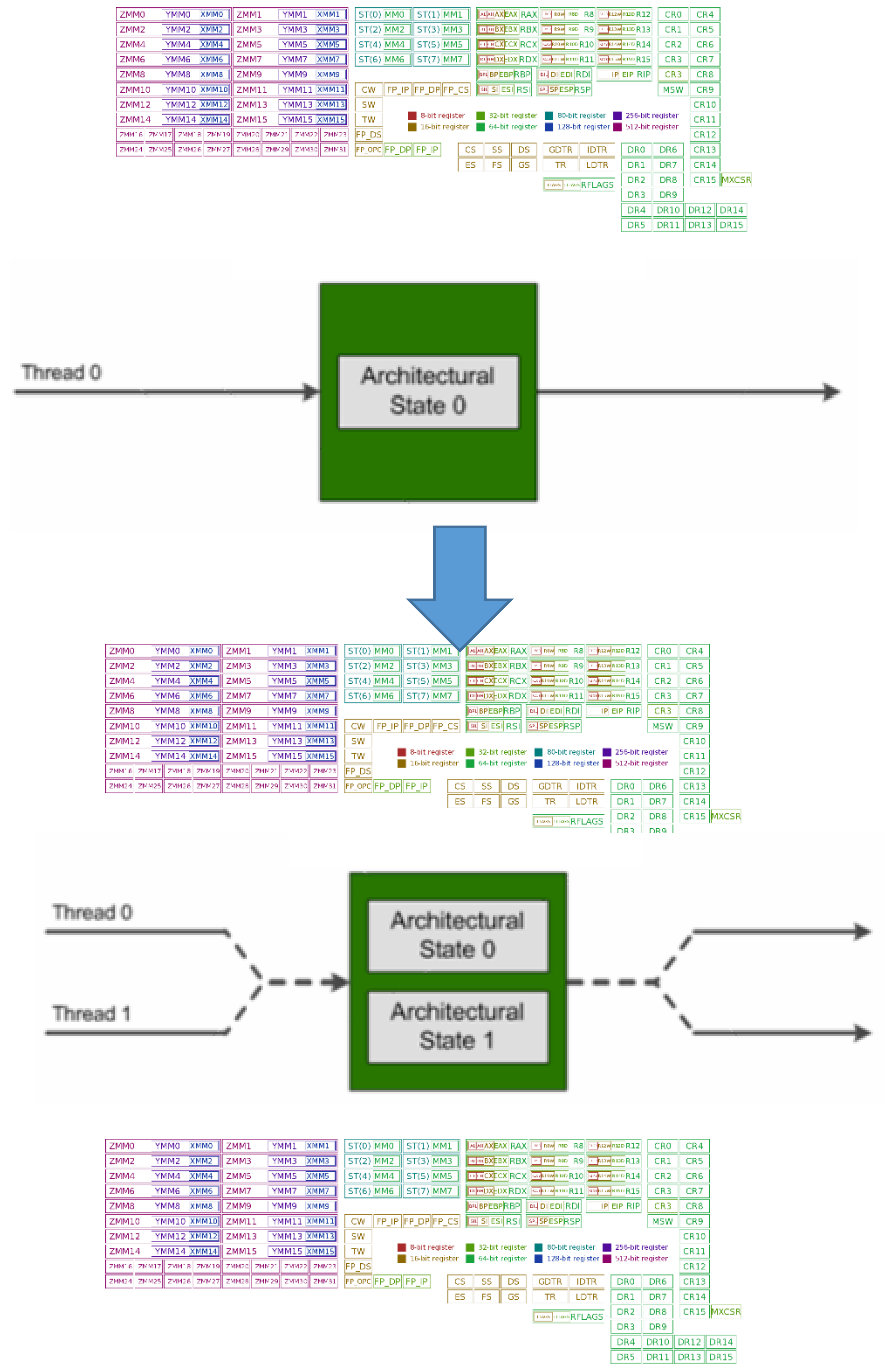

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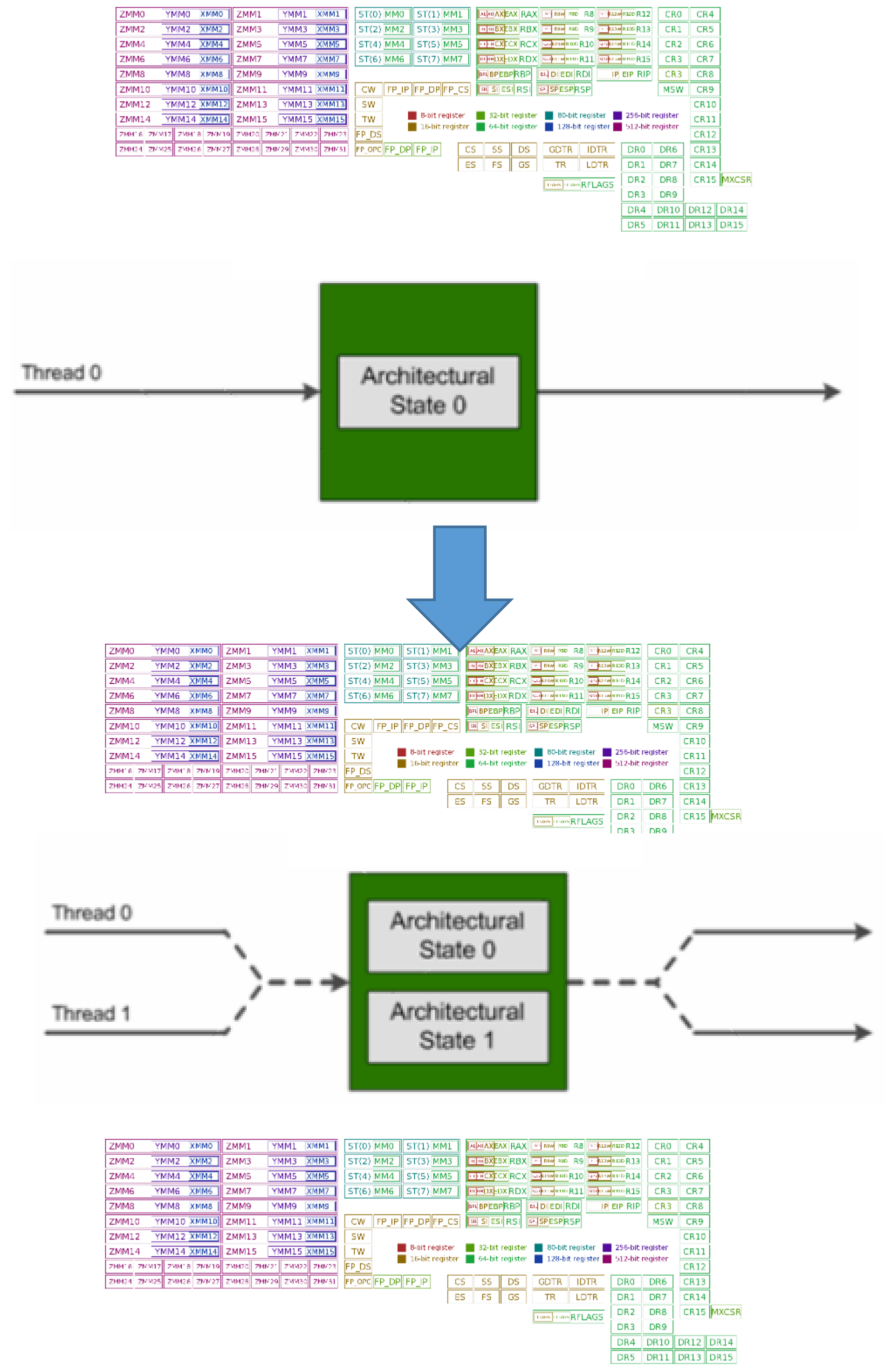
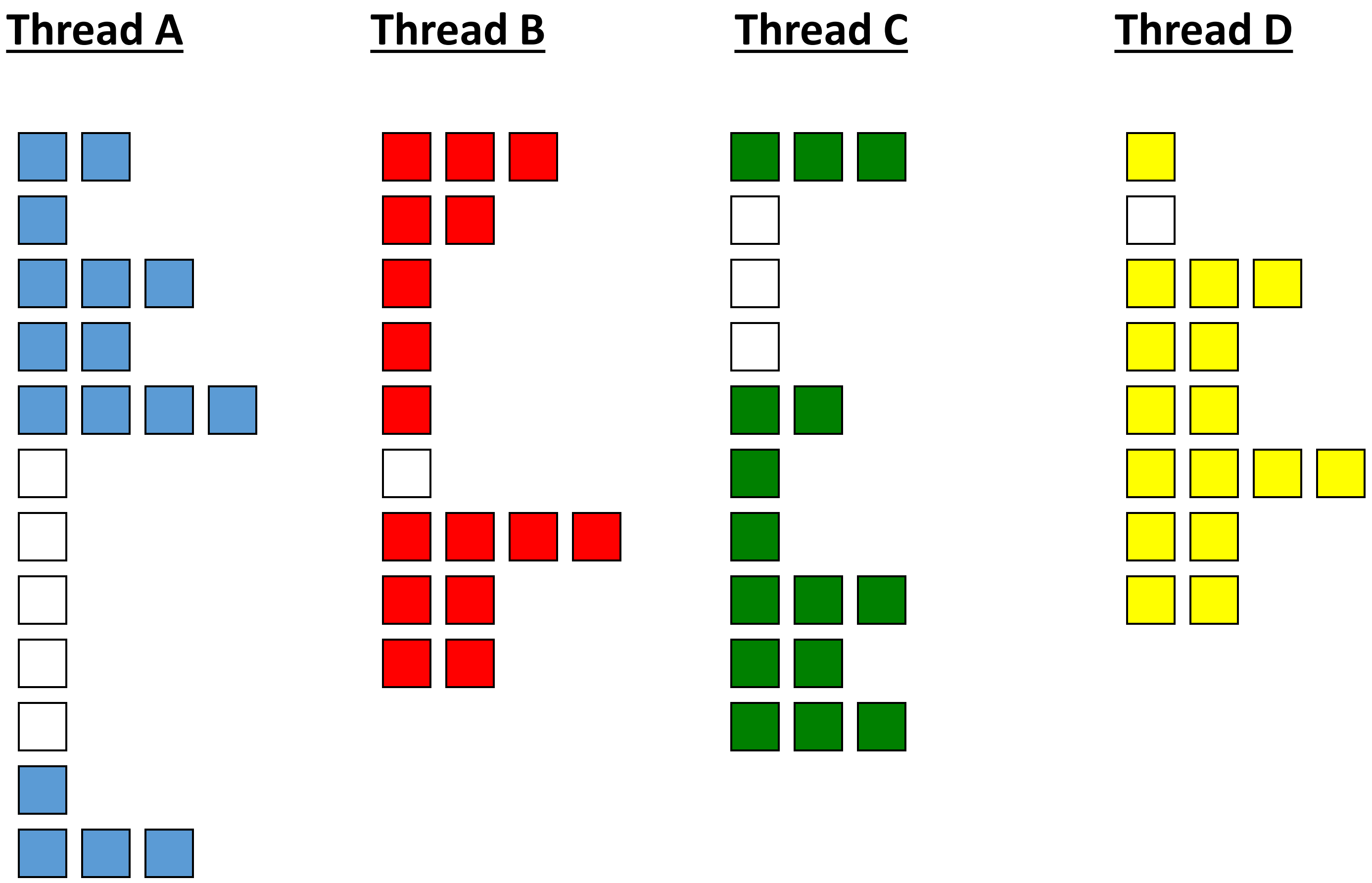
- Three variants:
  - Coarse
  - Fine-grain
  - Simultaneous



Running example



- Colors → pipeline full
- White → stall

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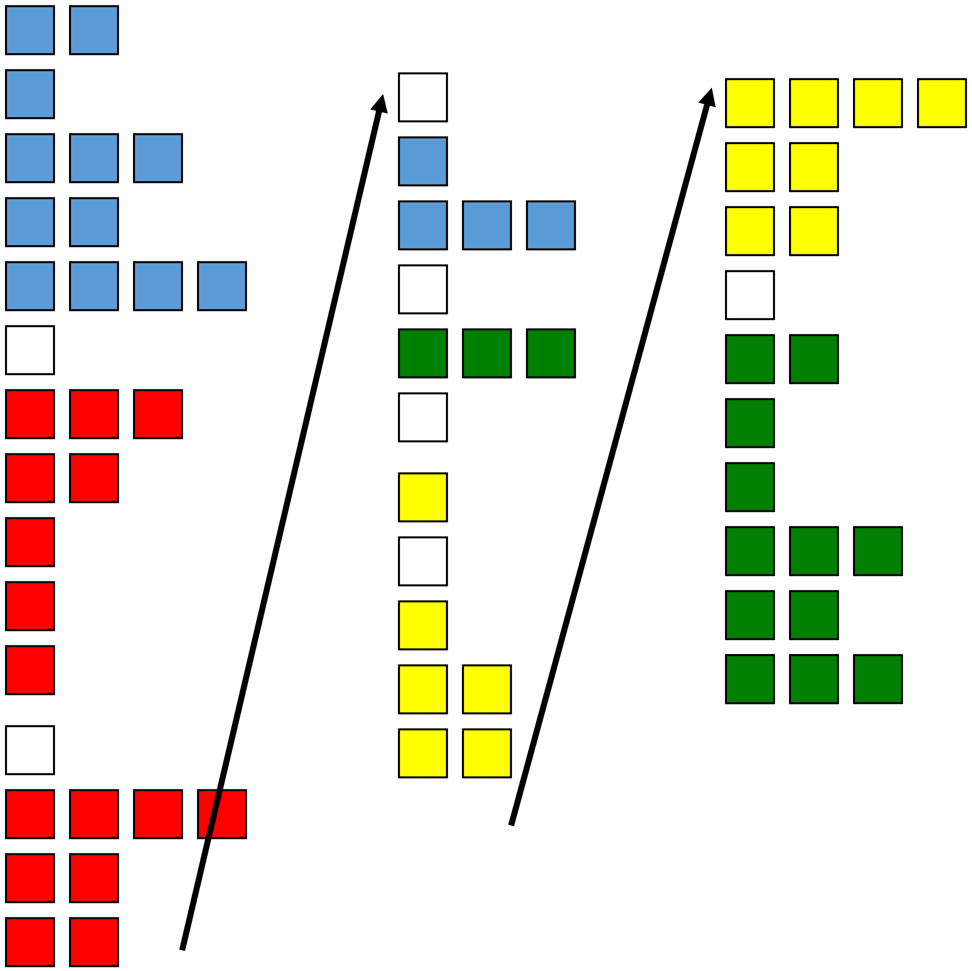

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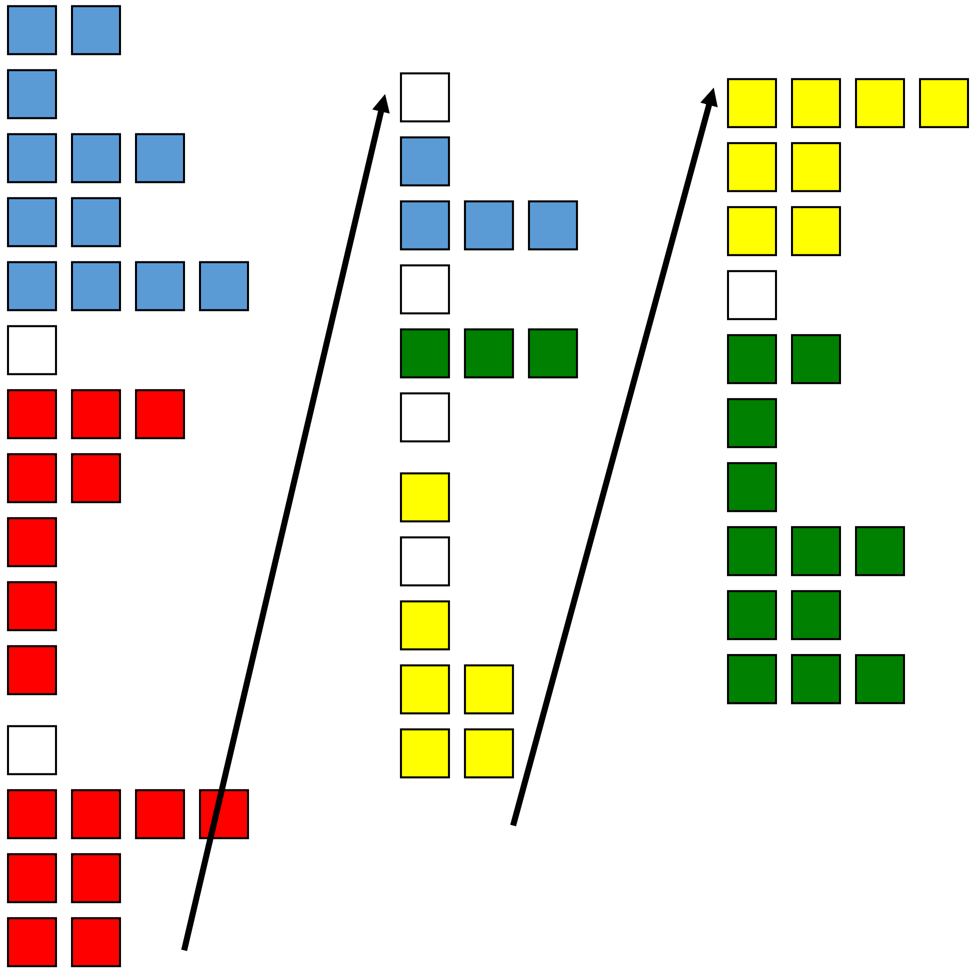

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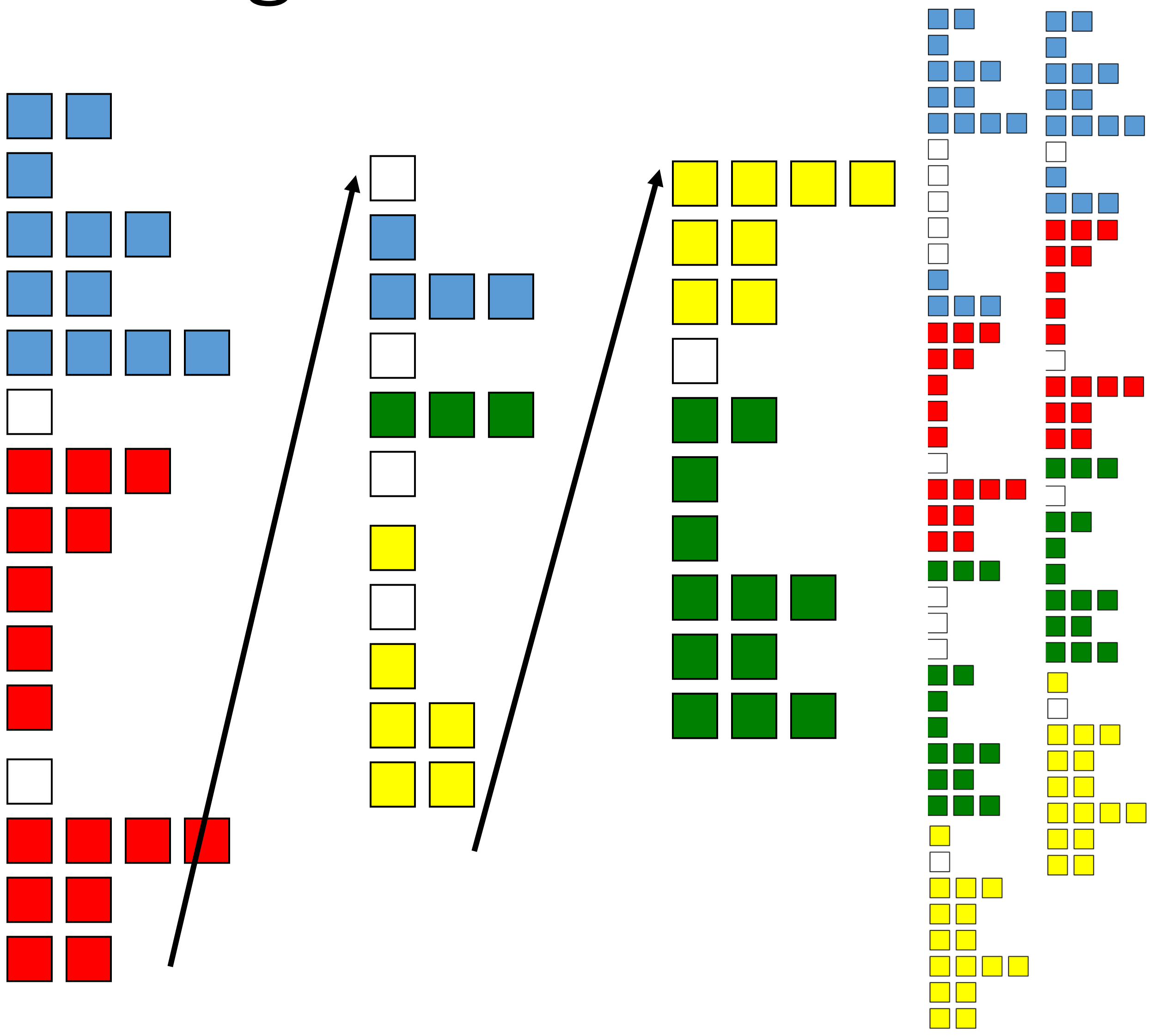

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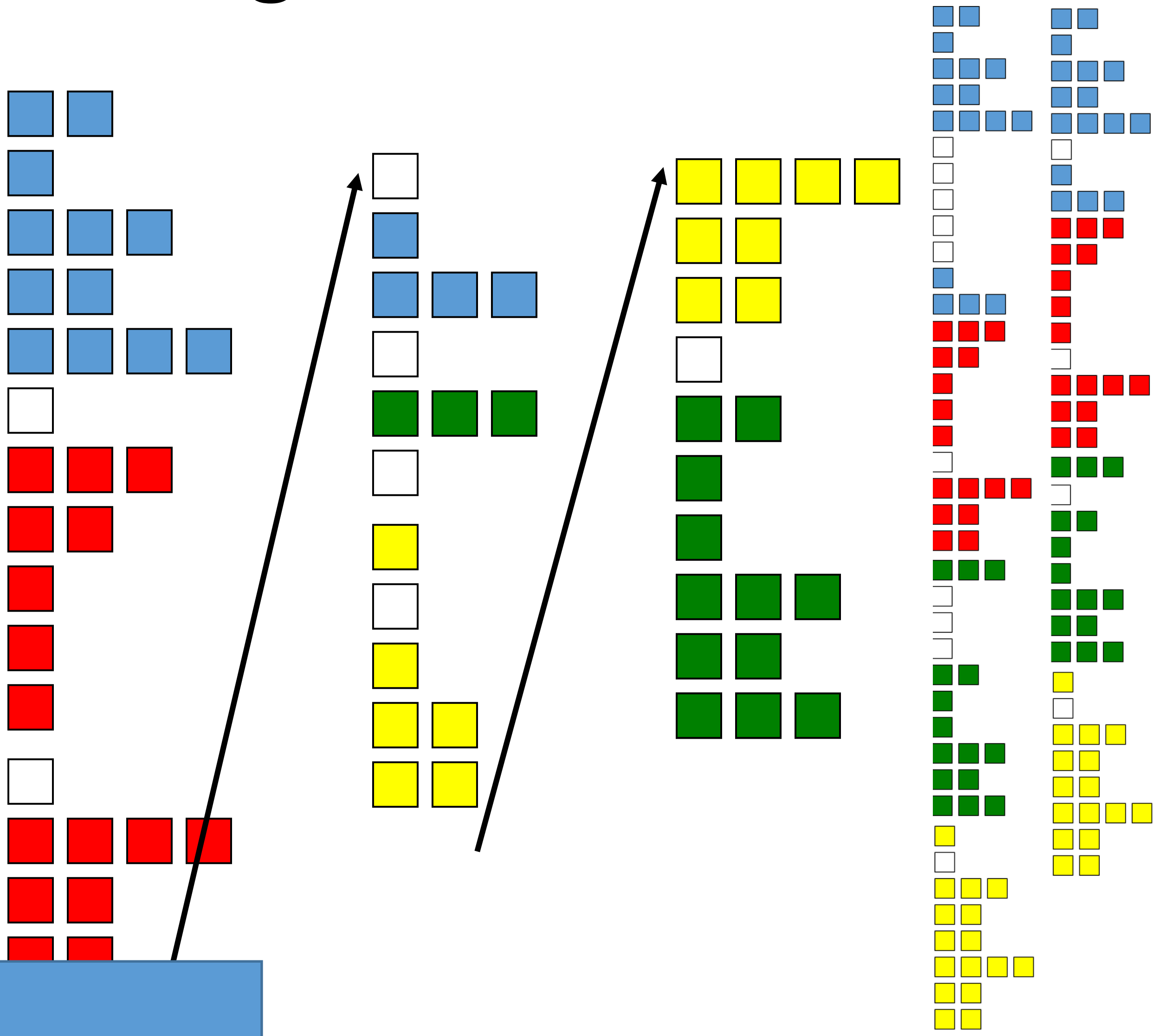

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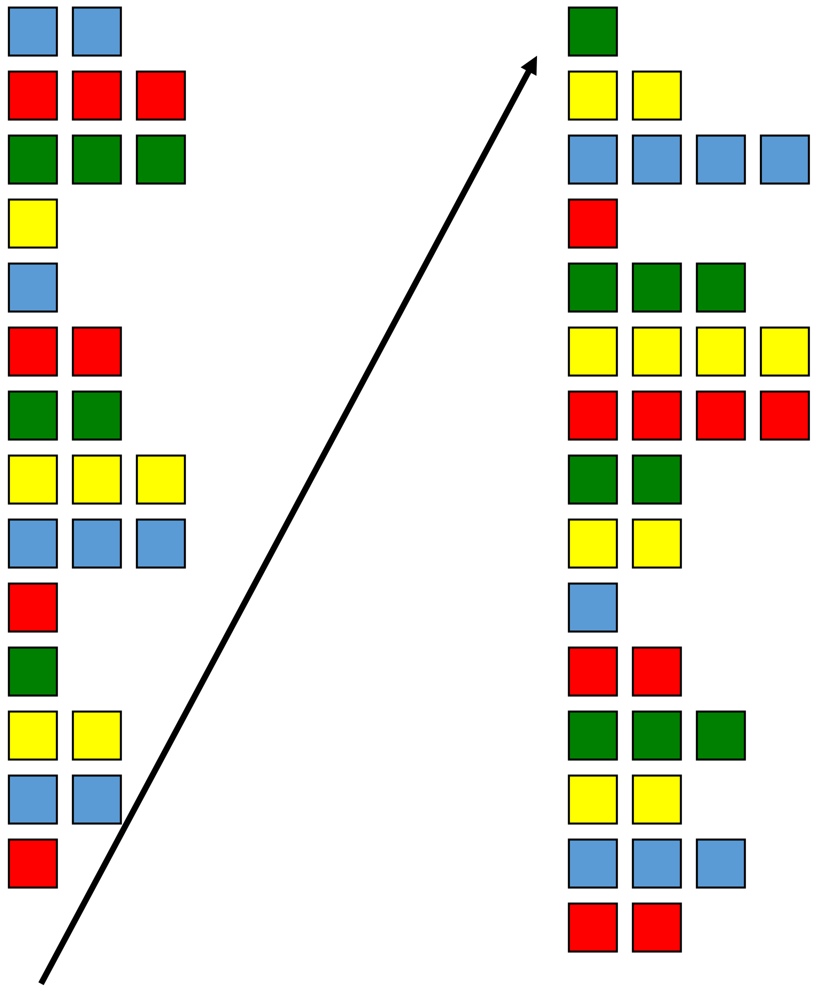

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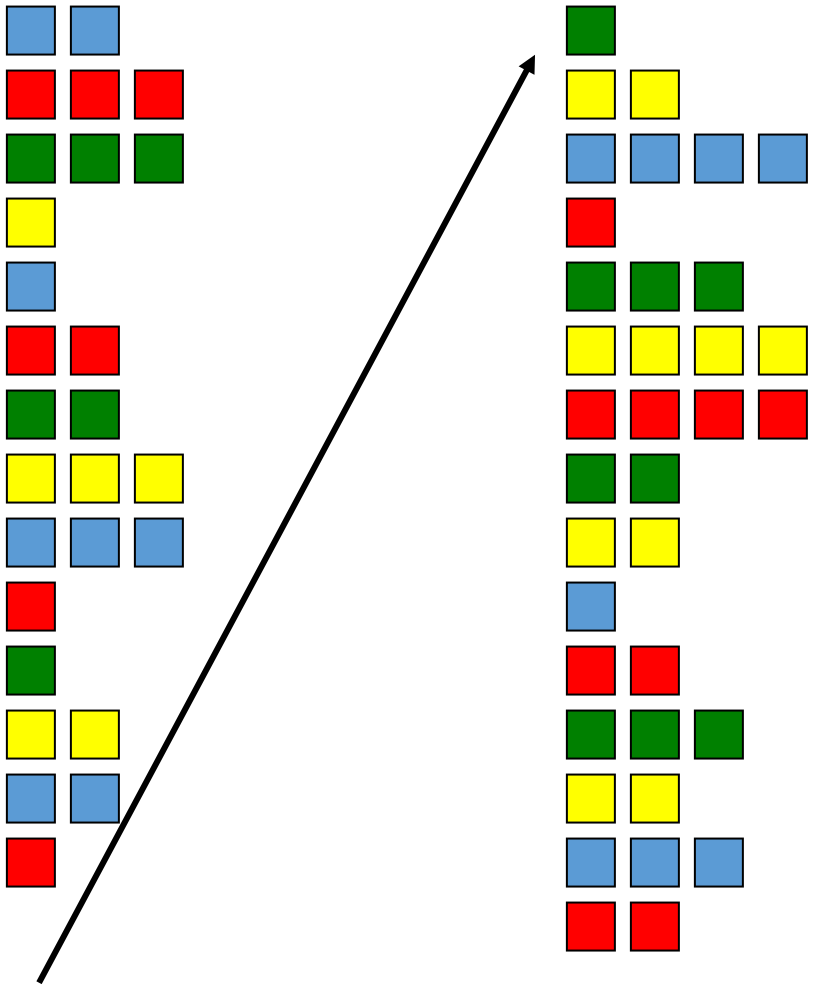

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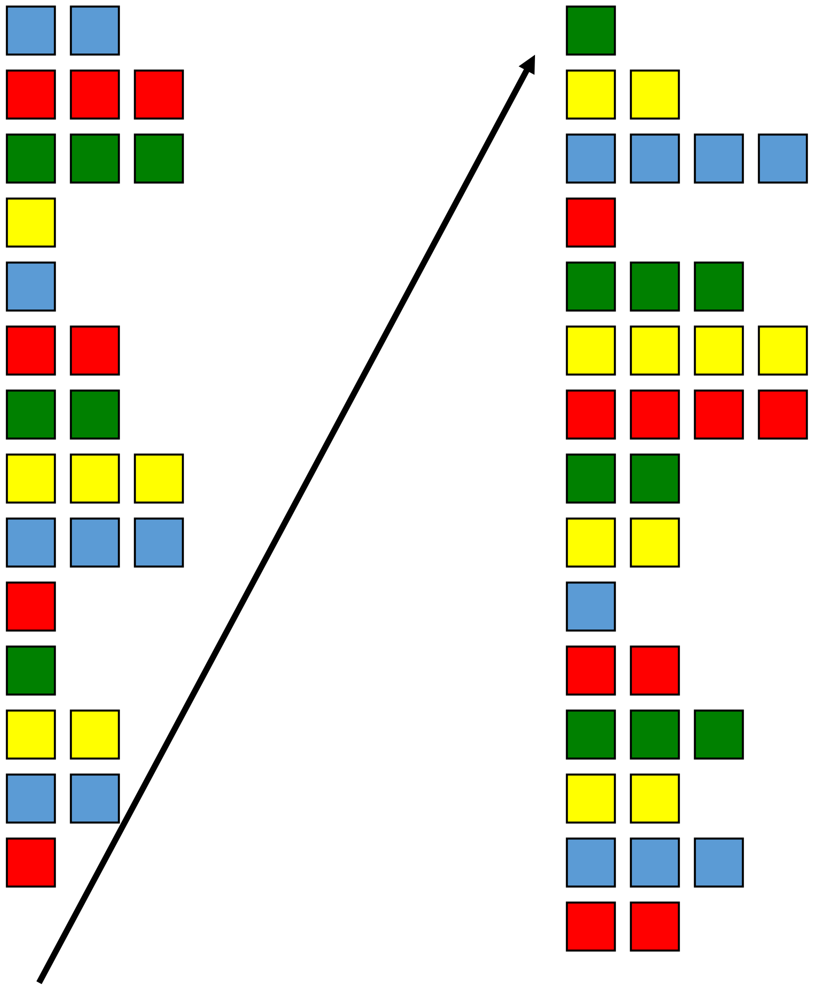

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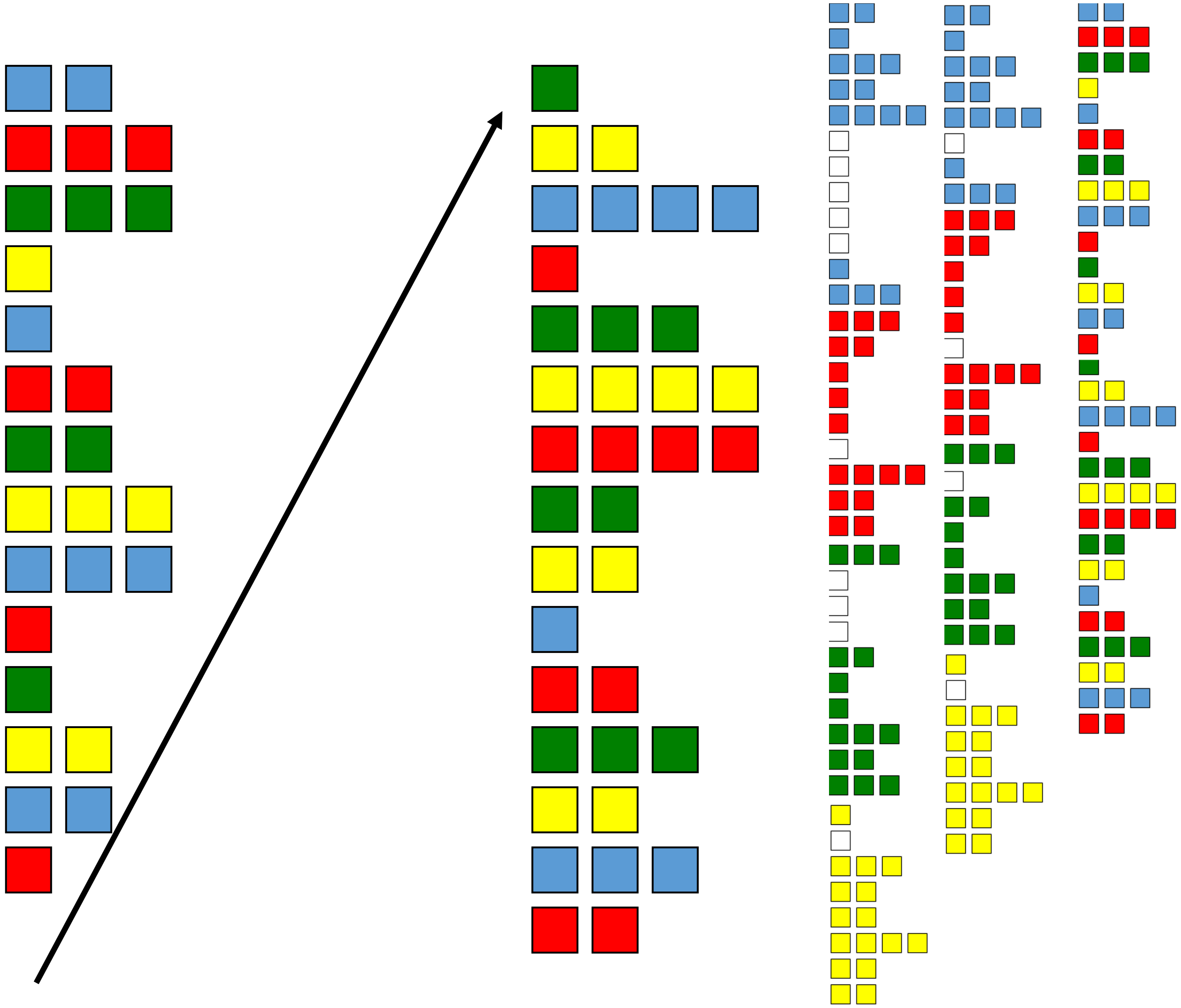

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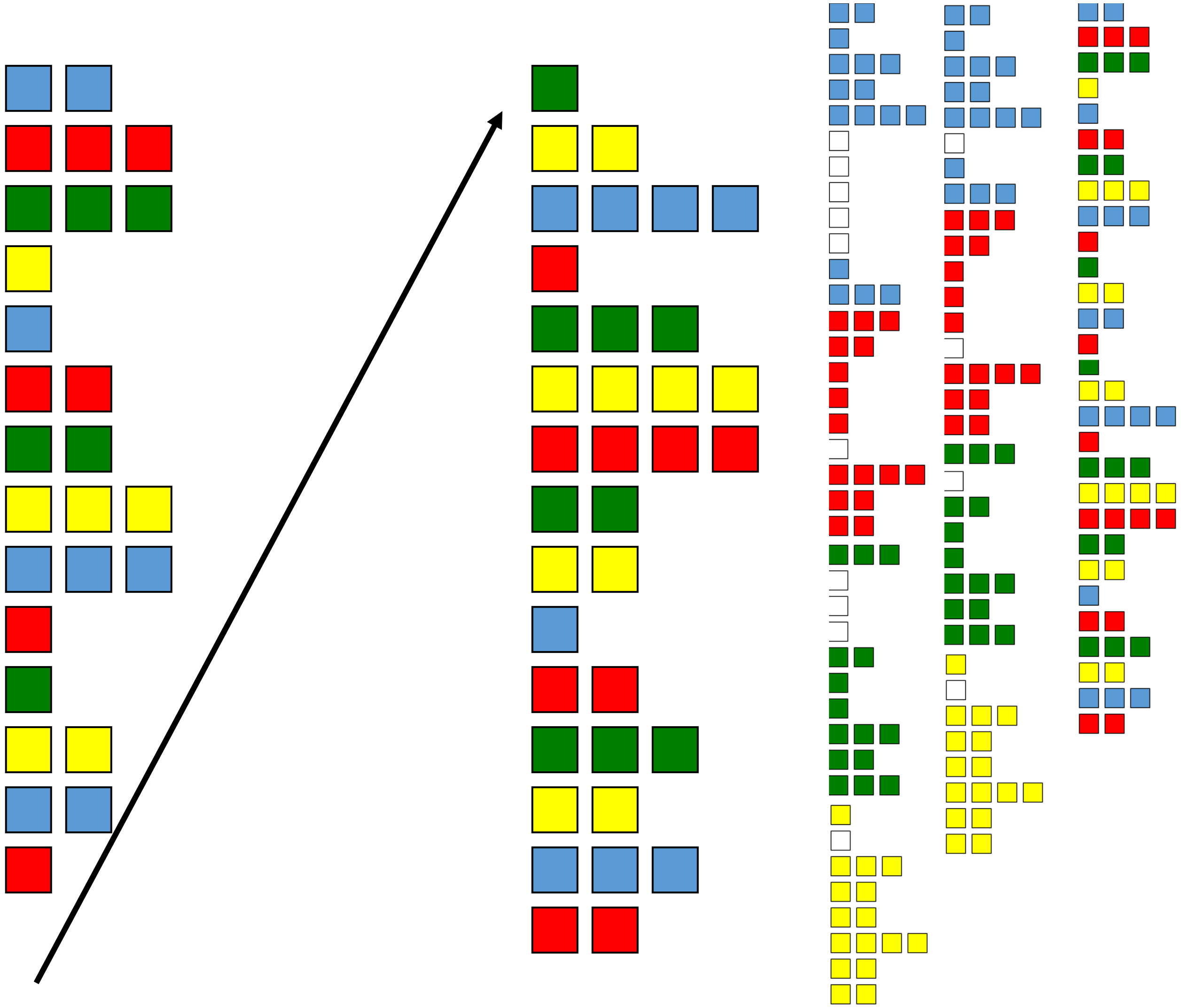

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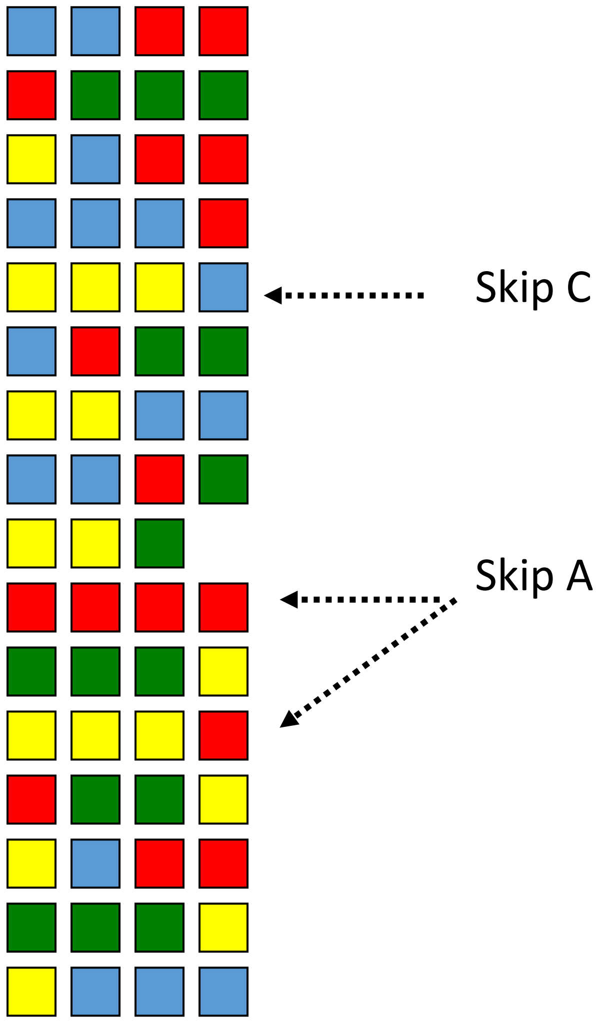

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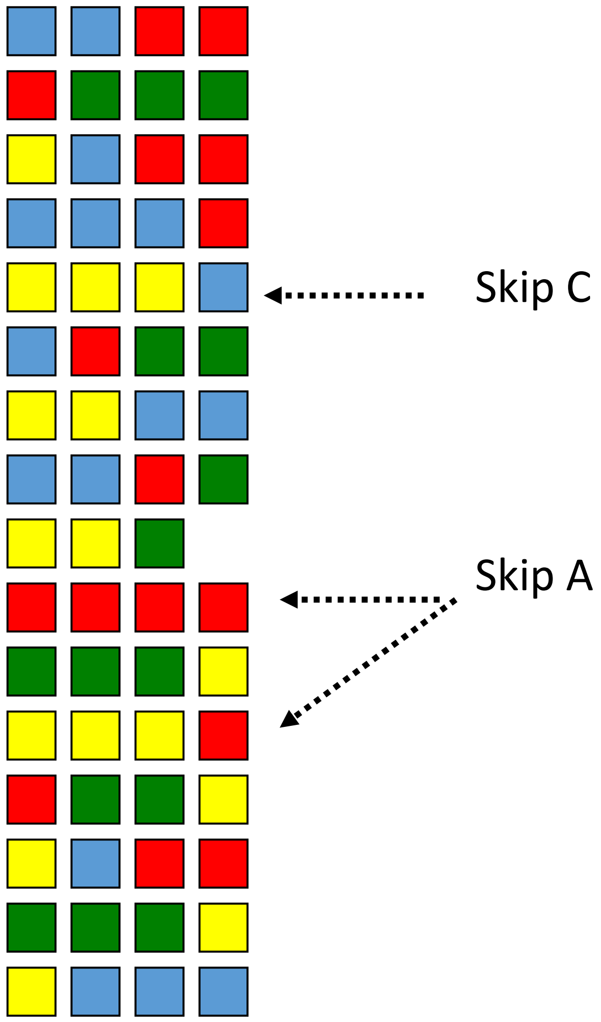

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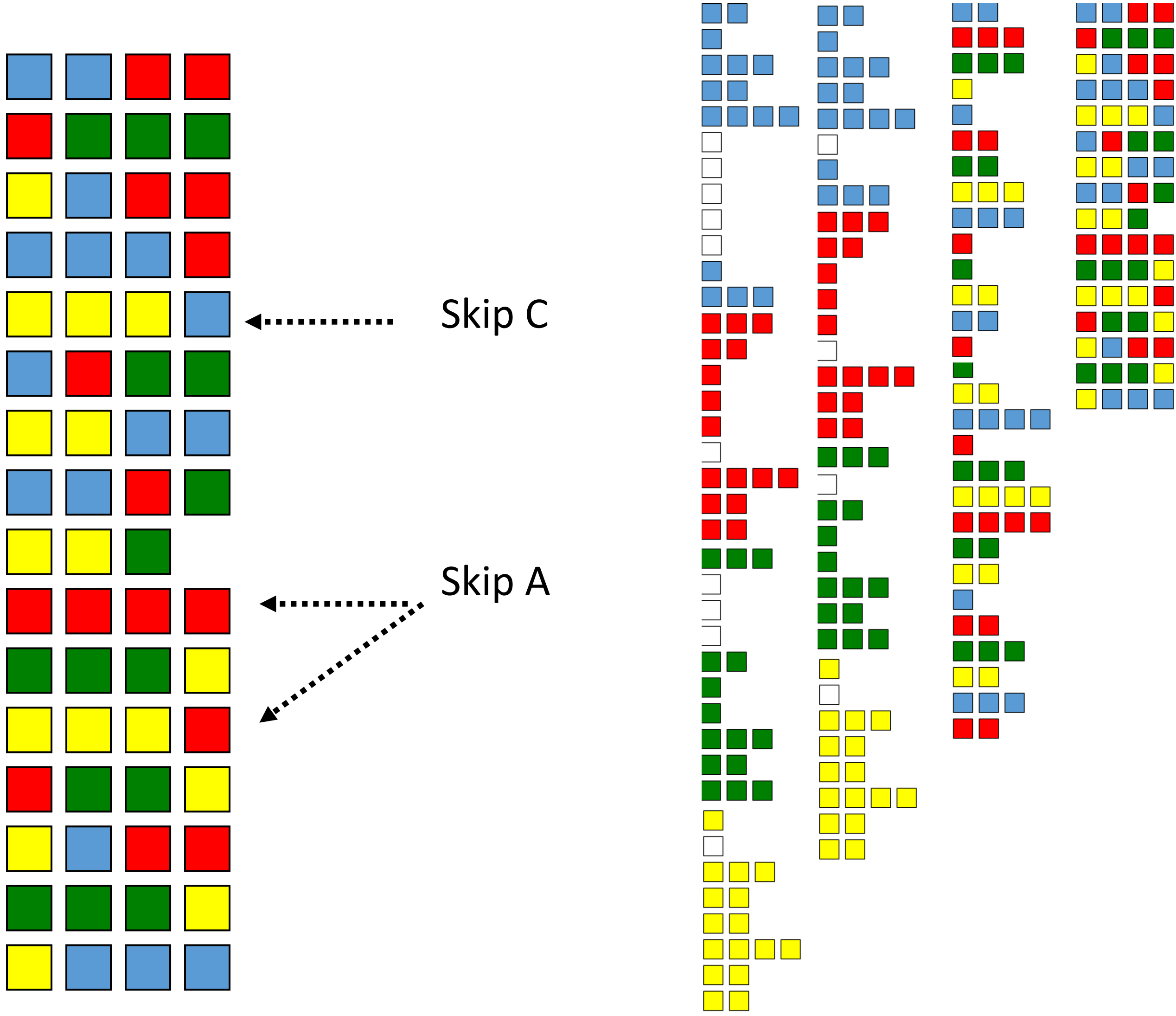

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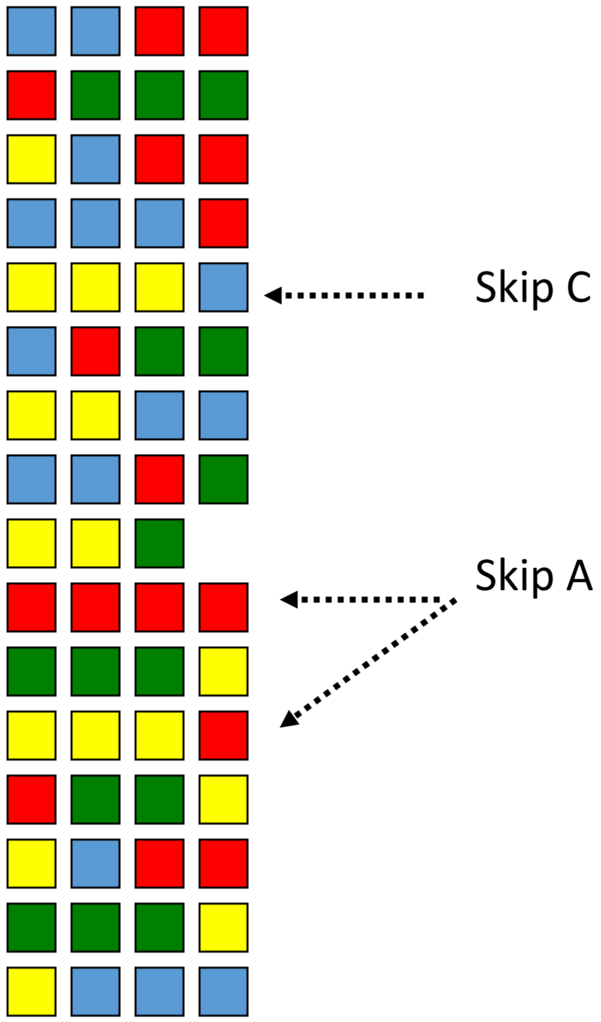

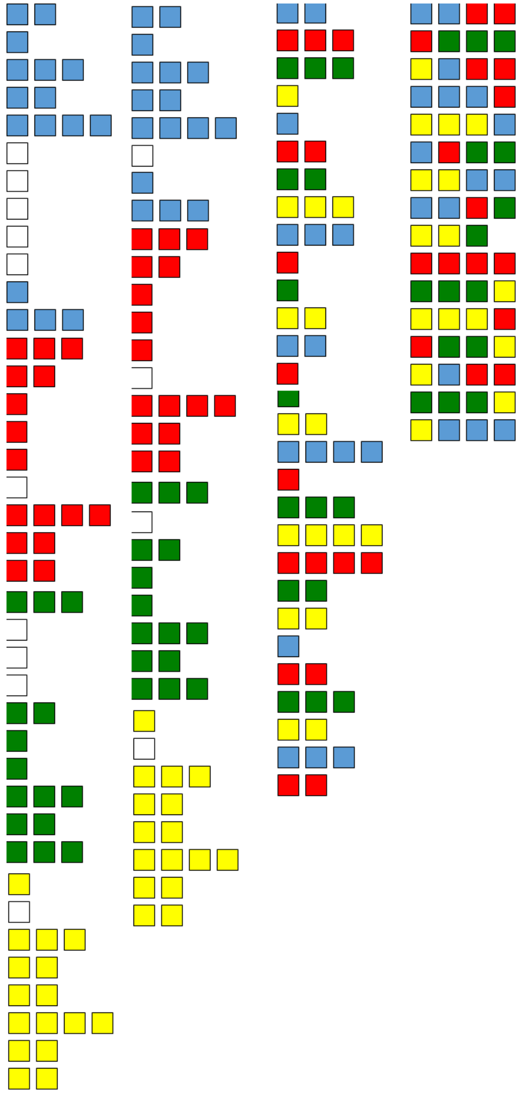

Why Vector and Multithreading Background?

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GPU:

- A very wide vector machine
- Massively multi-threaded to hide memory latency
- *Originally designed for graphics pipelines…*

Graphics ~= Rendering

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Inputs

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    - Geometry modeled w triangle meshes, surface normals
    - GPUs subdivide triangles into “fragments” (rasterization)
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- 2D projection seen from the view-point

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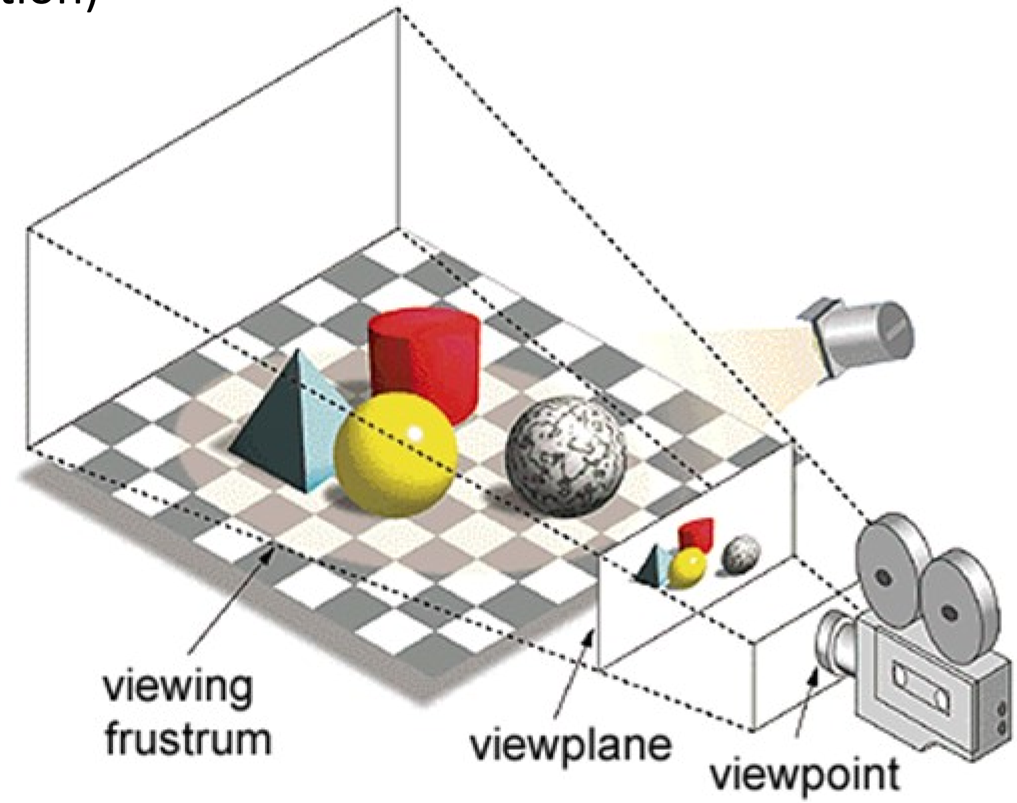
  - Attempt to model surface/light interactions with modeled objects/materials
- View point



Output

- 2D projection seen from the view-point

Grossly over-simplified rendering algorithm

Grossly over-simplified rendering algorithm

```
foreach(vertex v in model)
```

Grossly over-simplified rendering algorithm

```
foreach(vertex v in model)
        map $v_{model}$ → $v_{view}$
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```
foreach(vertex v in model)
        map $v_{model} \rightarrow v_{view}$
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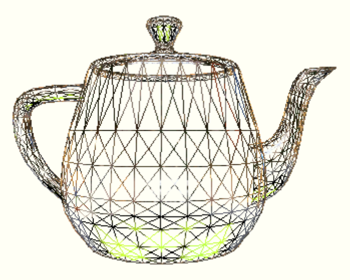
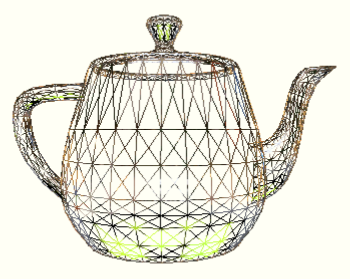
foreach triangle t ($v_{0,}$ $v_{1,}$ $v_2$)
```



Grossly over-simplified rendering algorithm



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foreach(vertex v in model)
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```

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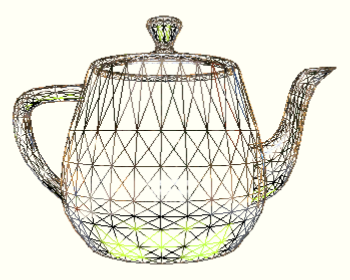

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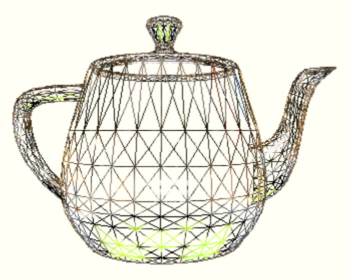

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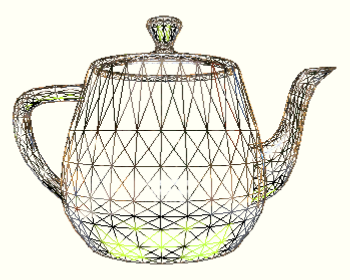

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fragment[] frags = {};
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        frags.add(rasterize(t));
foreach fragment f in frags
        choose_color(f);
display(visible_fragments(frags));
```

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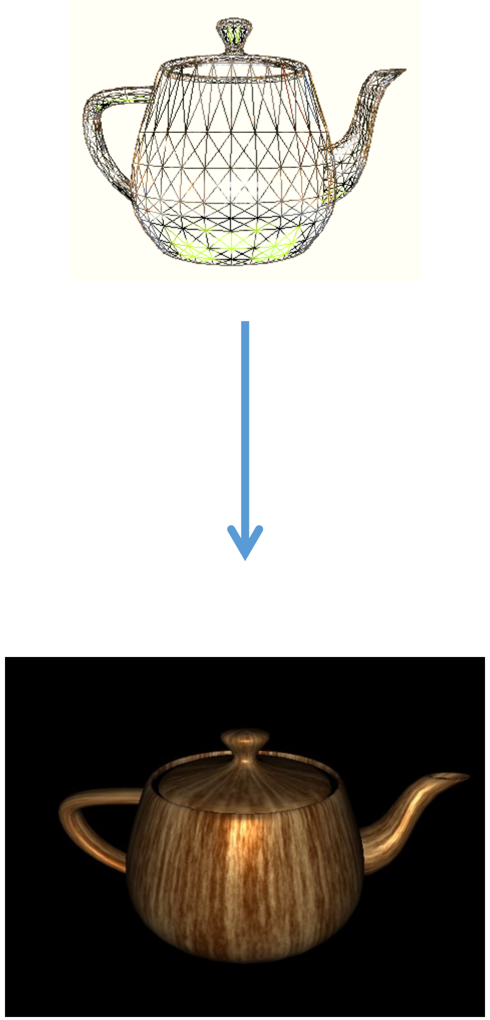

Algorithm → Graphics Pipeline

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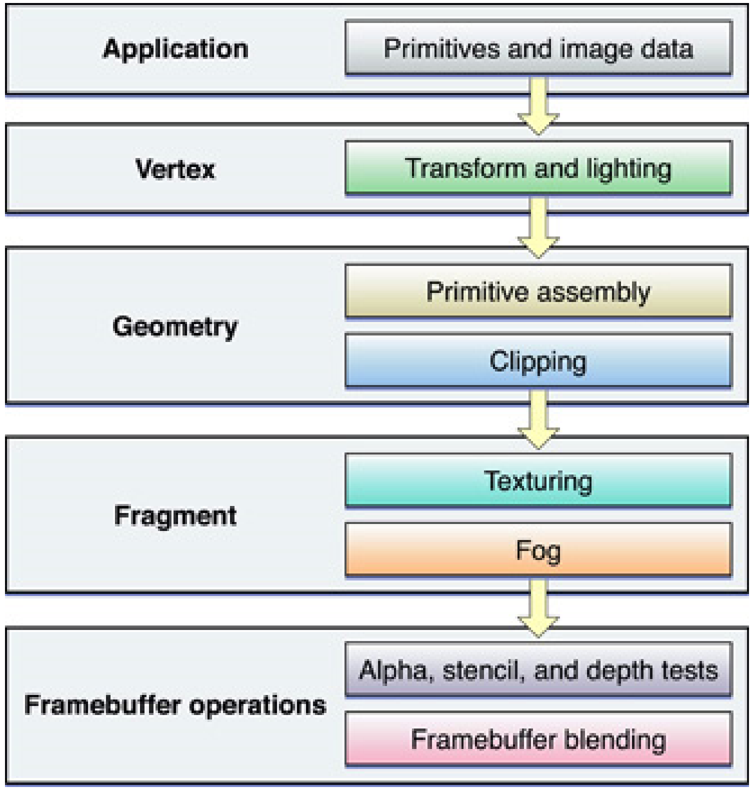

OpenGL pipeline

To first order, DirectX looks the same!

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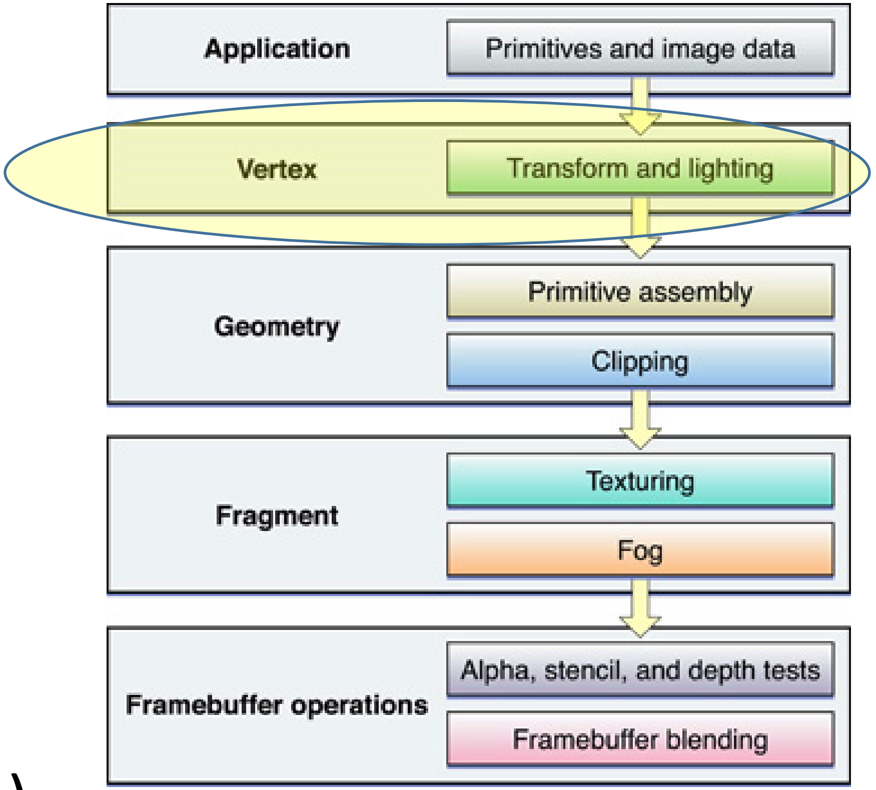

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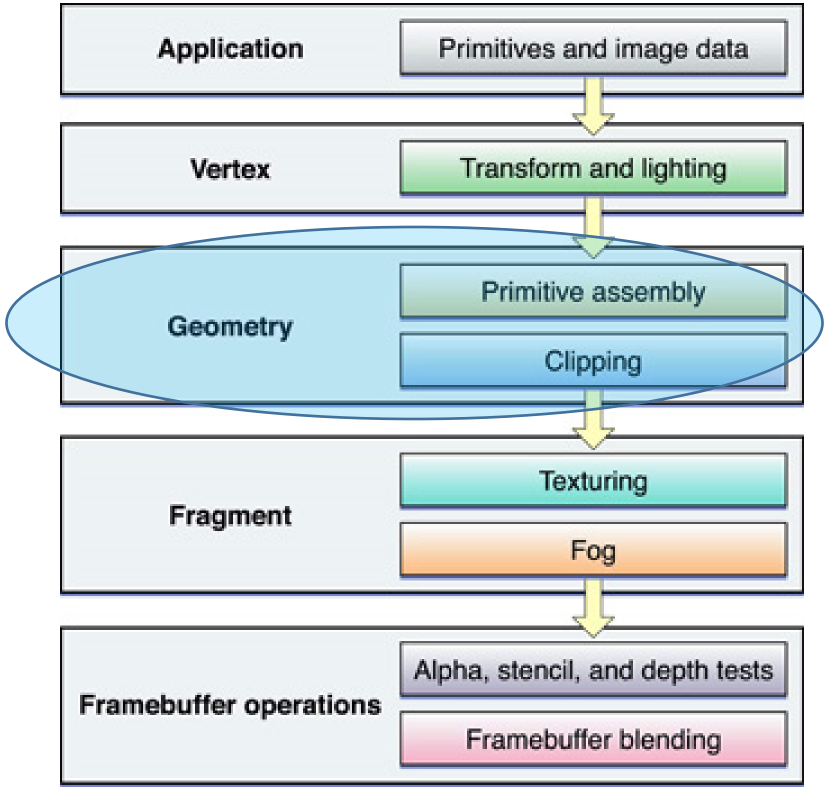

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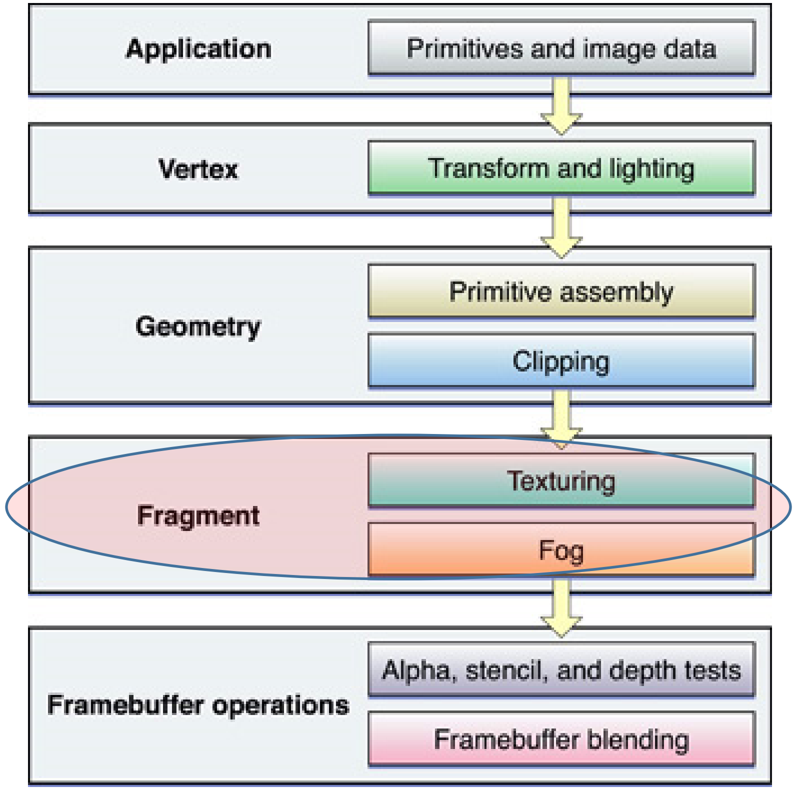

Application: Primitives and image data

Vertex: Transform and lighting

Geometry: Primitive assembly, Clipping

Fragment: Texturing, Fog

Framebuffer operations: Alpha, stencil, and depth tests, Framebuffer blending

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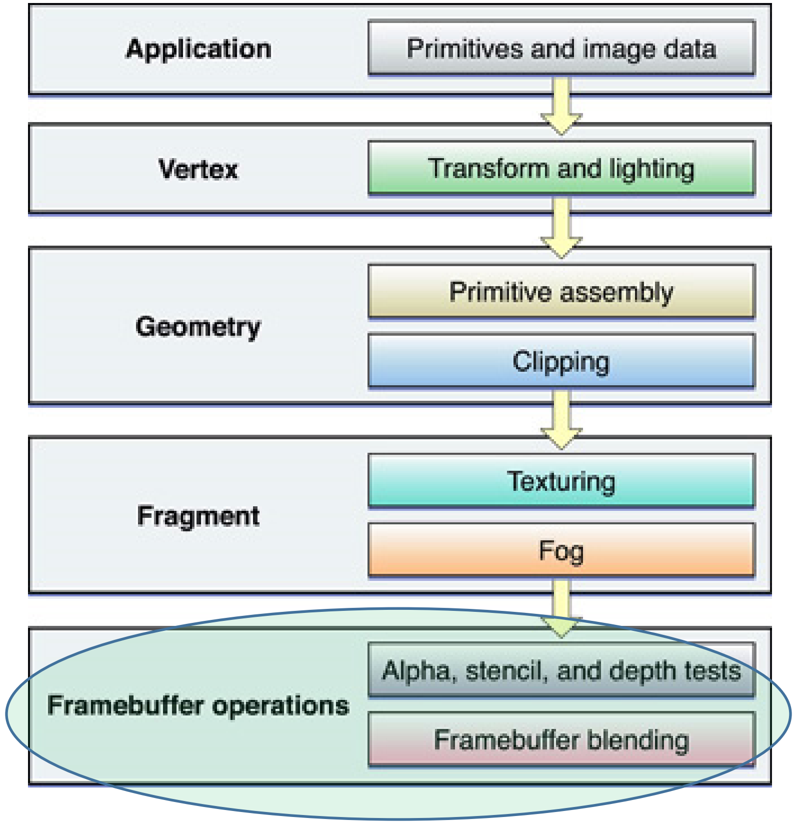

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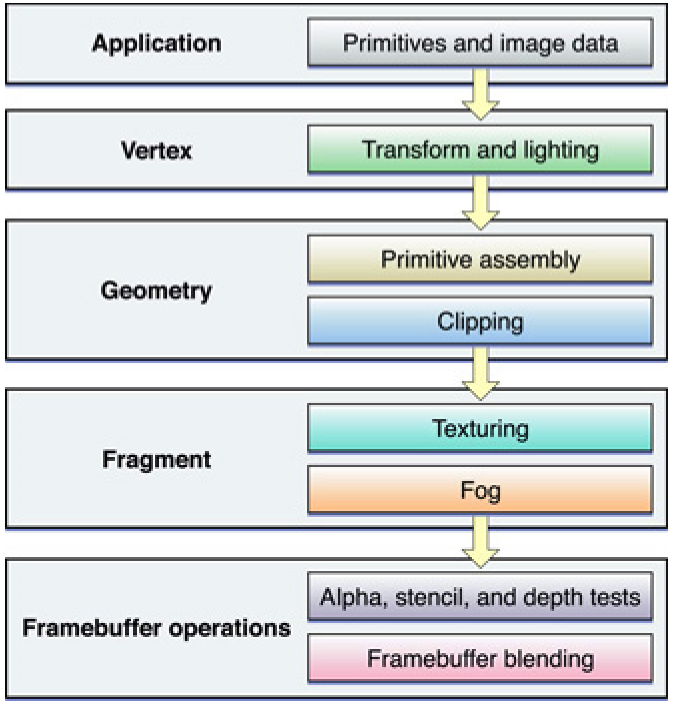
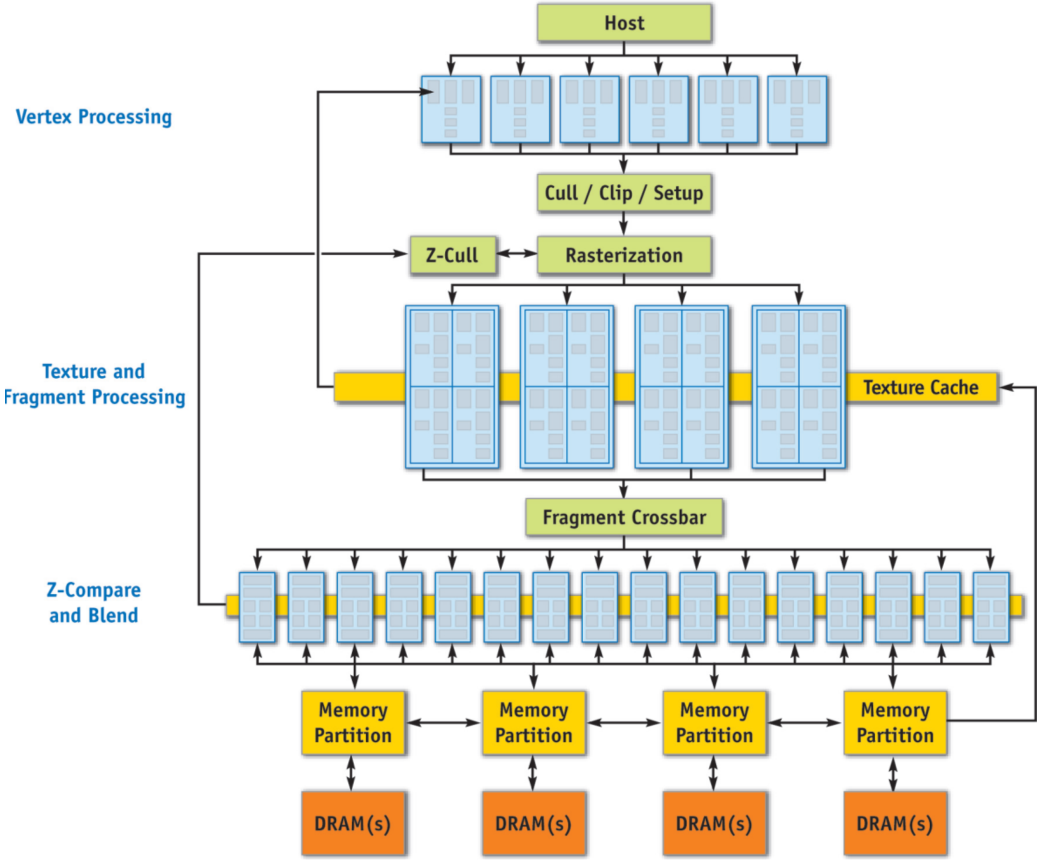
Graphics pipeline → GPU architecture

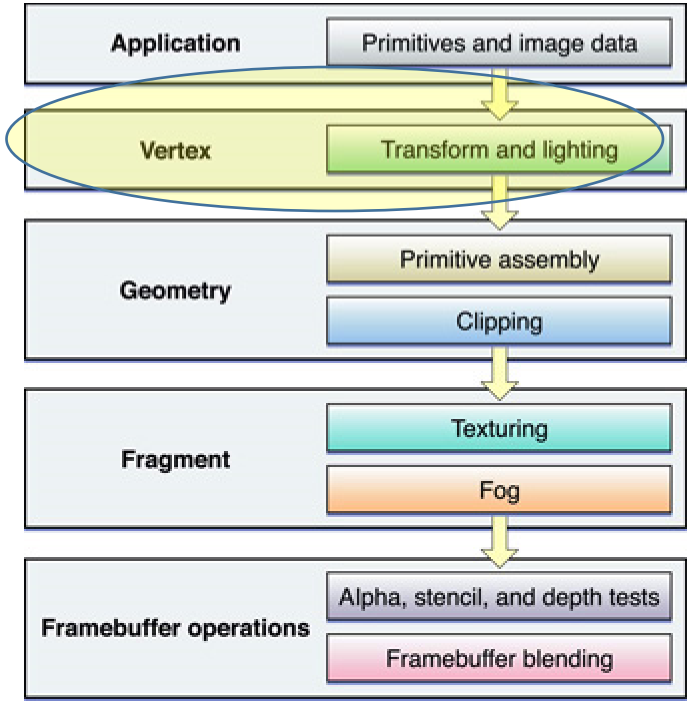


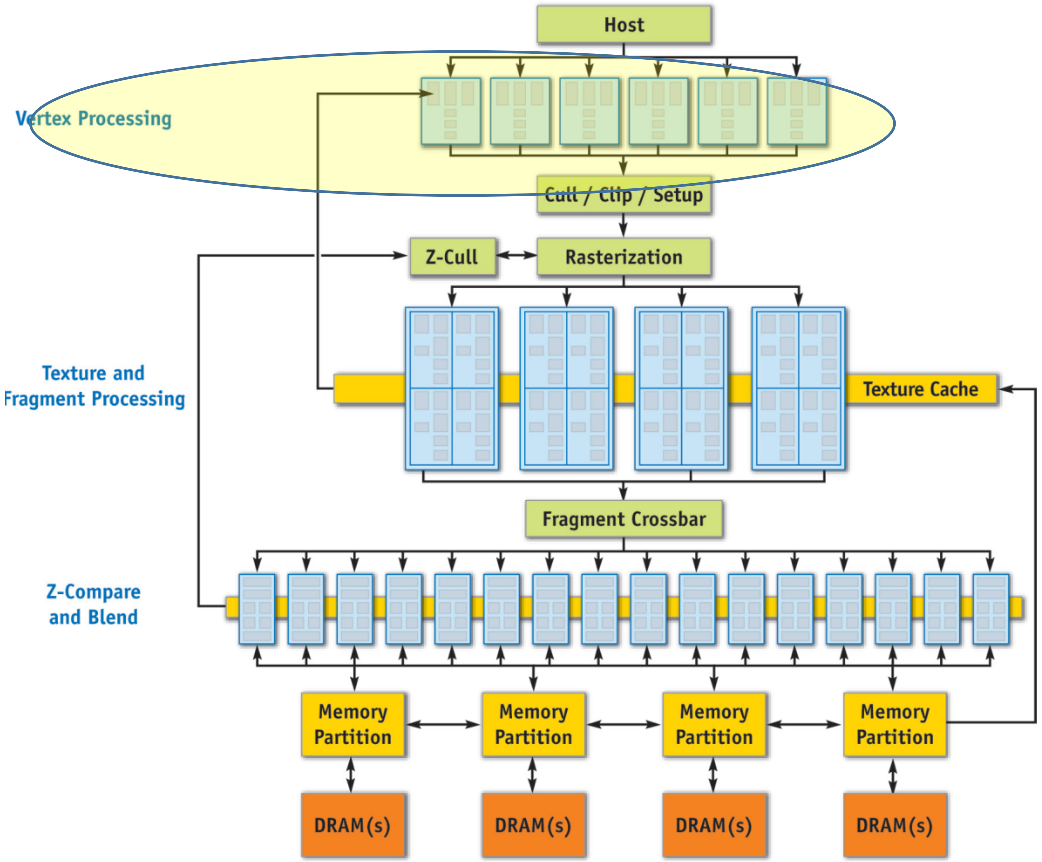


Limited "programmability" of shaders:
Minimal/no control flow
Maximum instruction count

GeForce 6 series

Graphics pipeline → GPU architecture





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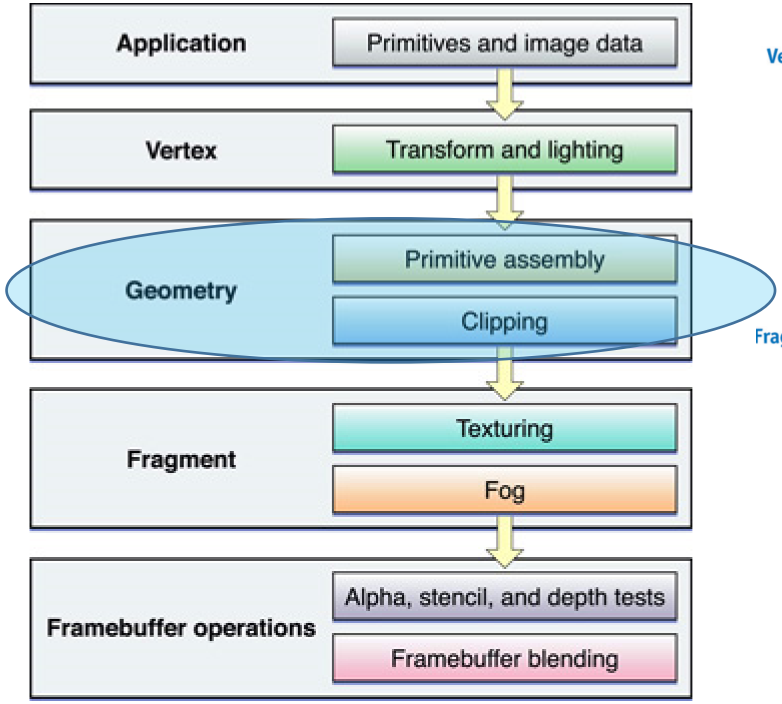

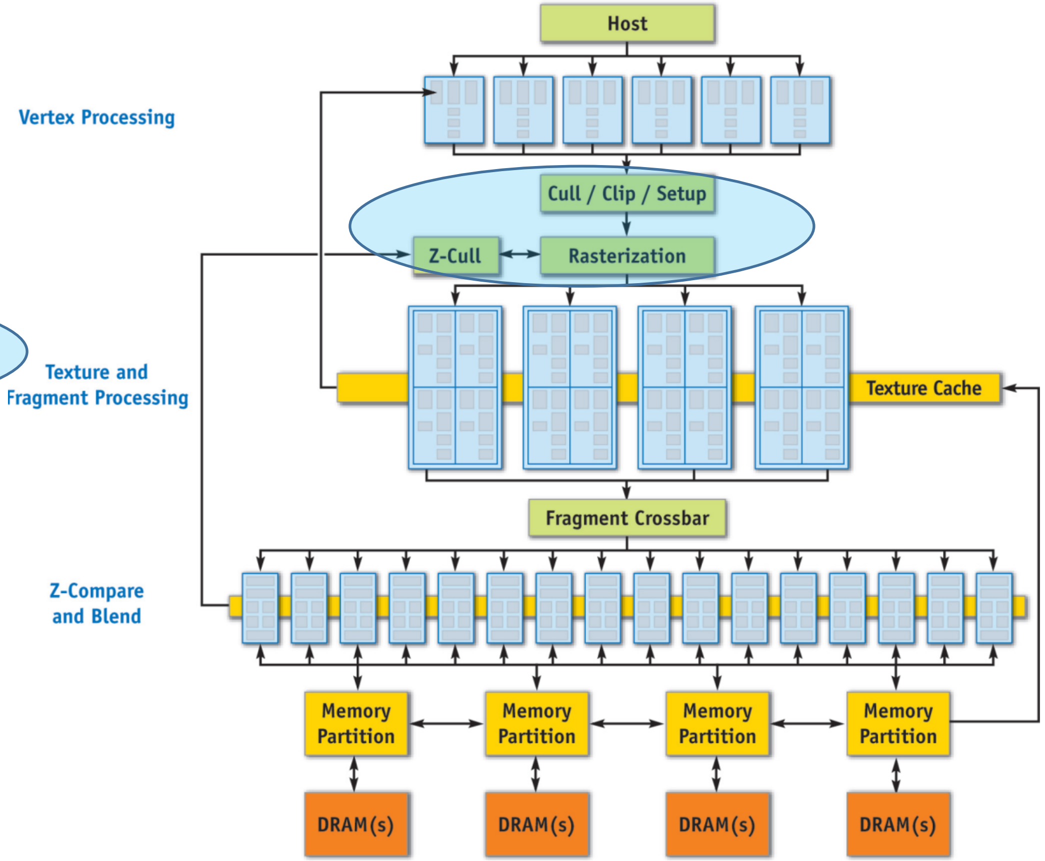

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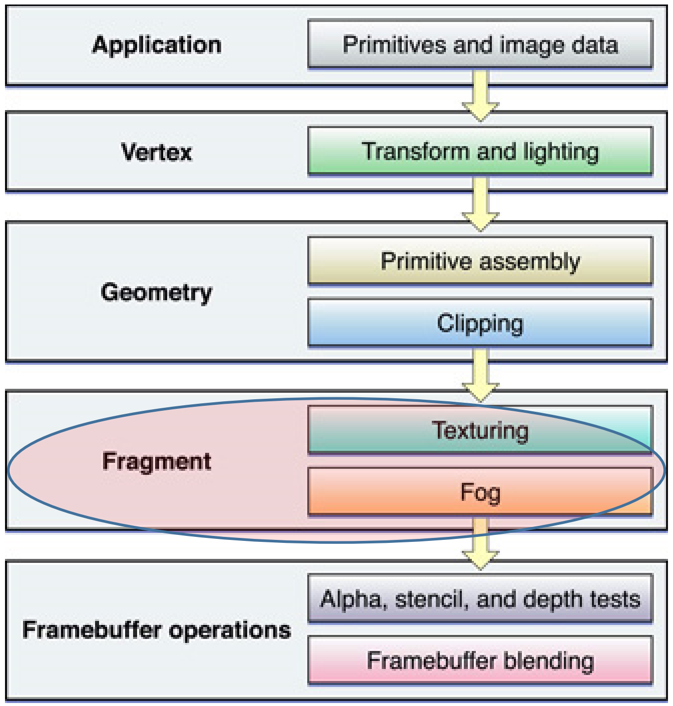

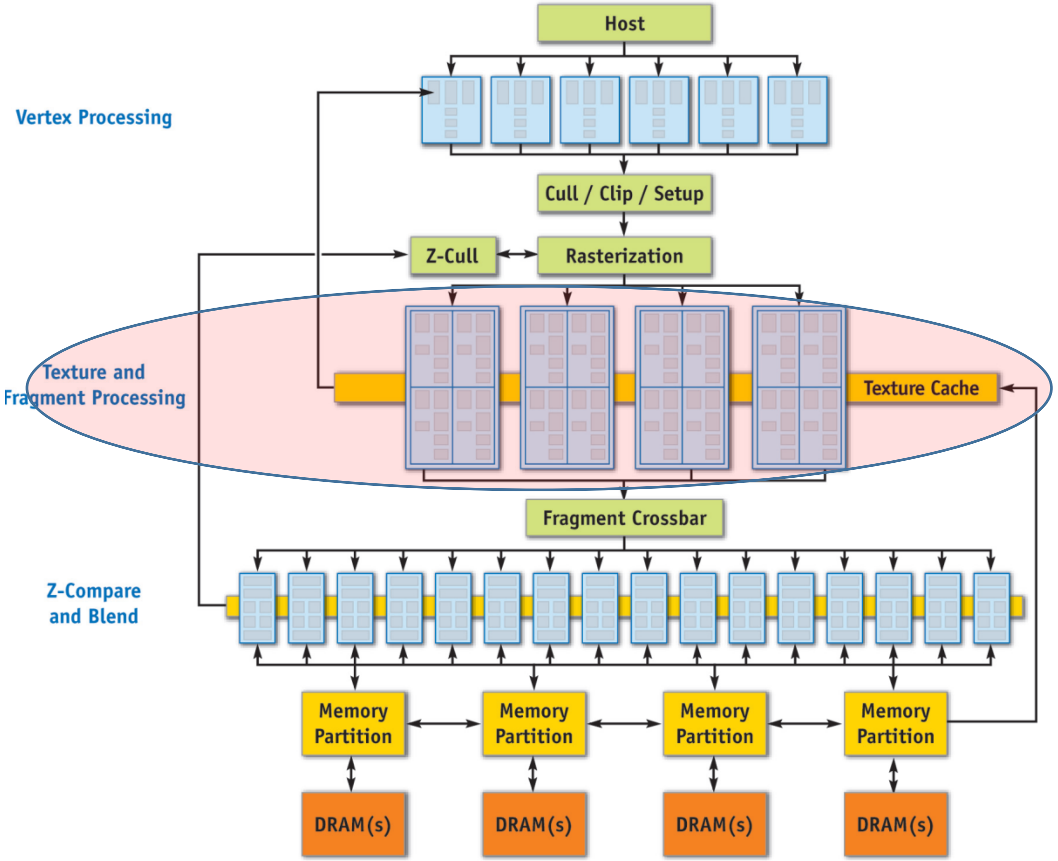

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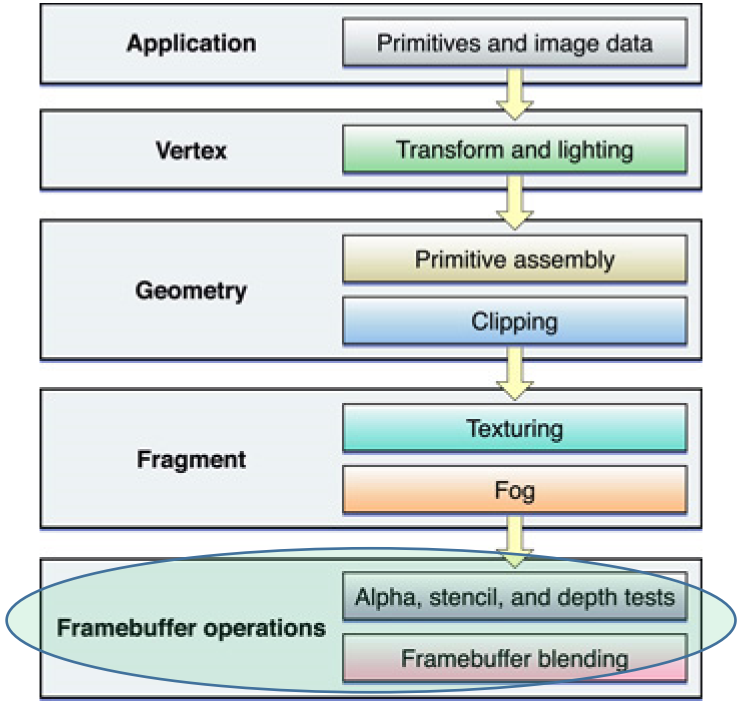

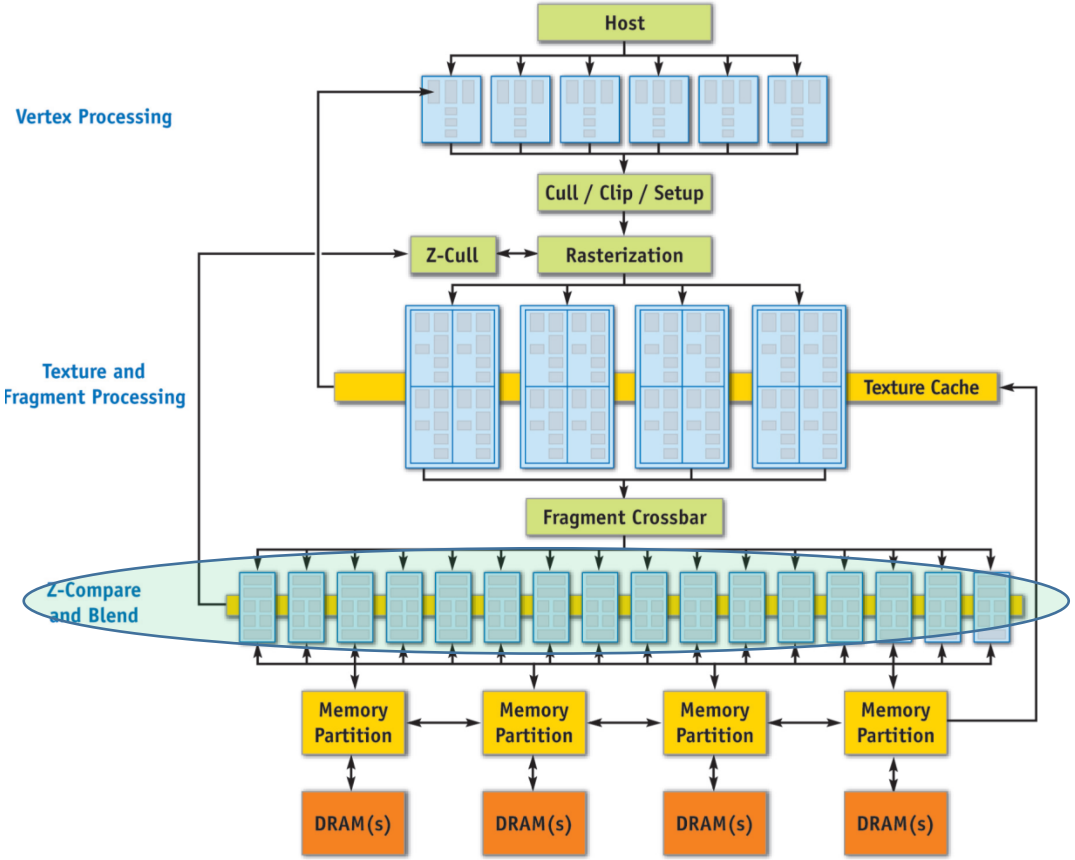

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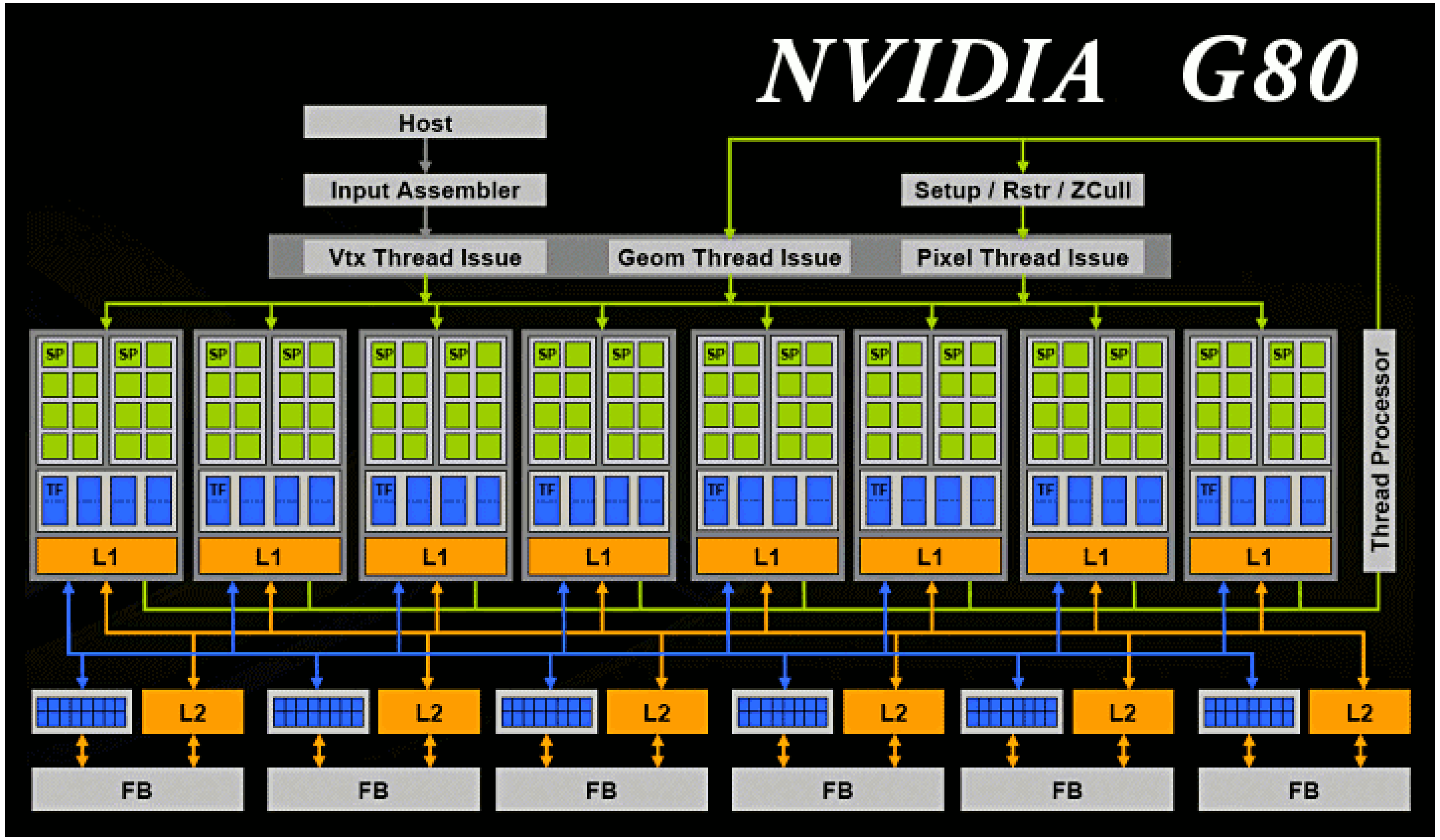
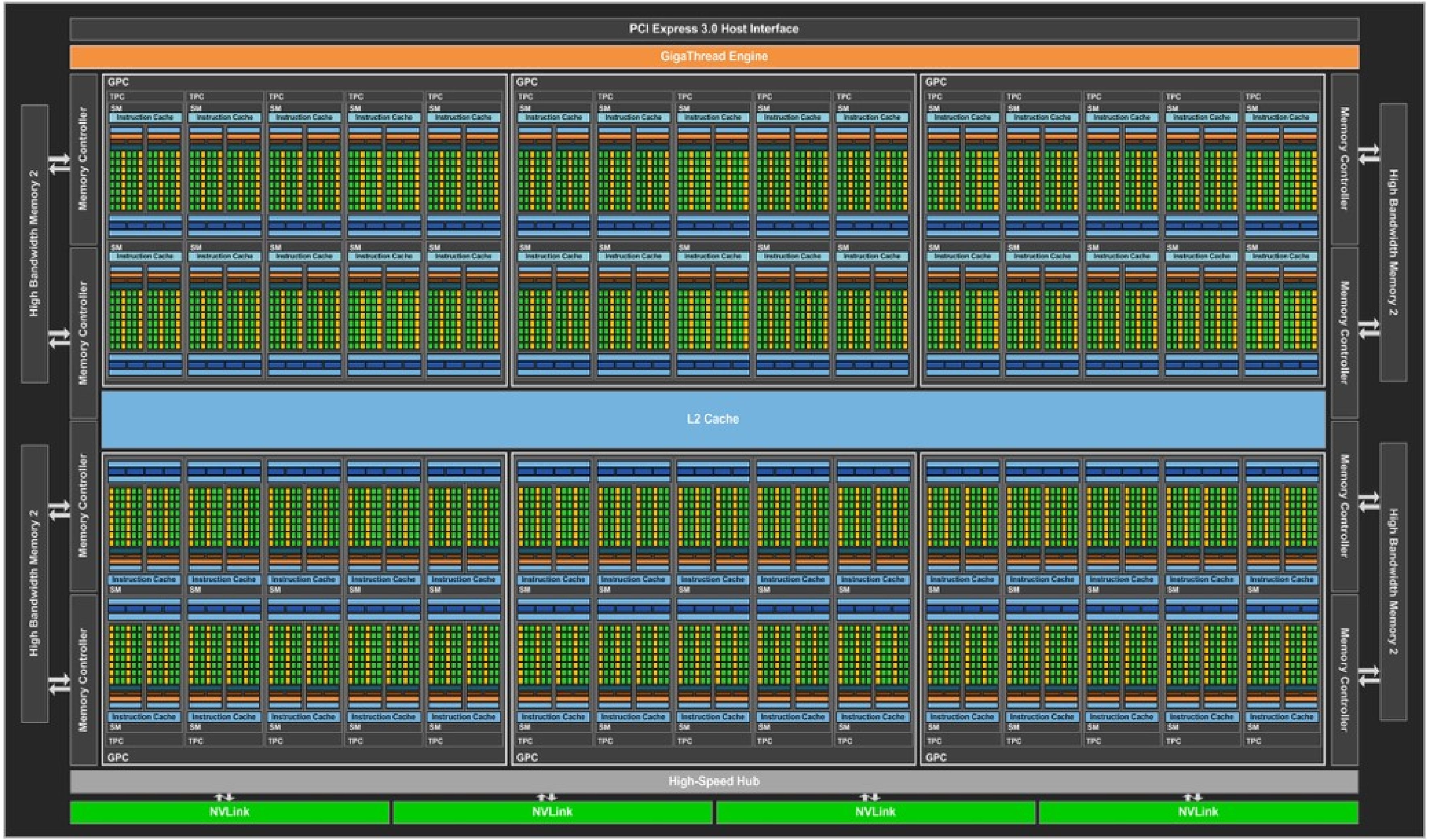
Late Modernity: unified shaders

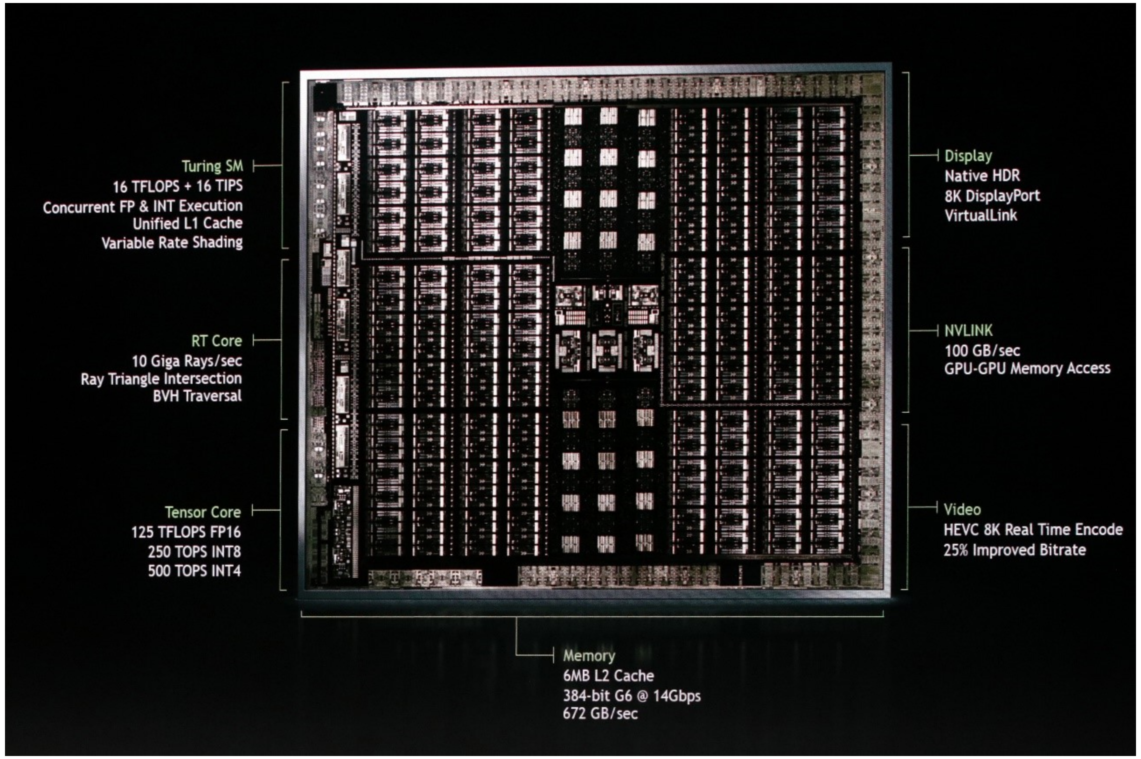


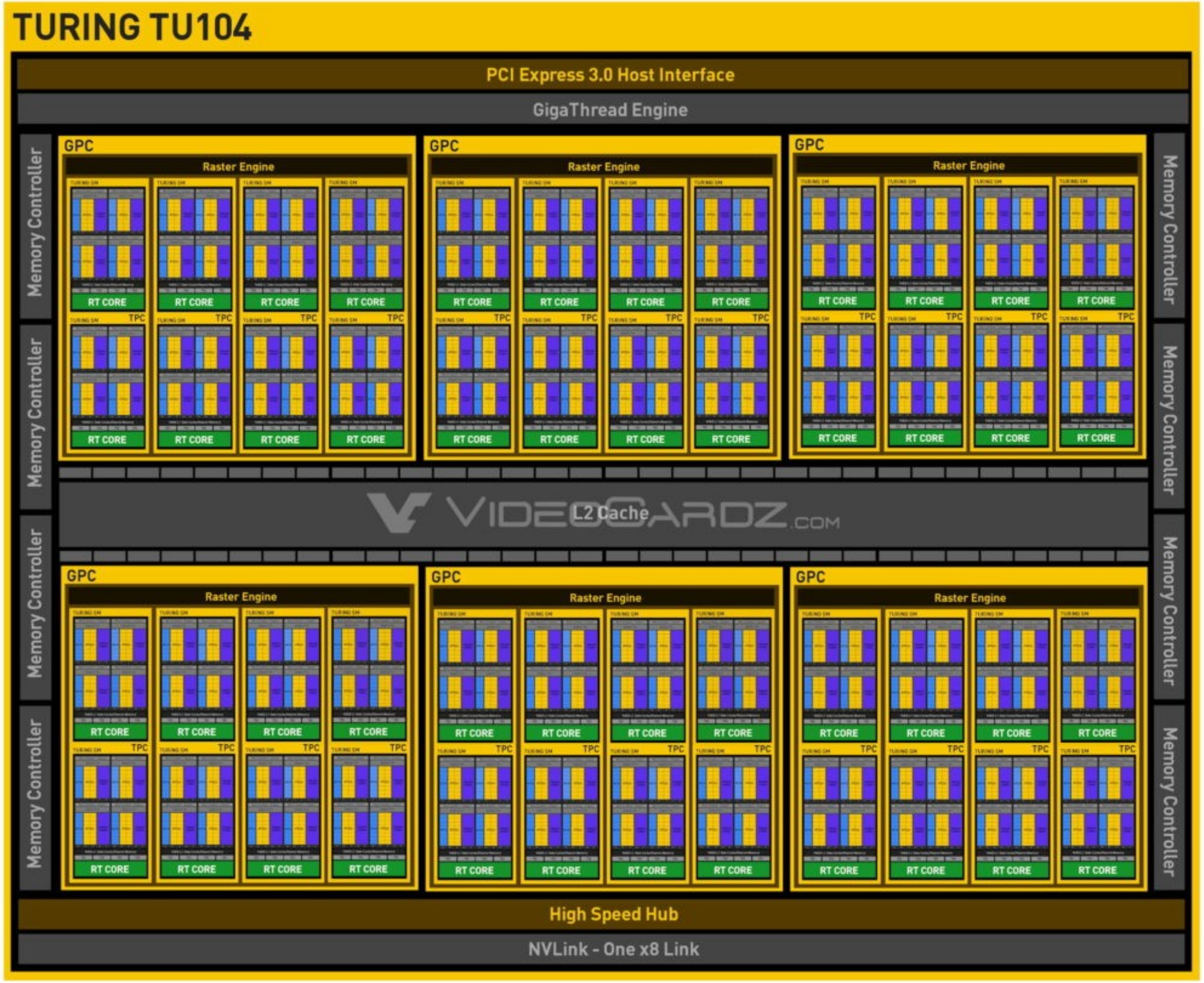
Mapping to Graphics pipeline no longer apparent
Processing elements no longer specialized to a particular role
Model supports *real* control flow, larger instr count

Mostly Modern: Pascal



Definitely Modern: Turing





Cross-generational GPU observations

GPUs designed for parallelism in graphics pipeline:

- Data
    - Per-vertex
    - Per-fragment
    - Per-pixel
- Task
    - Vertex processing
    - Fragment processing
    - Rasterization
    - Hidden-surface elimination
- MLP
    - HW multi-threading for hiding memory latency

- Simple cores
- Single instruction stream
    - Vector instructions (SIMD) OR
    - Implicit HW-managed sharing (SIMT)
- Hide memory latency with HW multi-threading

Cross-generational GPU observations

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OR

ring

n

Even as GPU architectures become more general, certain assumptions persist:

1. Data parallelism is ***trivially*** exposed
2. **All** problems look like painting a box with colored dots

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Even as GPU architectures become more general, certain assumptions persist:

1. Data parallelism is ***trivially*** exposed
2. **All** problems look like painting a box with colored dots

But what if my problem isn't painting a box?!!?!

Programming Model

- ***GPUs are I/O devices, managed by user-code***
- “kernels” == “shader programs”
- 1000s of HW-scheduled threads per kernel
- Threads grouped into independent blocks.
    - Threads in a block can synchronize (barrier)
    - This is the *only* synchronization
- “Grid” == “launch” == “invocation” of a kernel
    - a group of blocks (or warps)

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Need codes that are 1000s-X parallel….

Parallel Algorithms

- Sequential algorithms often do not permit easy parallelization
  - Does not mean there work has no parallelism
  - A different approach can yield parallelism
  - but often changes the algorithm
  - Parallelizing != just adding locks to a sequential algorithm
- Parallel Patterns
  - Map
  - Scatter, Gather
  - Reduction
  - Scan
  - Search, Sort

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  - Map
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  - Search, Sort

If you can express your algorithm using these patterns, an apparently fundamentally sequential algorithm can be made parallel

Map

- Inputs
  - Array A
  - Function f(x)
- map(A, f) → apply f(x) on all elements in A
- Parallelism trivially exposed
  - f(x) can be applied in parallel to all elements, in principle

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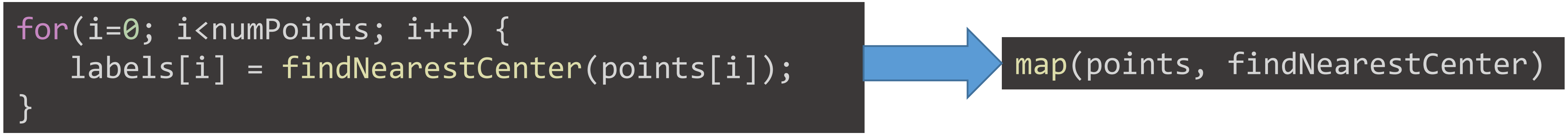

```
for(i=0; i<numPoints; i++) {
   labels[i] = findNearestCenter(points[i]);
}
```

```
map(points, findNearestCenter)
```

Scatter and Gather

Scatter and Gather

- Gather:
    - Read multiple items to single /packed location

Scatter and Gather

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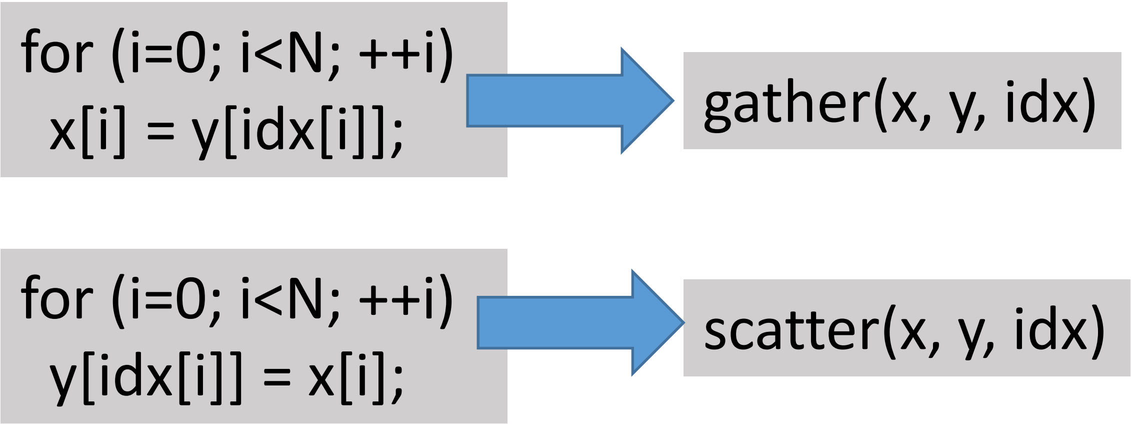

```
for (i=0; i<N; ++i)
  x[i] = y[idx[i]];
```

```
gather(x, y, idx)
```

```
for (i=0; i<N; ++i)
  y[idx[i]] = x[i];
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```
scatter(x, y, idx)
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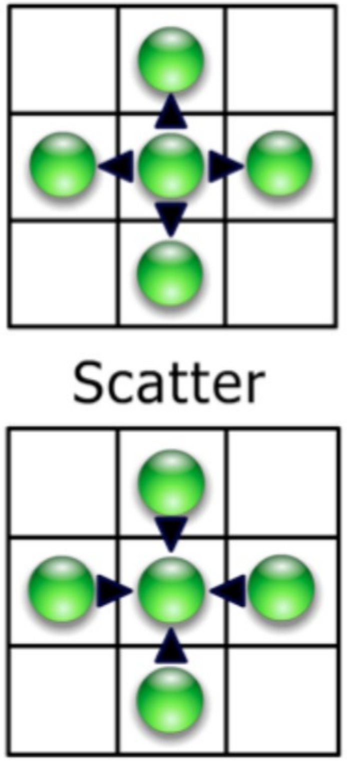

Scatter

Gather

Scatter and Gather

Why is this useful on a box-drawing machine?

- Gather:
  - Read multiple items to single /packed location
- Scatter:
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- Inputs: x, y, indeces, N

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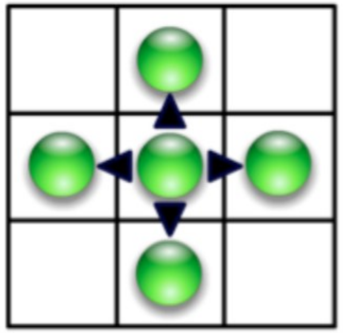
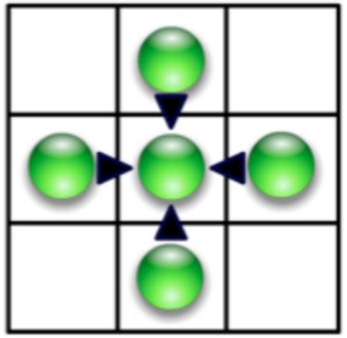


Scatter



Gather

Reduce

- Input
  - Associative operator op
  - Ordered set s = [a, b, c, … z]
- Reduce(op, s) returns a op b op c … op z

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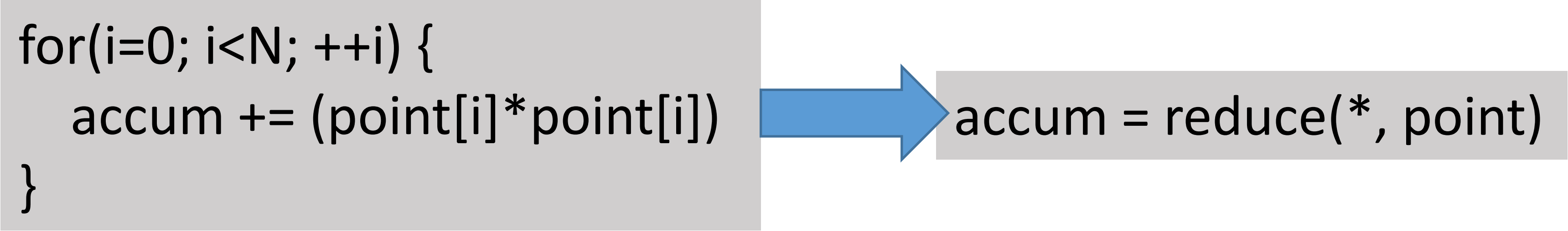

```
for(i=0; i<N; ++i) {
   accum += (point[i]*point[i])
}
```

```
accum = reduce(*, point)
```

Reduce

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  - Associative operator op
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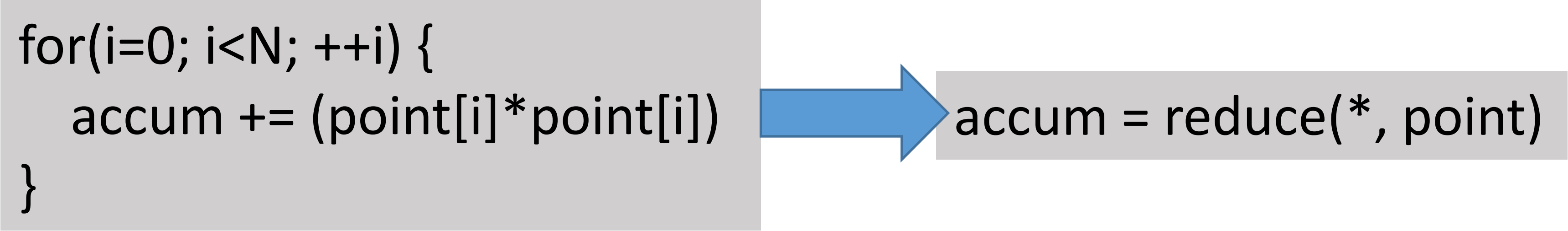

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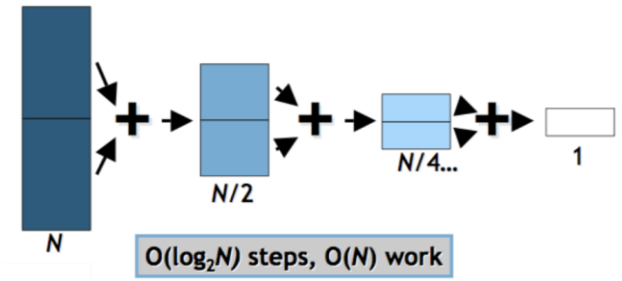

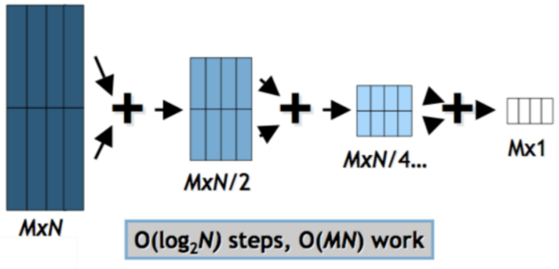

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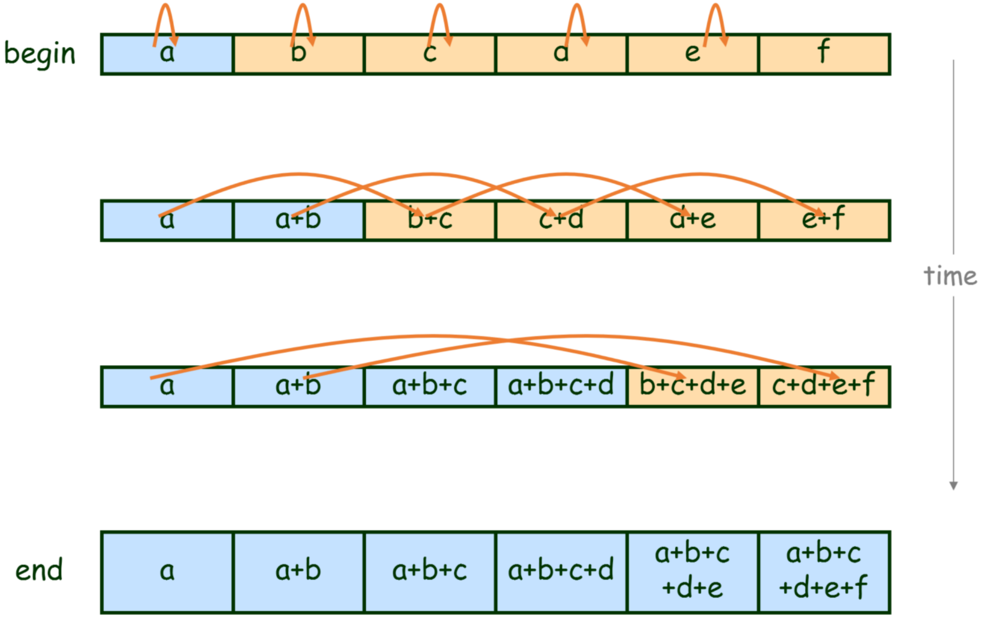
Why must op be associative?

Scan (prefix sum)

- Input
  - Associative operator op
  - Ordered set s = [a, b, c, … z]
  - Identity I

- scan(op, s) = [I, a, (a op b), (a op b op c) …]

- Scan is the workhorse of parallel algorithms:
  - Sort, histograms, sparse matrix, string compare, …



begin a b c d e f

a a+b b+c c+d d+e e+f

a a+b a+b+c a+b+c+d b+c+d+e c+d+e+f

end a a+b a+b+c a+b+c+d a+b+c +d+e a+b+c +d+e+f

GroupBy

- Group a collection by key
- Lambda function maps elements → key

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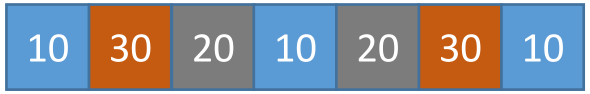

10	30	20	10	20	30	10

GroupBy

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var res = ints.GroupBy(x => x);
```
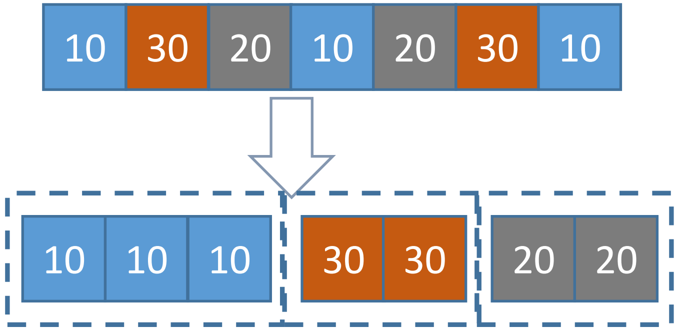
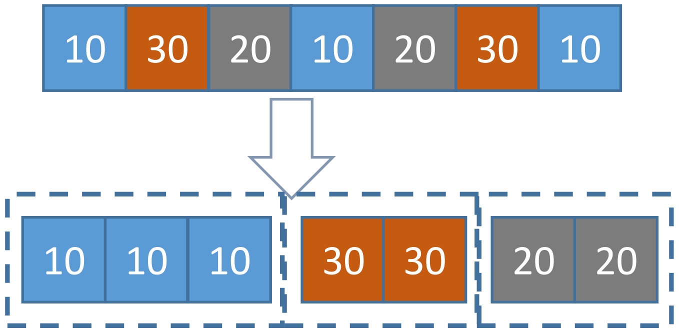



GroupBy

- Group a collection by key
- Lambda function maps elements → key

```
var res = ints.GroupBy(x => x);
```

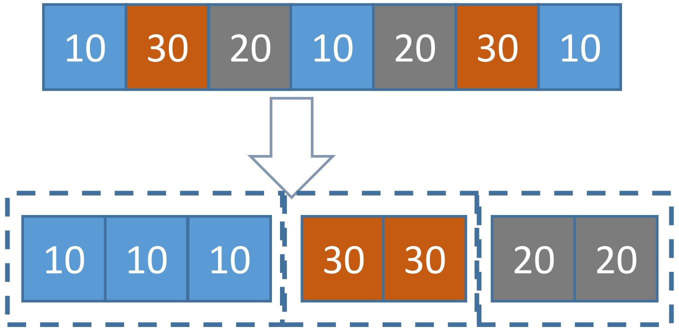


```
foreach(T elem in ints)
{
   key   = KeyLambda(elem);

   group = GetGroup(key);

   group.Add(elem);
}
```

GroupBy

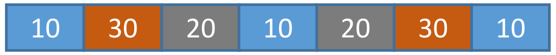
- Group a collection by key
- Lambda function maps elements → key

```
var res = ints.GroupBy(x => x);
```



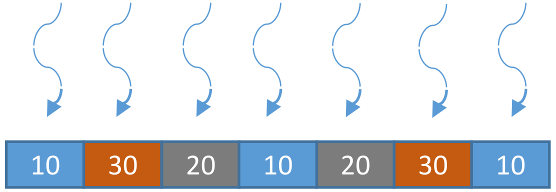
```
foreach(T elem in PF(ints))
{
   key   = KeyLambda(elem);

   group = GetGroup(key);

   group.Add(elem);
}
```

GroupBy using parallel primitives



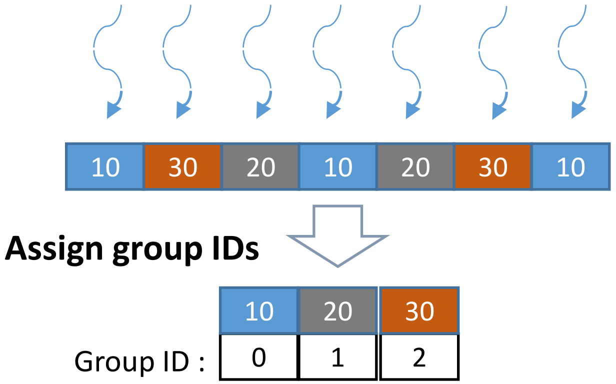
10	30	20	10	20	30	10

GroupBy using parallel primitives



10	30	20	10	20	30	10

GroupBy using parallel primitives

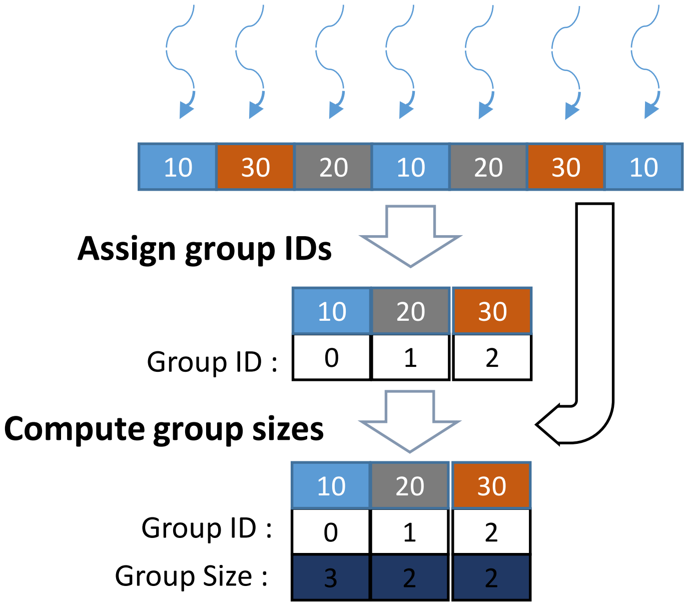


10	30	20	10	20	30	10

Assign group IDs

	10	20	30
Group ID :	0	1	2

GroupBy using parallel primitives



10	30	20	10	20	30	10

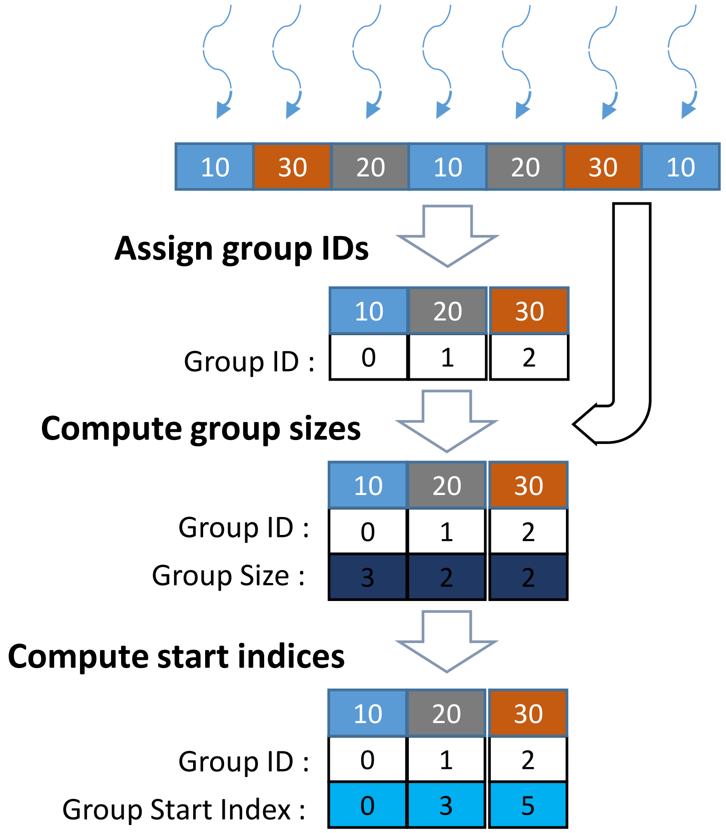
Assign group IDs

	10	20	30
Group ID :	0	1	2

Compute group sizes

	10	20	30
Group ID :	0	1	2
Group Size :	3	2	2

GroupBy using parallel primitives



10	30	20	10	20	30	10

Assign group IDs

	10	20	30
Group ID :	0	1	2

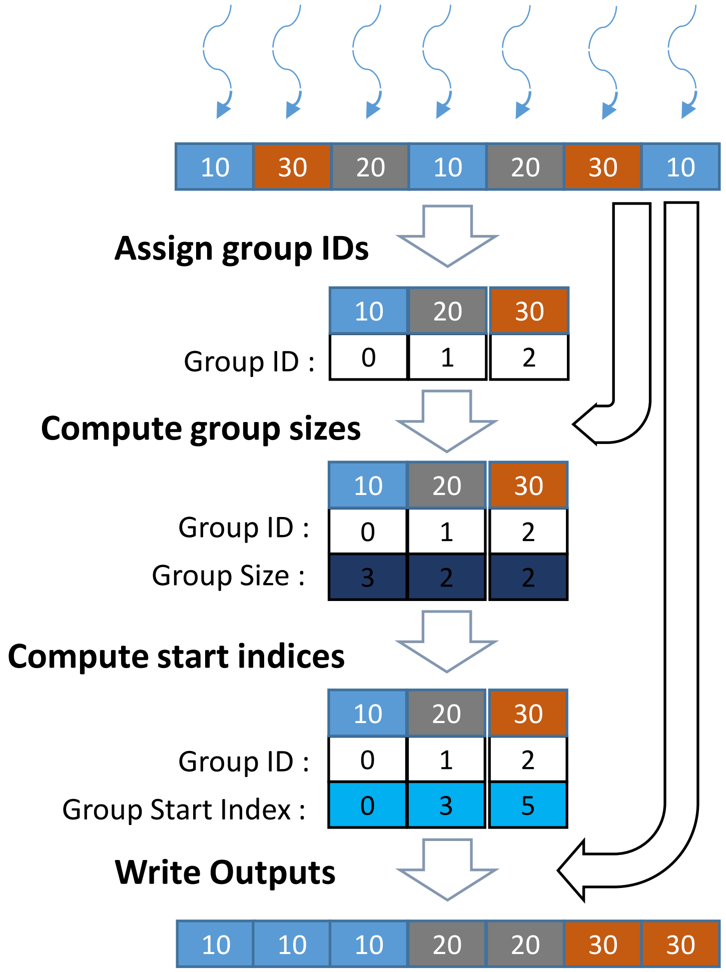
Compute group sizes

	10	20	30
Group ID :	0	1	2
Group Size :	3	2	2

Compute start indices

	10	20	30
Group ID :	0	1	2
Group Start Index :	0	3	5

GroupBy using parallel primitives



10 | 30 | 20 | 10 | 20 | 30 | 10

Assign group IDs

	10	20	30
Group ID :	0	1	2

Compute group sizes

	10	20	30
Group ID :	0	1	2
Group Size :	3	2	2

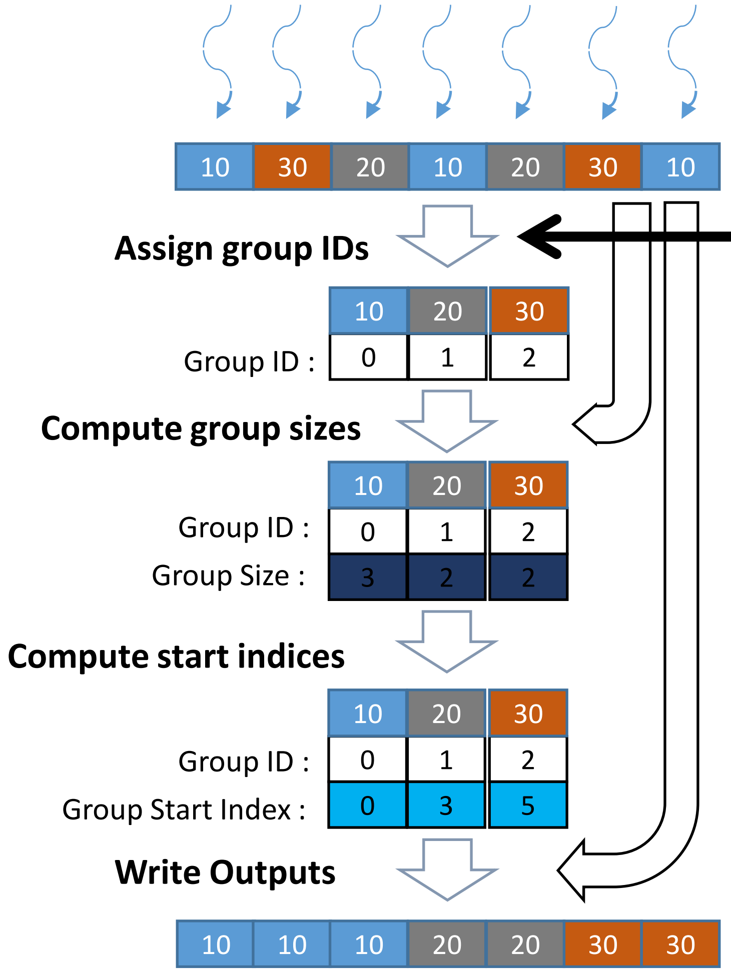
Compute start indices

	10	20	30
Group ID :	0	1	2
Group Start Index :	0	3	5

Write Outputs

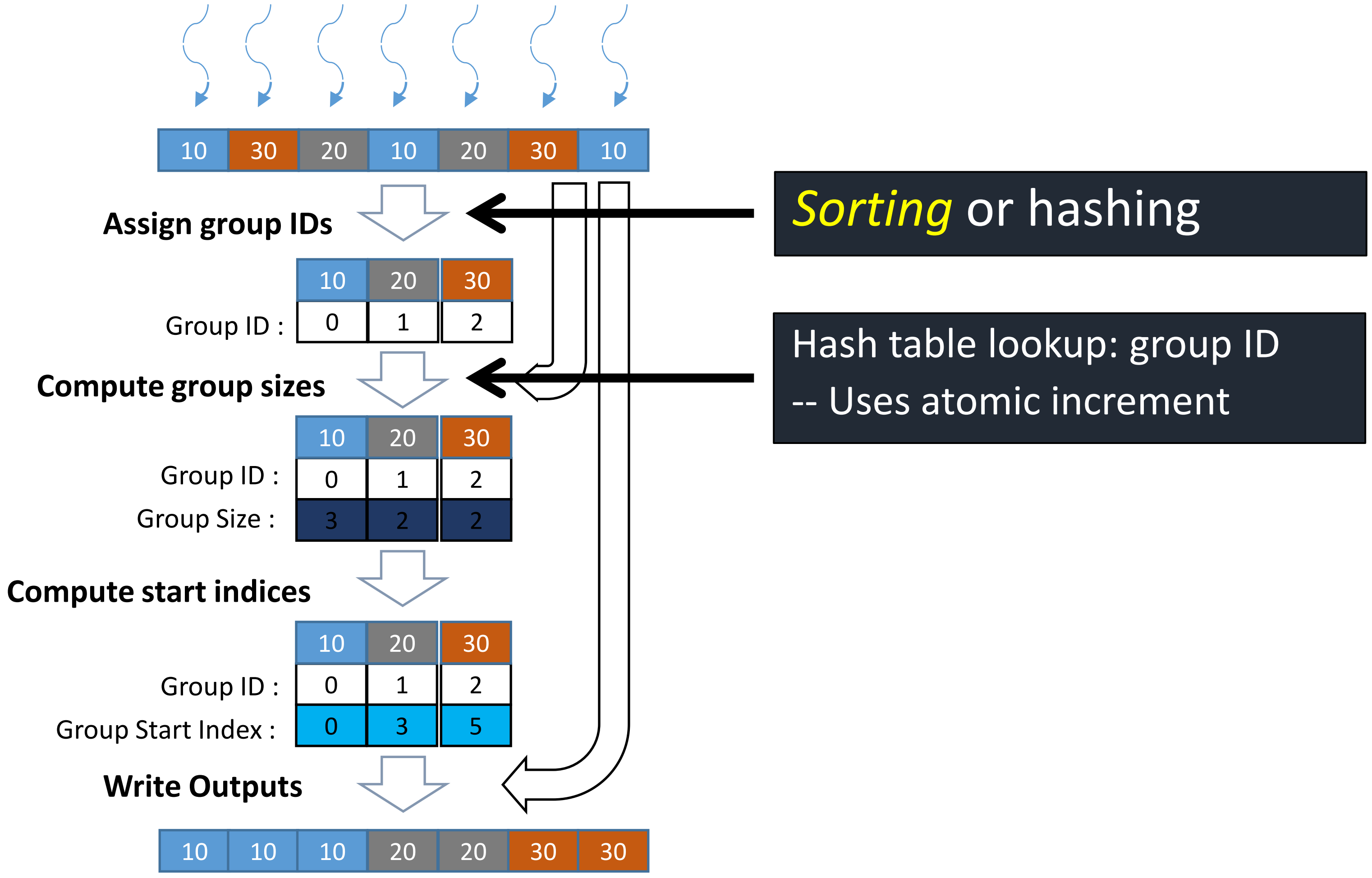
10 | 10 | 10 | 20 | 20 | 30 | 30

GroupBy using parallel primitives



Sorting or hashing

GroupBy using parallel primitives



10	30	20	10	20	30	10

Assign group IDs

	10	20	30
Group ID :	0	1	2

Compute group sizes

	10	20	30
Group ID :	0	1	2
Group Size :	3	2	2

Compute start indices

	10	20	30
Group ID :	0	1	2
Group Start Index :	0	3	5

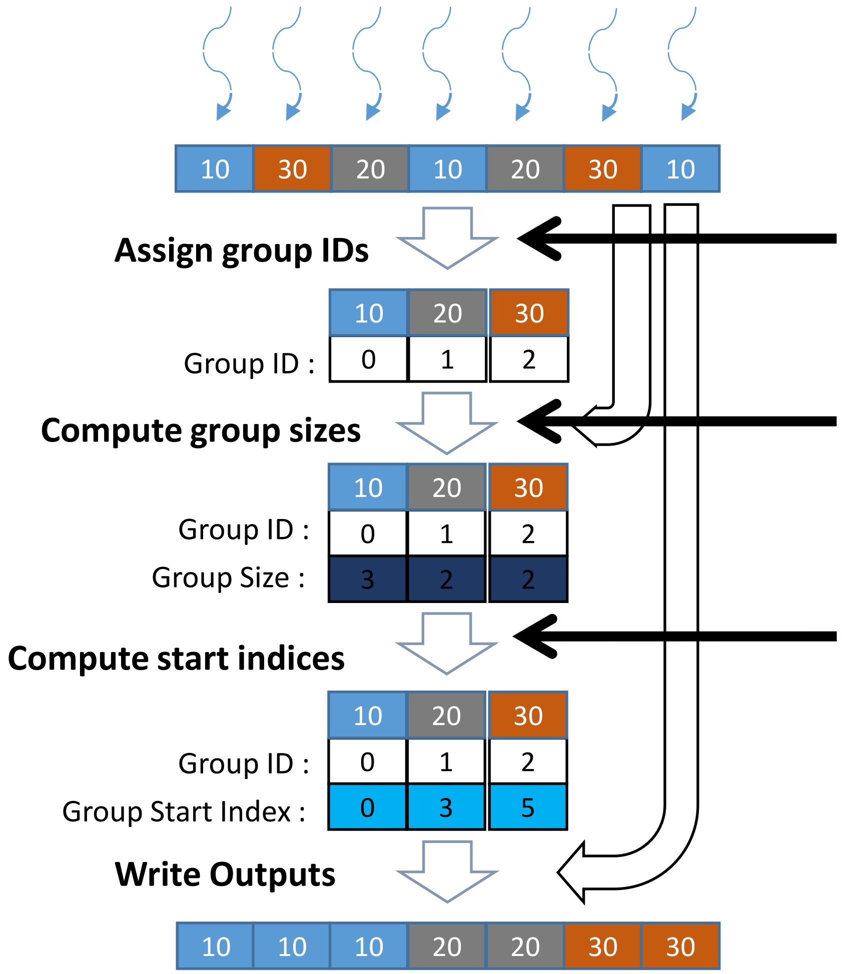
Write Outputs

10	10	10	20	20	30	30

Sorting or hashing

Hash table lookup: group ID
-- Uses atomic increment

GroupBy using parallel primitives



10	30	20	10	20	30	10

Assign group IDs

	10	20	30
Group ID :	0	1	2

Compute group sizes

	10	20	30
Group ID :	0	1	2
Group Size :	3	2	2

Compute start indices

	10	20	30
Group ID :	0	1	2
Group Start Index :	0	3	5

Write Outputs

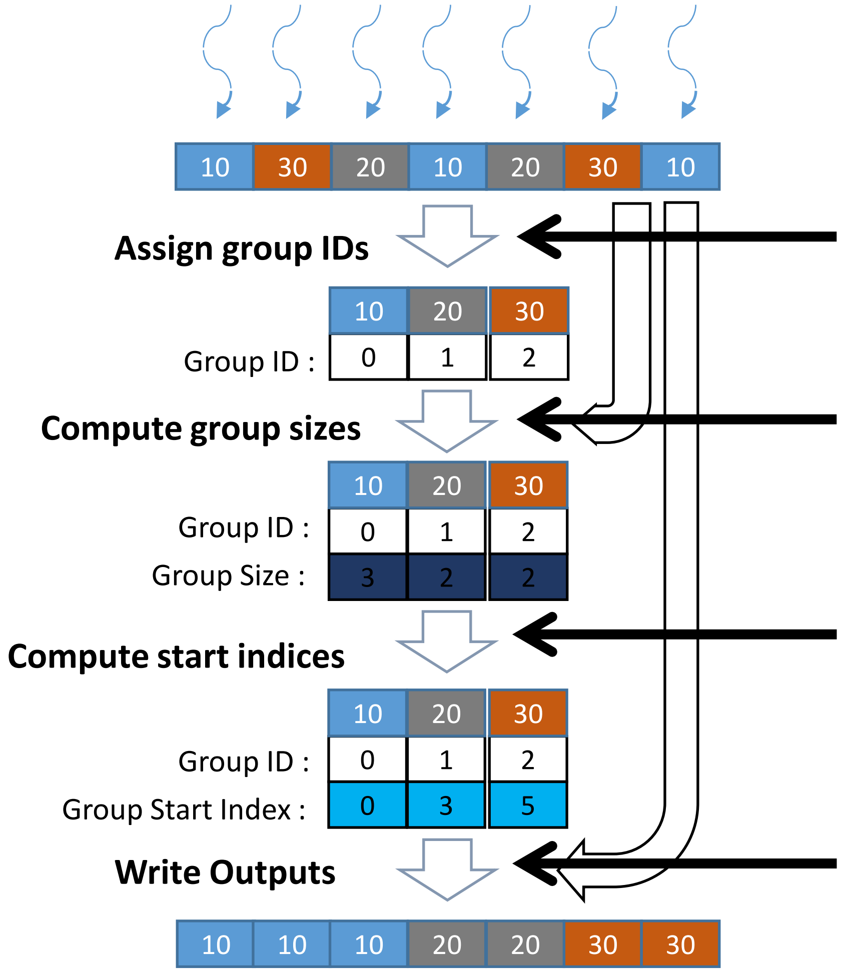
10	10	10	20	20	30	30

Sorting or hashing

Hash table lookup: group ID
-- Uses atomic increment

prefix sum of group sizes

GroupBy using parallel primitives



10	30	20	10	20	30	10

Assign group IDs

	10	20	30
Group ID :	0	1	2

Compute group sizes

	10	20	30
Group ID :	0	1	2
Group Size :	3	2	2

Compute start indices

	10	20	30
Group ID :	0	1	2
Group Start Index :	0	3	5

Write Outputs

10	10	10	20	20	30	30

Sorting or hashing

Hash table lookup: group ID
-- Uses atomic increment

prefix sum of groups sizes

Write to output location
- Uses atomic increment
- ***Scatter gather***