



# Parallel Algorithms

Chris Rossbach and Calvin Lin

cs380p

# Outline

Over the next few classes:

Background from many areas

Architecture

Vector processors

Hardware multi-threading

Graphics

Graphics pipeline

Graphics programming models

Algorithms

parallel architectures → parallel algorithms

Programming GPUs

CUDA

Basics: getting something working

Advanced: making it perform

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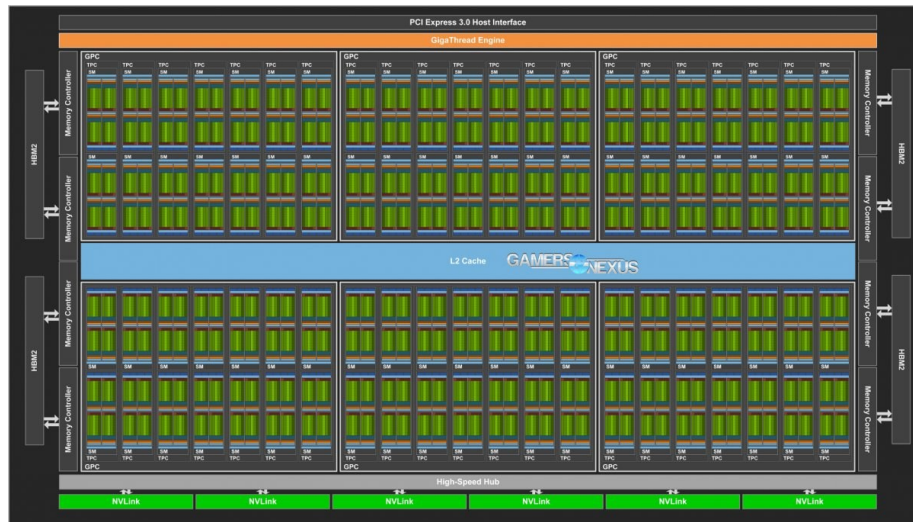


# Review



Each SM has multiple vector units (4)  
32 lanes wide → warp size

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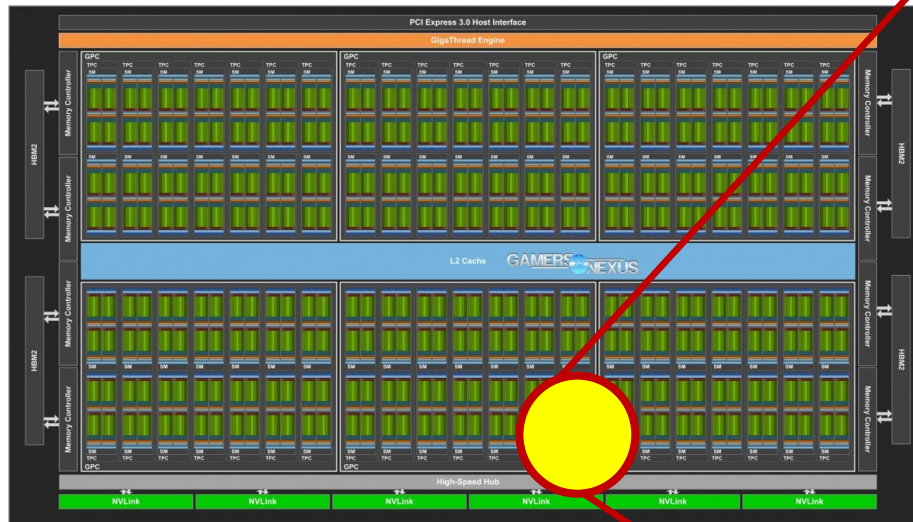
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Each TB has some number of threads

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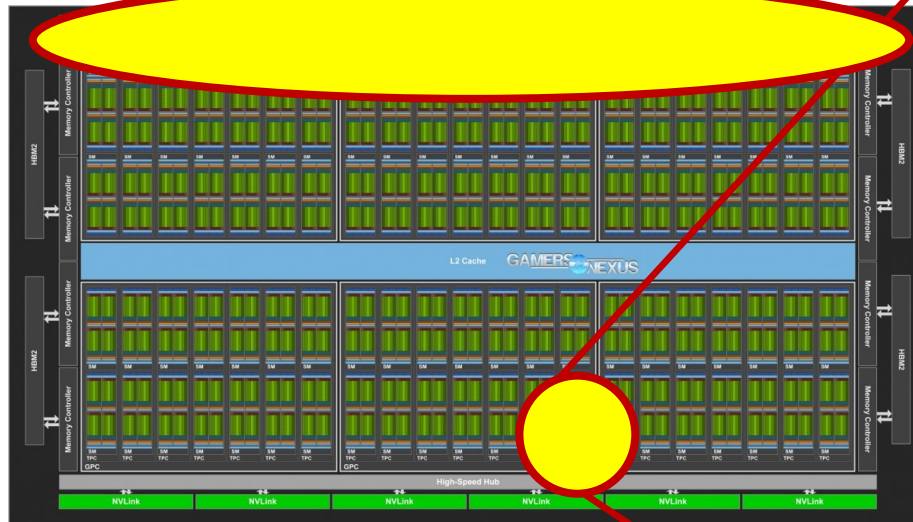


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Thread block scheduler



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Thread block scheduler warp (thread) scheduler



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# Programming Model

“kernels” == “shader programs”

1000s of HW-scheduled threads per kernel

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Threads in a block can synchronize (barrier)

This is the *\*only\** synchronization

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***Need codes that are 1000s-X  
parallel...***

# Parallel Algorithms

Sequential algorithms often do not permit easy parallelization

Does not mean there work has no parallelism

A different approach can yield parallelism

but often changes the algorithm

Parallelizing != just adding locks to a sequential algorithm

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If you can express your algorithm using these patterns, an apparently fundamentally sequential algorithm can be made parallel

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Map

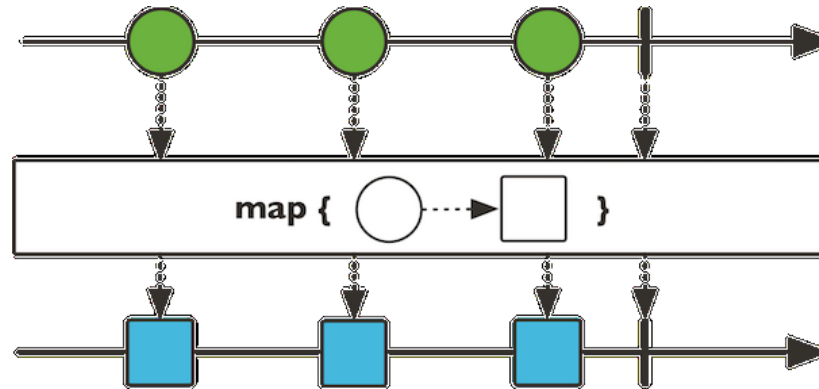
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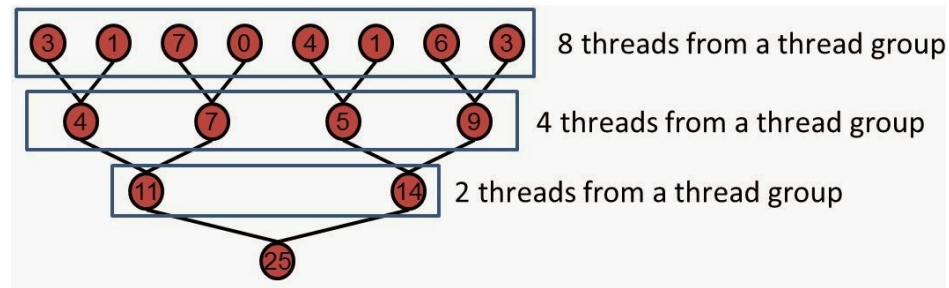
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Examples:

Map

Reductions



# Parallel Algorithms

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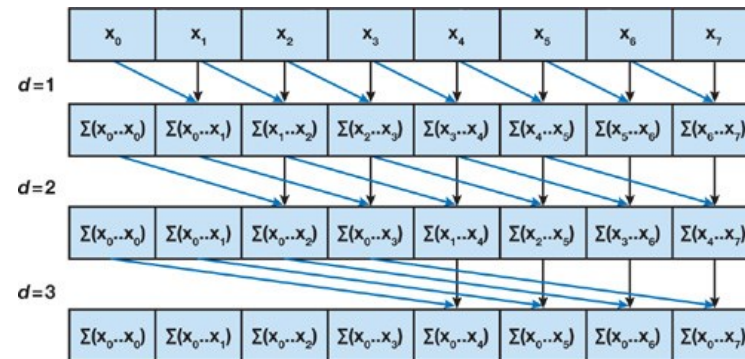
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Examples:

Map

Reductions

Scans



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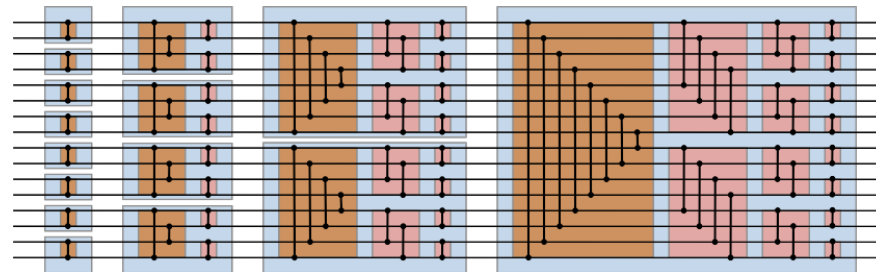
Examples:

Map

Reductions

Scans

Re-orderings (scatter/gather/sort)





# Map

## Inputs

Array A

Function  $f(x)$

$\text{map}(A, f) \rightarrow$  apply  $f(x)$  on all elements in A

Parallelism trivially exposed

$f(x)$  can be applied in parallel to all elements, in principle

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Array A

Function f(x)

`map(A, f)` → apply f(x) on all elements in A

Parallelism trivially exposed

f(x) can be applied in parallel to all elements, in principle

```
for(i=0; i<numPoints; i++) {  
    labels[i] = findNearestCenter(points[i]);  
}
```



```
map(points, findNearestCenter)
```



# Scatter and Gather

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Gather:

Read multiple items to single /packed location

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```
for (i=0; i<N; ++i)  
  x[i] = y[idx[i]]; → gather(x, y, idx)
```

```
for (i=0; i<N; ++i)  
  y[idx[i]] = x[i]; → scatter(x, y, idx)
```

# Scatter and Gather

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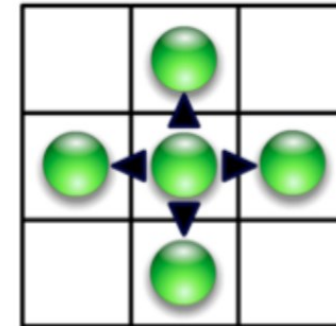


gather(x, y, idx)

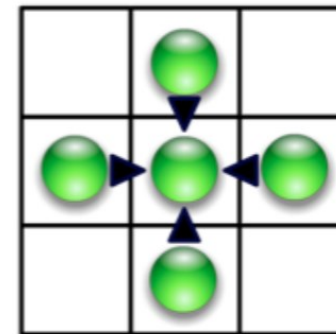
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for (i=0; i<N; ++i)  
  y[idx[i]] = x[i];
```



scatter(x, y, idx)



Scatter



Gather



# Reduce

## Input

Associative operator **op**

Ordered set  $s = [a, b, c, \dots z]$

Reduce(**op**,  $s$ ) returns

$a \text{ op } b \text{ op } c \dots \text{ op } z$

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```
for(i=0; i<N; ++i) {  
    accum += (point[i]*point[i])  
}
```



```
accum = reduce(*, point)
```

# Reduce

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$\text{accum} = \text{reduce}(*, \text{point})$

Why must **op** be associative?

# Reduce

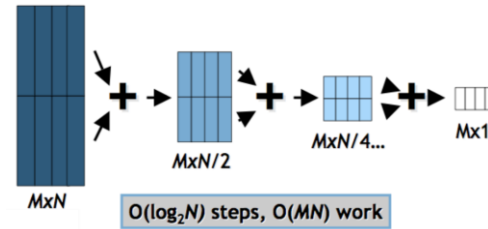
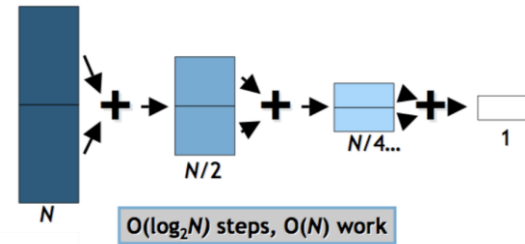
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# Scan (Prefix Sum)

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Associative operator **op**

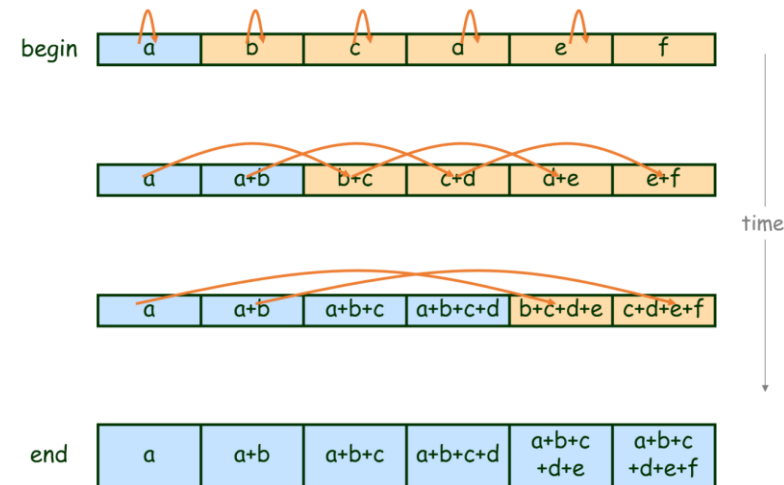
Ordered set  $s = [a, b, c, \dots z]$

Identity  $I$

$$\text{scan}(\text{op}, s) = [I, a, (a \text{ op } b), (a \text{ op } b \text{ op } c) \dots]$$

Scan is the workhorse of parallel algorithms:

Sort, histograms, sparse matrix, string compare, ...



# Example: Parallel GroupBy

Group a collection by key

Lambda function maps elements → key

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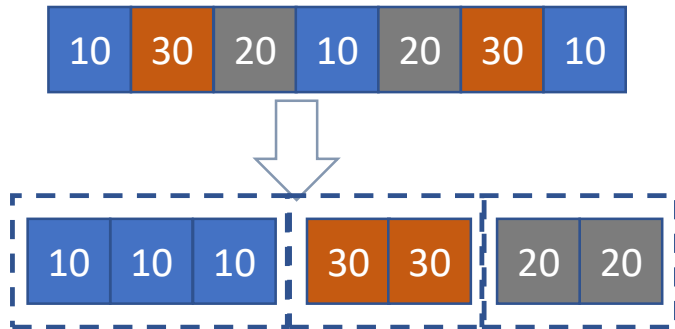


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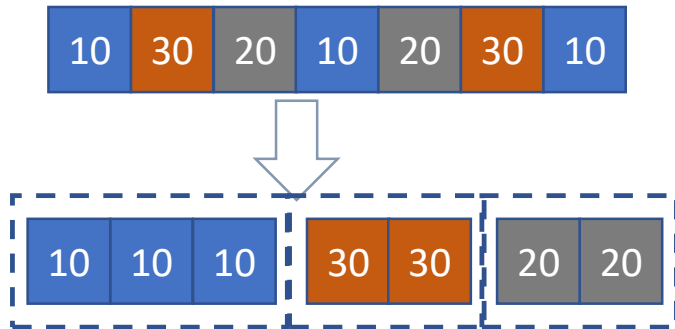


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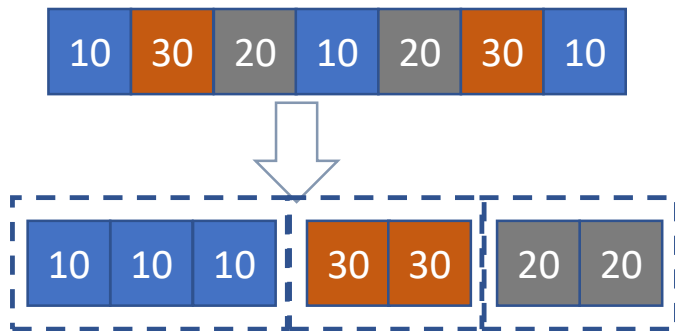
```
foreach(T elem in ints)  
{  
    key    = KeyLambda(elem);  
  
    group = GetGroup(key);  
  
    group.Add(elem);  
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# Example: Parallel GroupBy

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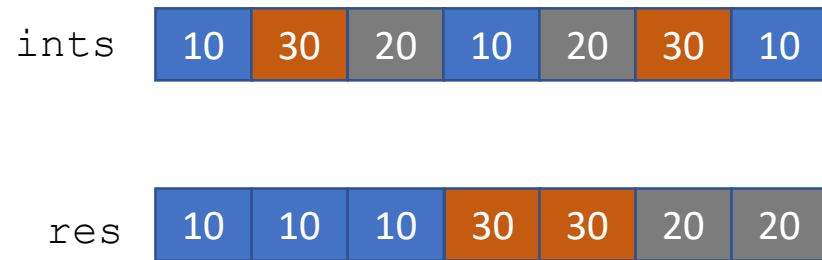
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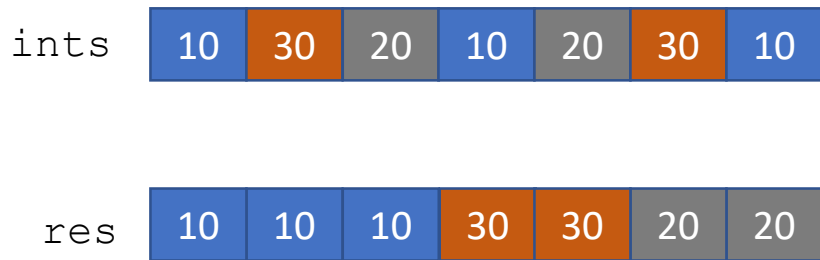


# Parallel GroupBy

## Process each input element in parallel

grouping ~ shuffling

input item → output offset such that groups are contiguous

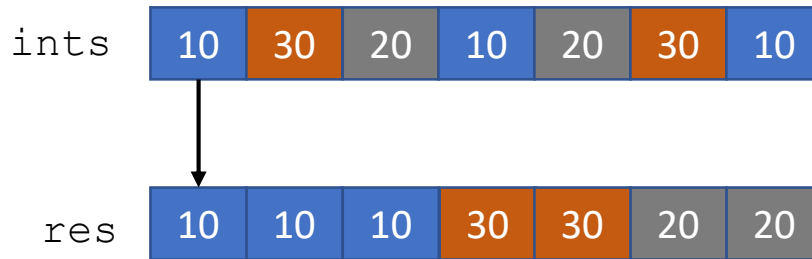


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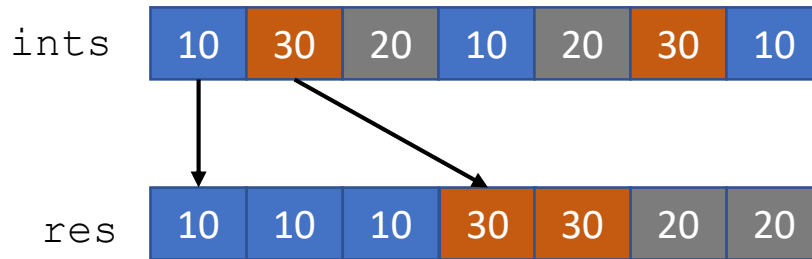


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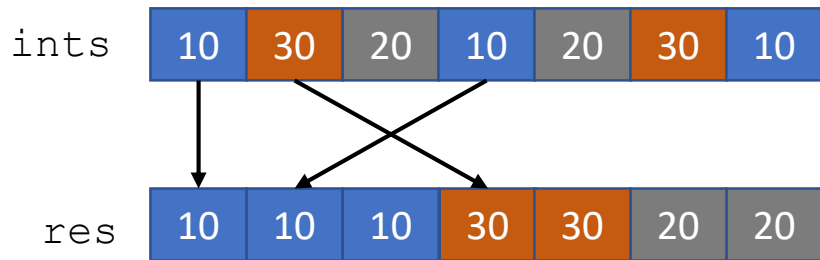


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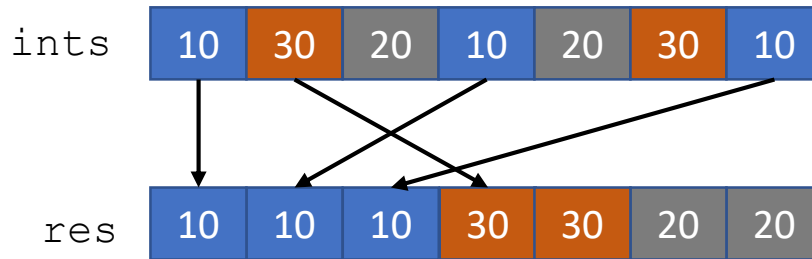


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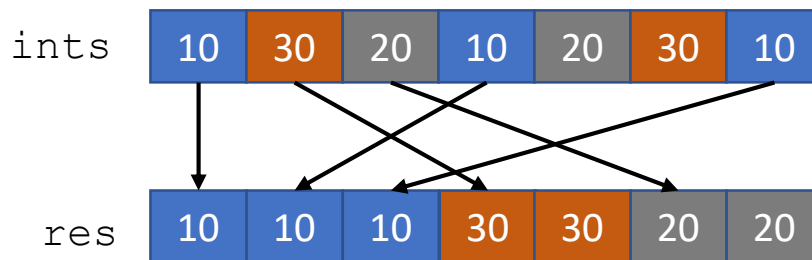


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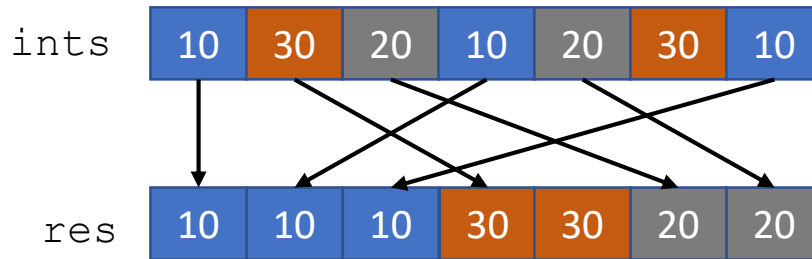


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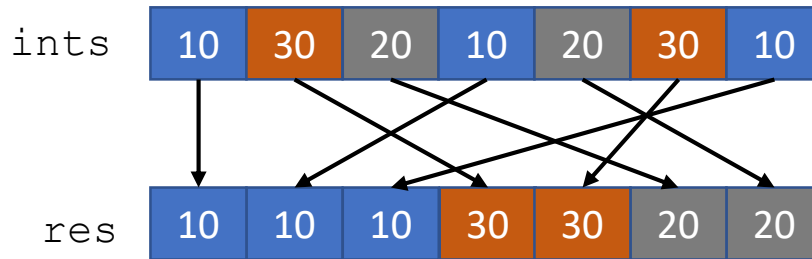


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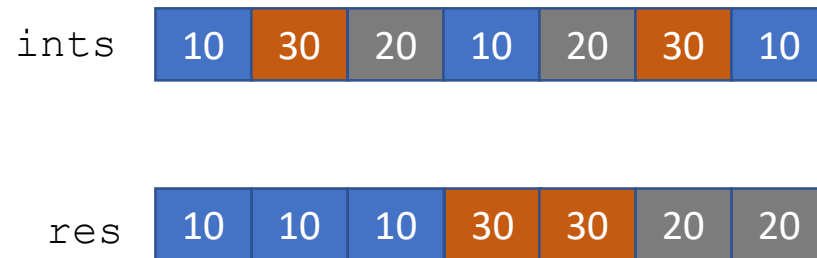


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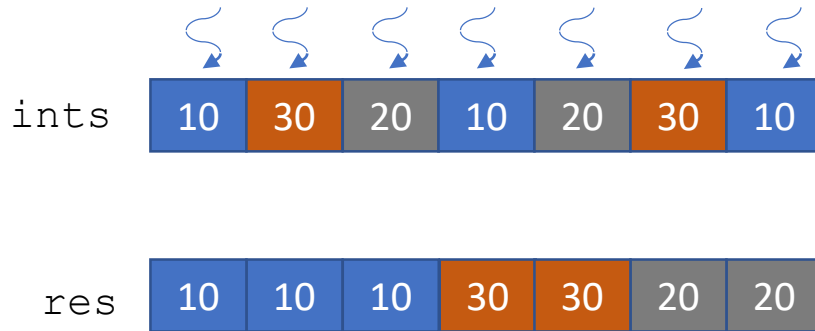


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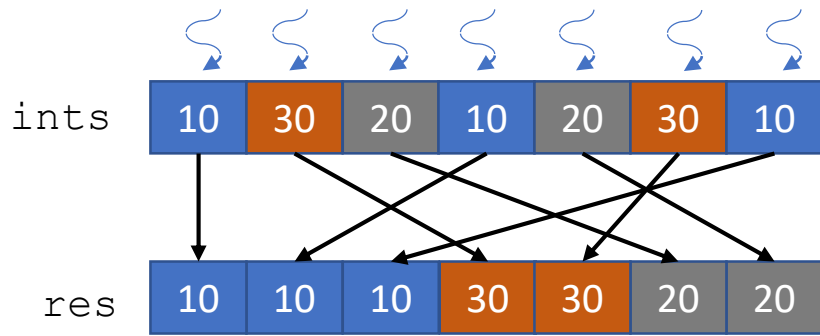


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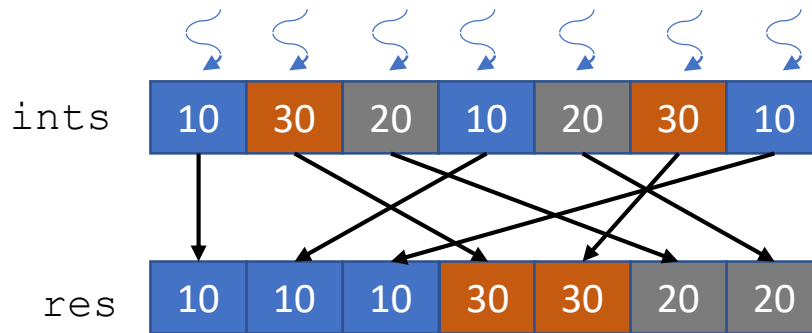
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output offset = group offset + item number

... but how to get the group offset, item number?





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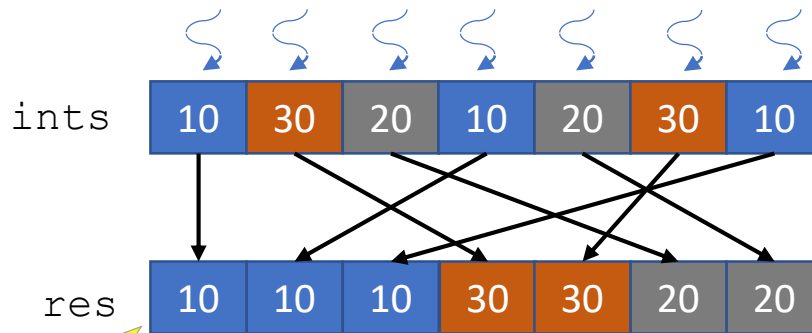
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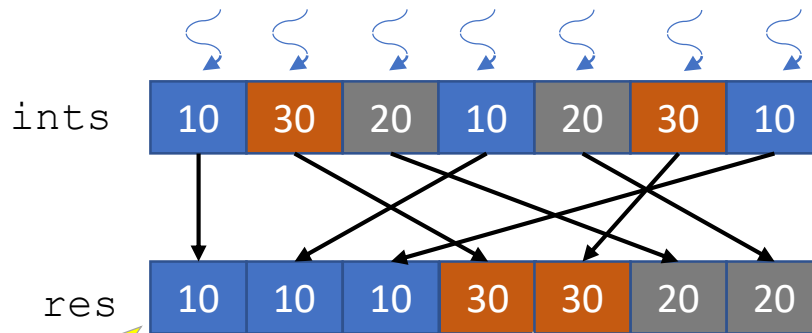
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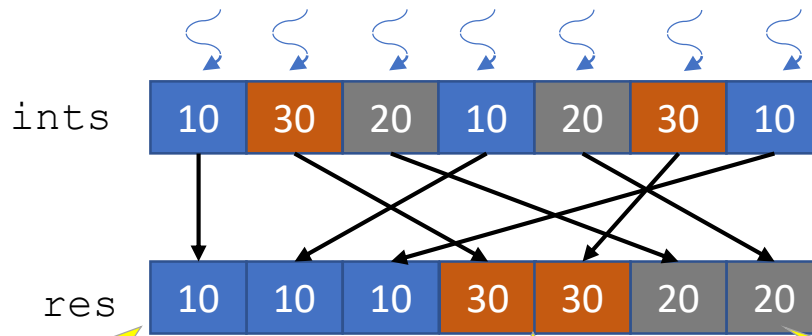
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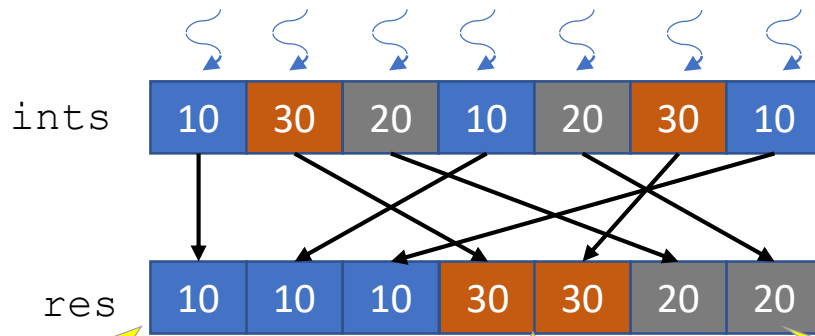
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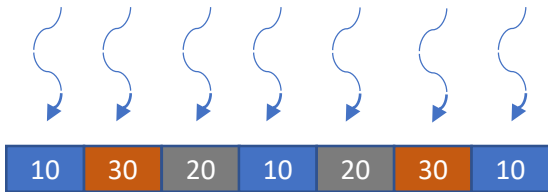
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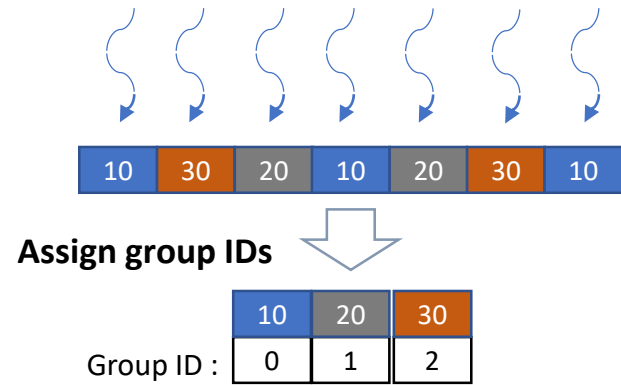
# GroupBy with Parallel Primitives

10	30	20	10	20	30	10
----	----	----	----	----	----	----

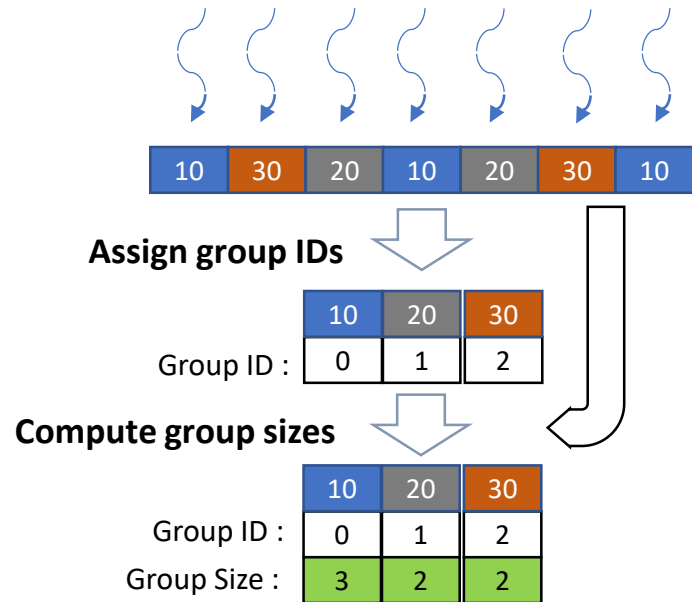
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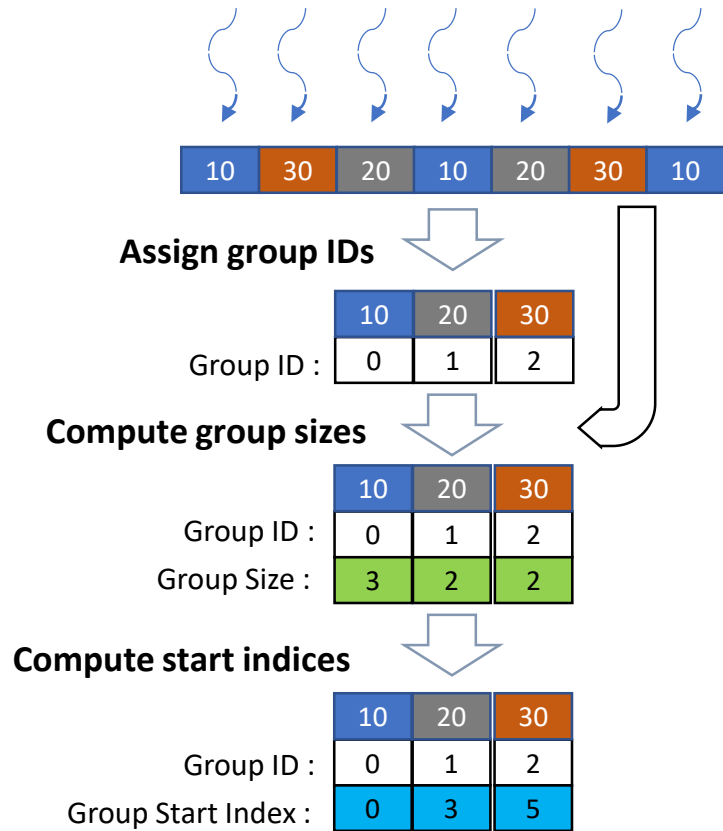


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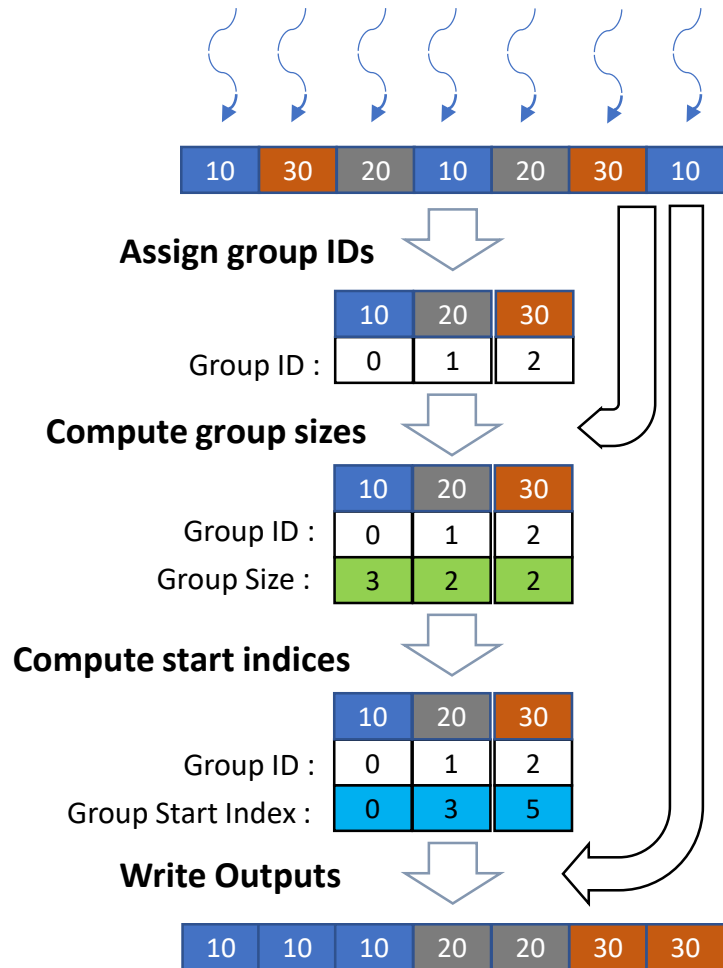




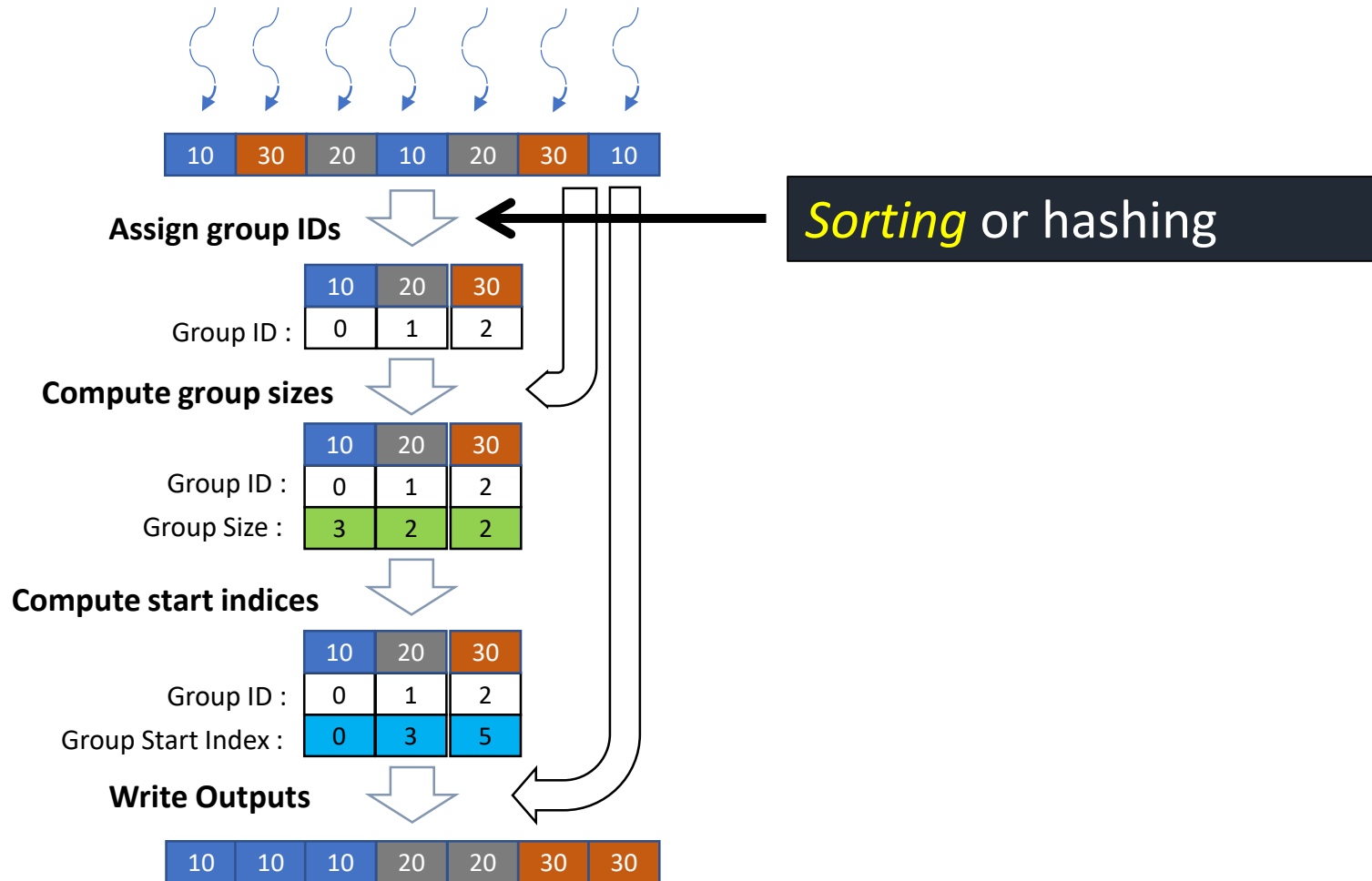
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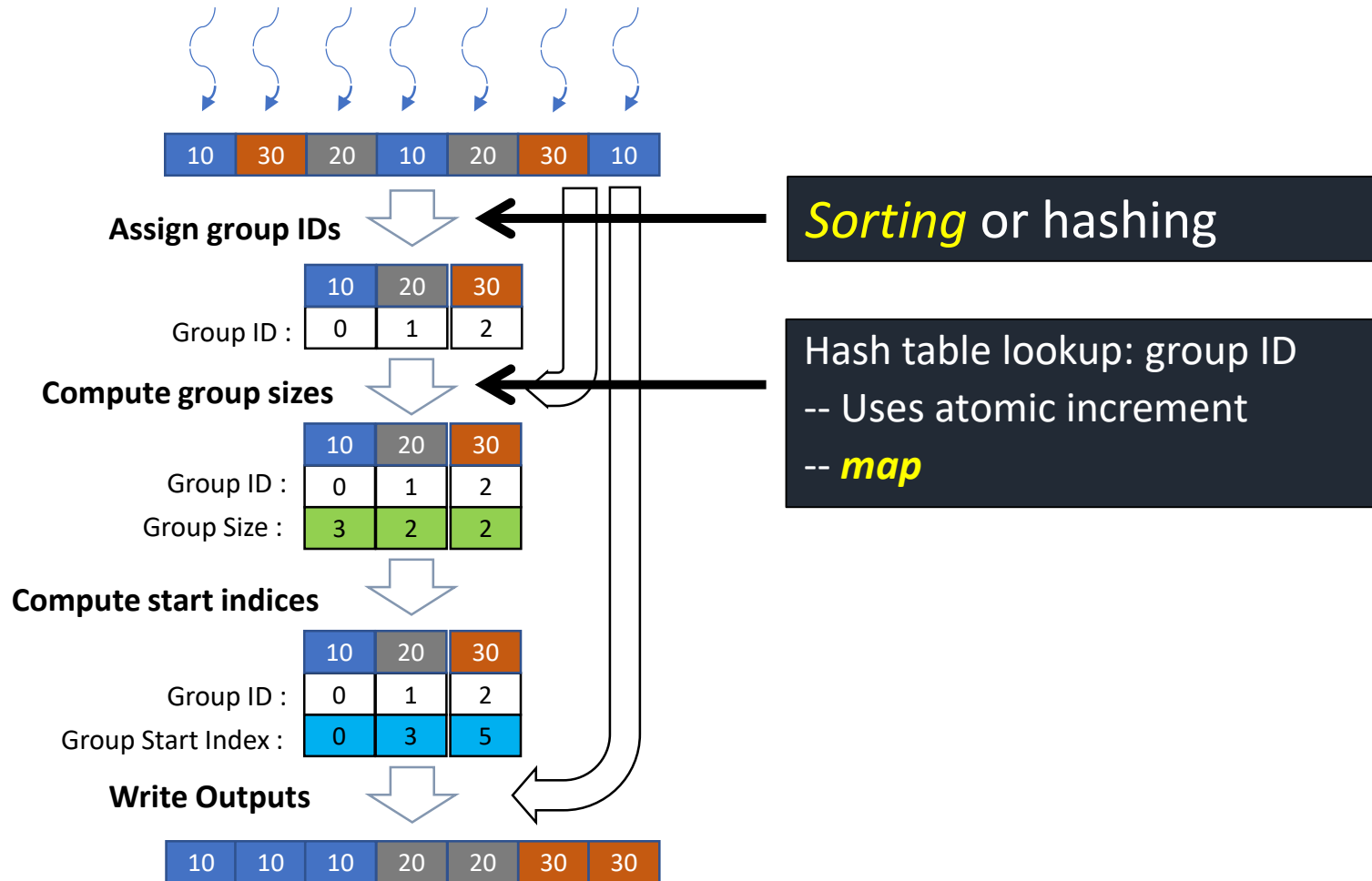
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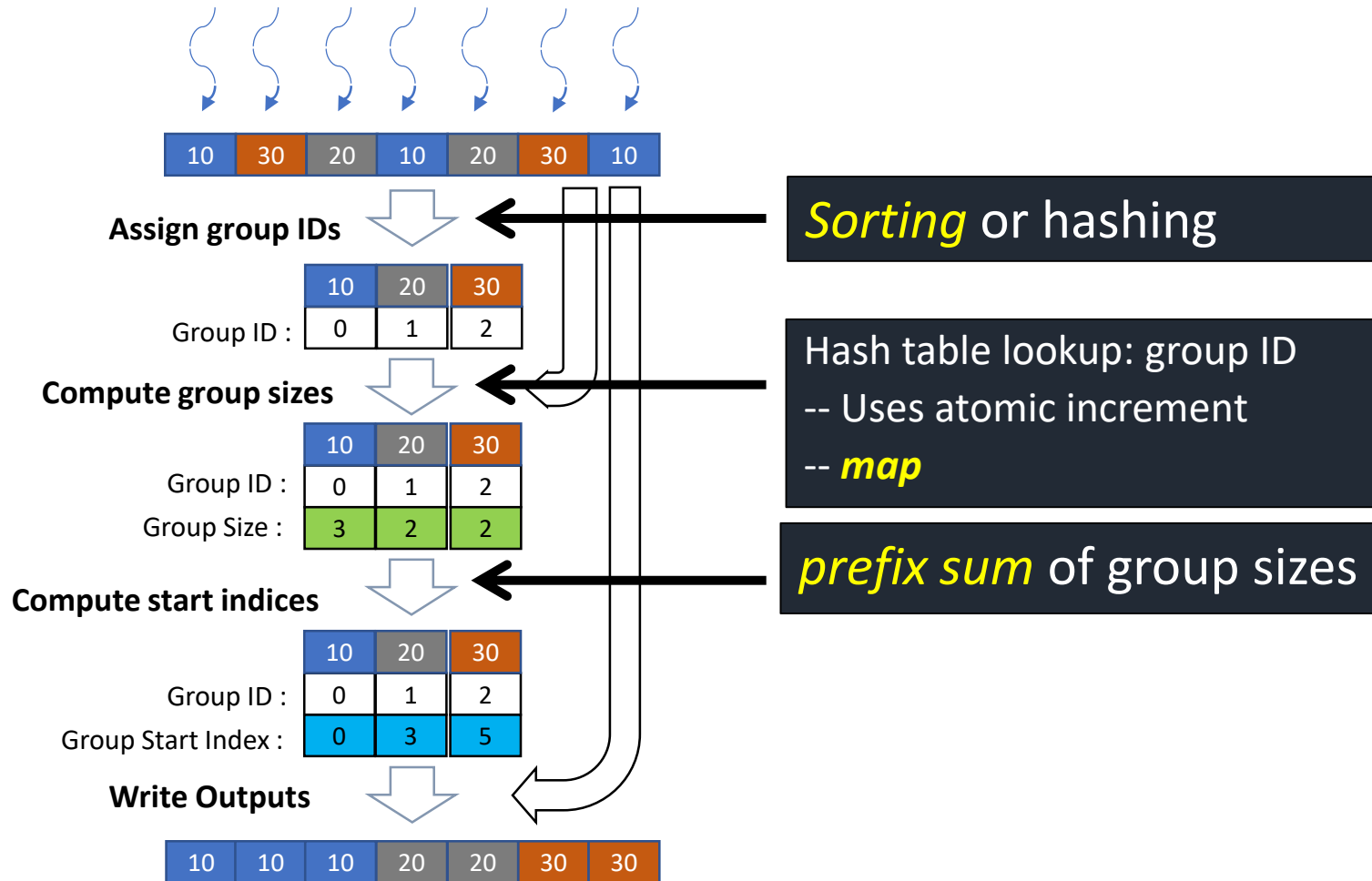
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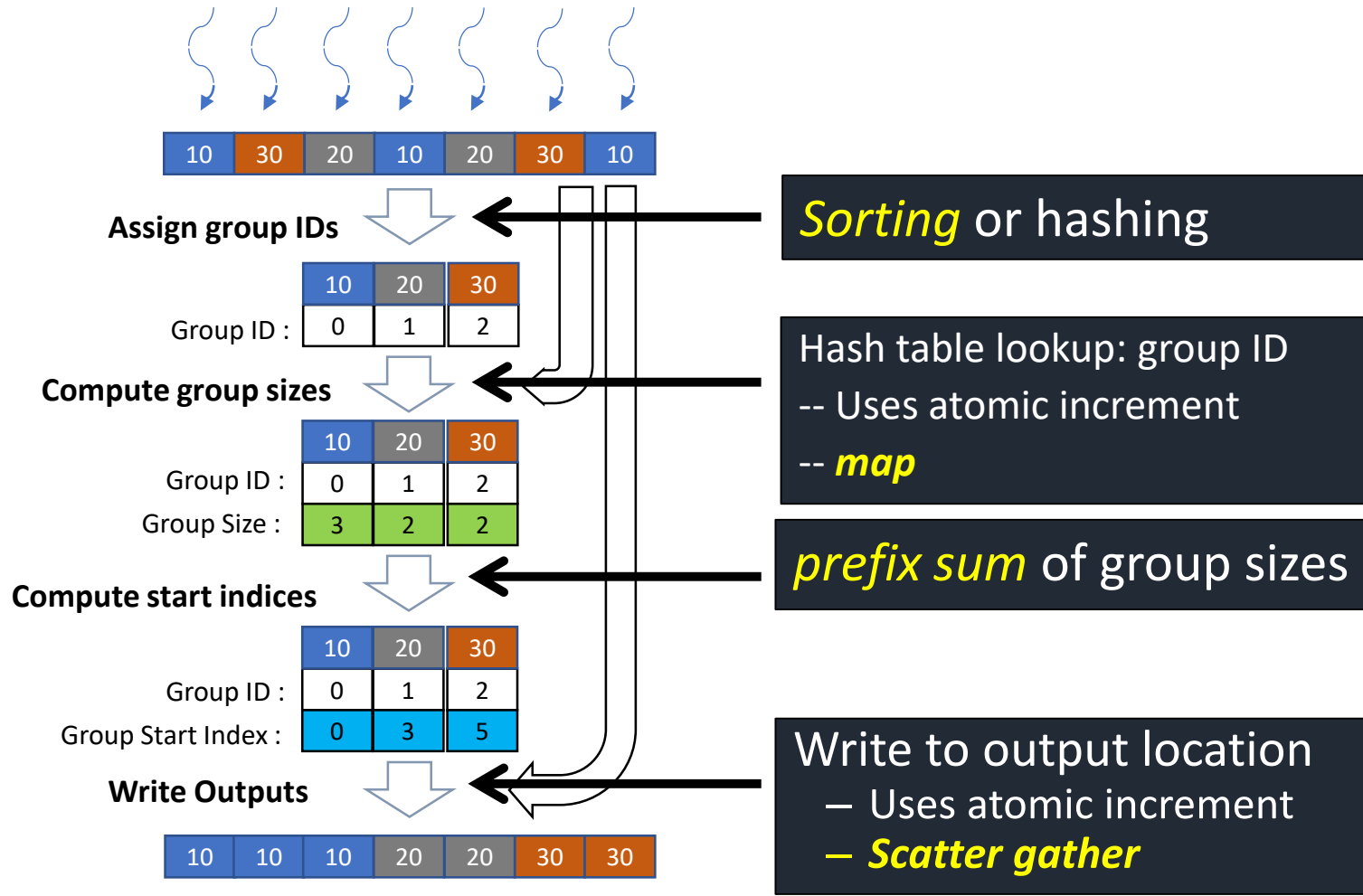
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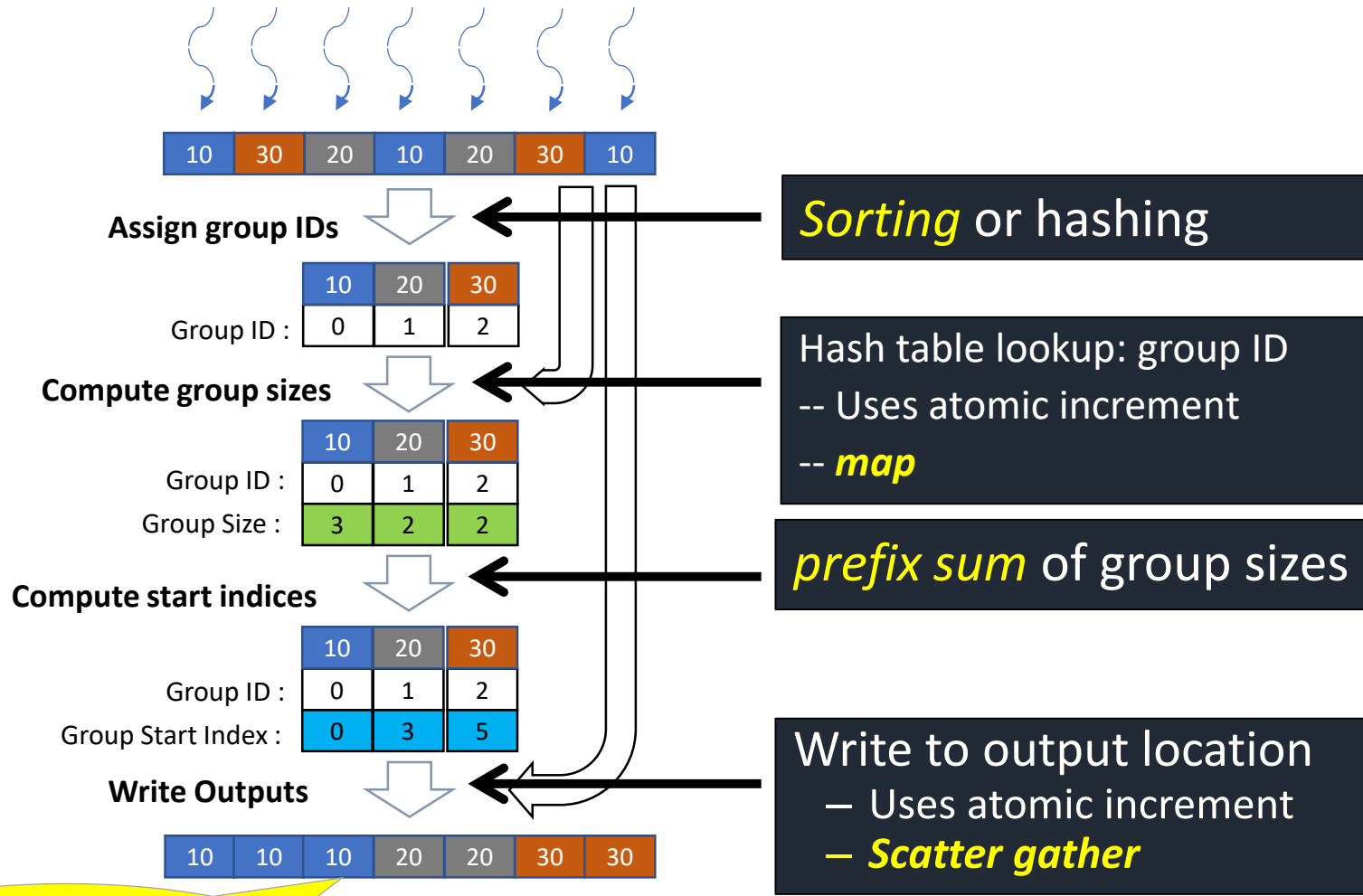
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# GroupBy with Parallel Primitives



We'll revisit after more CUDA background...



# Parallel Patterns



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## Thrust:

Large set of algorithms

~75 functions

~125 variations

## Flexible

User-defined types

User-defined operators

Algorithm	Description
<code>reduce</code>	Sum of a sequence
<code>find</code>	First position of a value in a sequence
<code>mismatch</code>	First position where two sequences differ
<code>inner_product</code>	Dot product of two sequences
<code>equal</code>	Whether two sequences are equal
<code>min_element</code>	Position of the smallest value
<code>count</code>	Number of instances of a value
<code>is_sorted</code>	Whether sequence is in sorted order
<code>transform_reduce</code>	Sum of transformed sequence



# Parallel Patterns

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Dwarf Popularity (Red Hot → Blue Cool)

	HPC	Embed	SPEC	ML	Games	DB
1 Dense Matrix	Red	Red	Red	Red	Red	Yellow
2 Sparse Matrix	Red	Yellow	Yellow	Red	Red	Light Blue
3 Spectral (FFT)	Red	Yellow	Light Blue	Yellow	Yellow	Light Blue
4 N-Body	Red	Light Blue	Yellow	Light Blue	Yellow	Light Blue
5 Structured Grid	Red	Red	Red	Light Blue	Yellow	Light Blue
6 Unstructured	Red	Light Blue	Light Blue	Yellow	Yellow	Light Blue
7 MapReduce	Red	Light Blue	Green	Red	Light Blue	Red
8 Combinational	Light Blue	Red	Light Blue	Green	Light Blue	Green
9 Graph Traversal	Light Blue	Red	Yellow	Red	Yellow	Yellow
10 Dynamic Prog	Light Blue	Yellow	Light Blue	Red	Light Blue	Red
11 Backtrack/ B&B	Light Blue	Light Blue	Light Blue	Red	Light Blue	Yellow
12 Graphical Models	Light Blue	Light Blue	Light Blue	Red	Light Blue	Yellow
13 FSM	Light Blue	Red	Red	Yellow	Yellow	Red



# Parallel Patterns

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TBB is a collection of components for parallel programming:

- Basic algorithms: `parallel_for`, `parallel_reduce`, `parallel_scan`
- Advanced algorithms: `parallel_while`, `parallel_do`, `parallel_pipeline`, `parallel_sort`
- Containers: `concurrent_queue`, `concurrent_priority_queue`, `concurrent_vector`, `concurrent_hash_map`
- Memory allocation: `scalable_malloc`, `scalable_free`, `scalable_realloc`, `scalable_calloc`, `scalable_allocator`, `cache_aligned_allocator`
- Mutual exclusion: `mutex`, `spin_mutex`, `queuing_mutex`, `spin_rw_mutex`, `queuing_rw_mutex`, `recursive_mutex`
- Atomic operations: `fetch_and_add`, `fetch_and_increment`, `fetch_and_decrement`, `compare_and_swap`, `fetch_and_store`
- Timing: portable fine grained global time stamp
- Task scheduler: direct access to control the creation and activation of tasks



# Parallel Patterns

# Summary

Re-expressing apparently sequential algorithms as combinations of parallel patterns is a common technique when targeting GPUs

## Examples

- Reductions

- Scans

- Re-orderings (scatter/gather)

- Sort

- Map

What is the *right* set of parallel patterns to support?