Topic 24
Heaps

"You think you know when you can **learn**,
are more sure when you can **write,**
even more when you can **teach**,
but certain when you can **program**."
- Alan Perlis

Priority Queue

- Recall priority queue
  - elements enqueued based on priority
  - dequeue removes the highest priority item
- Options?
  - List? Binary Search Tree? Clicker 1

	Linked List enqueue	BST enqueue
A.	O(N)	O(1)
B.	O(N)	O(logN)
C.	O(N)	O(N)
D.	O(logN)	O(logN)
E.	O(1)	O(logN)

Another Option

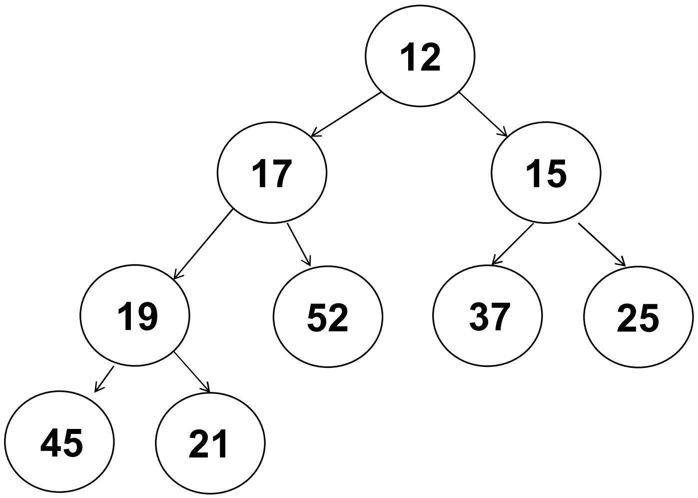
- The *heap* data structure
  - not to be confused with the runtime heap (portion of memory for dynamically allocated variables)
- A complete binary tree
  - all levels have maximum number of nodes except deepest where nodes are filled in from left to right
- Maintains the *heap order property*
  - in a min heap the value in the root of any subtree is less than or equal to all other values in the subtree

Clicker 2

- In a max heap with no duplicates where is the largest value?

A. the root of the tree
B. in the left-most node
C. in the right-most node
D. a node in the lowest level
E. none of these

Example Min Heap

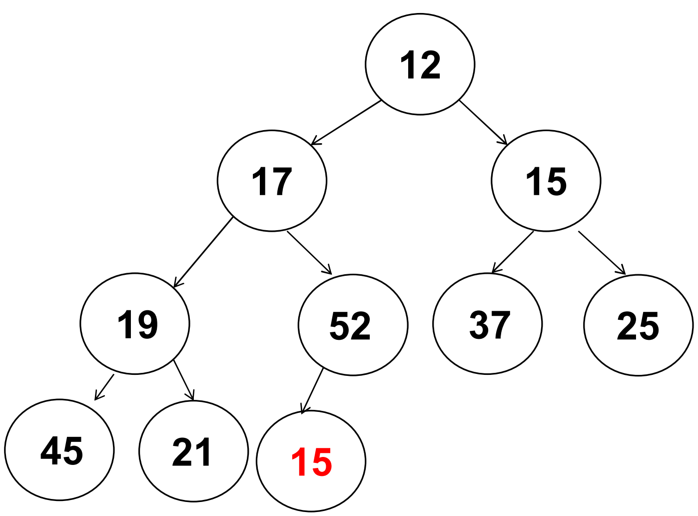


Add Operation

- Add new element to next open spot in array
- Swap with parent if new value is less than parent
- Continue back up the tree as long as the new value is less than new parent node

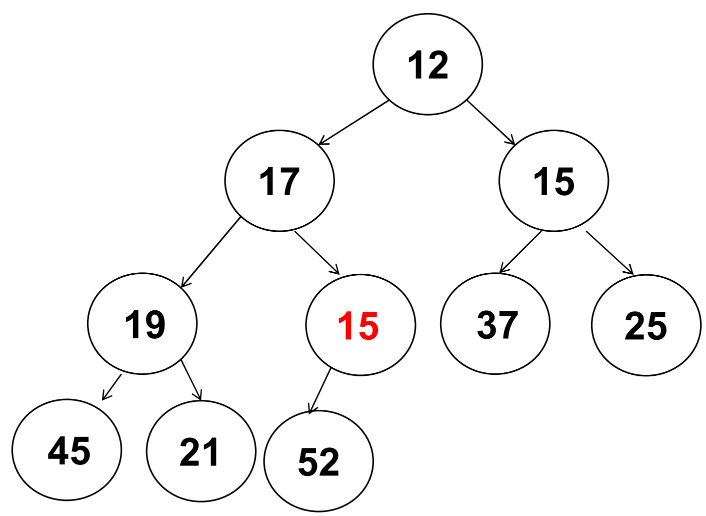
Add Example

- Add 15 to heap (initially next left most node)



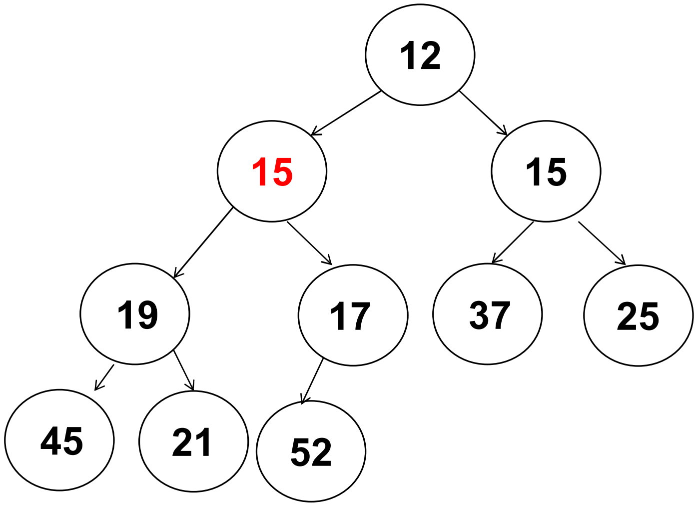
Add Example

- Swap 15 and 52



Enqueue Example

- Swap 15 and 17, then stop

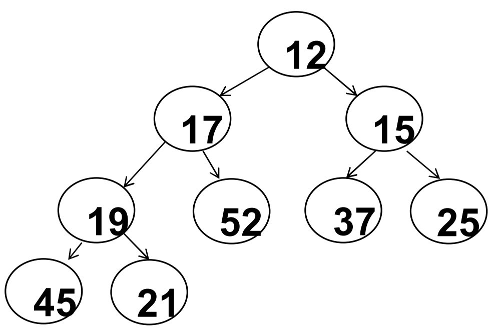


Add Example

- Insert the following values 1 at a time into a min heap:
  16 9 5 8 13 8 8 5 5 19 27 9 3

Internal Storage

- Interestingly heaps are often implemented with an array instead of nodes



for element at index i:
parent index: *i / 2*
left child index: *i * 2*
right child index: *i * 2 + 1*

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**17**	**15**	**19**	**52**	**37**	**25**	**45**	**21**						

In Honor of Elijah, The Meme King, Spring 2020

PriorityQueue Class

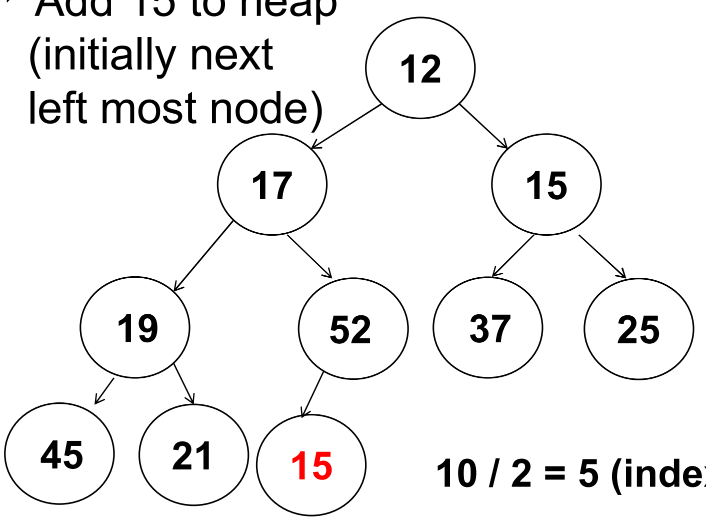
```
public class PriorityQueue<E extends Comparable<? super E>>
{

    private int size;
    private E[] con;

    public PriorityQueue() {
        con = getArray(2);
    }

    private E[] getArray(int size) {
        return (E[]) (new Comparable[size]);
    }
```


PriorityQueue enqueue / add

```
public void enqueue(E val) {
    if ( size >= con.length - 1 )
        enlargeArray(con.length * 2);

    size++;
    int indexToPlace = size;
    while ( indexToPlace > 1
            && val.compareTo( con[indexToPlace / 2] ) < 0 ) {

        con[indexToPlace] = con[indexToPlace / 2]; // swap
        indexToPlace /= 2; // change indexToPlace to parent
    }
    con[indexToPlace] = val;
}

private void enlargeArray(int newSize) {
    E[] temp = getArray(newSize);
    System.arraycopy(con, 1, temp, 1, size);
    con = temp;
}
```


Enqueue / add Example With Array Shown

- Add 15 to heap (initially next left most node)

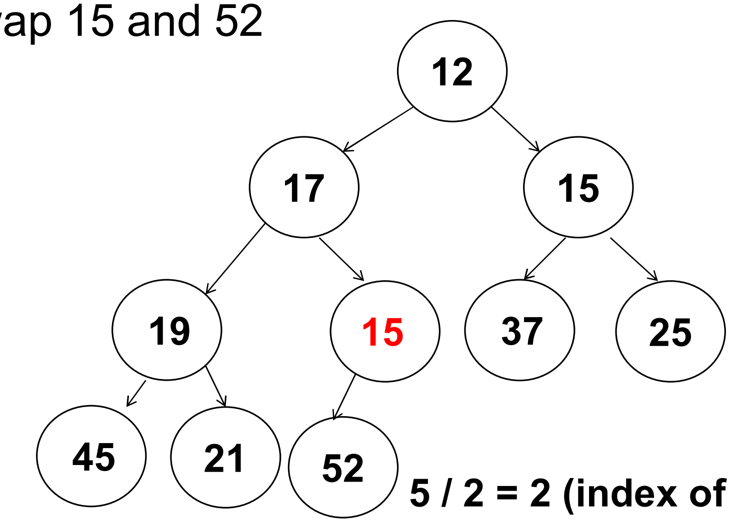


10 / 2 = 5 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**17**	**15**	**19**	**52**	**37**	**25**	**45**	**21**	**15**					

Enqueue Example With Array Shown

- Swap 15 and 52

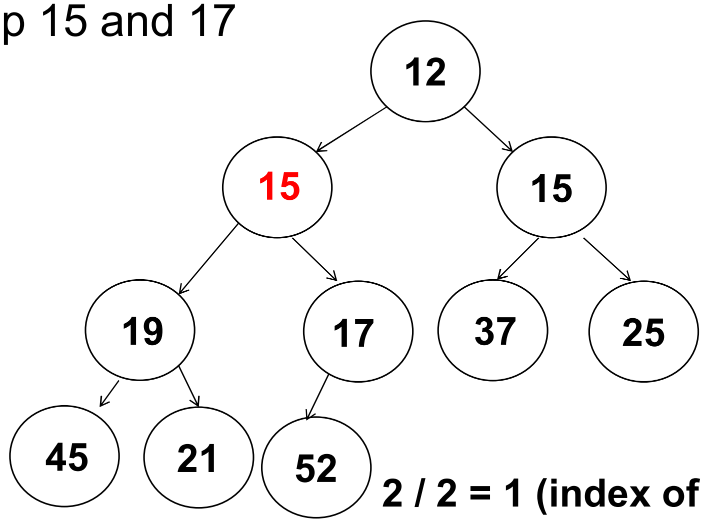


5 / 2 = 2 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**17**	**15**	**19**	**15**	**37**	**25**	**45**	**21**	**52**					

Enqueue Example With Array Shown

- Swap 15 and 17

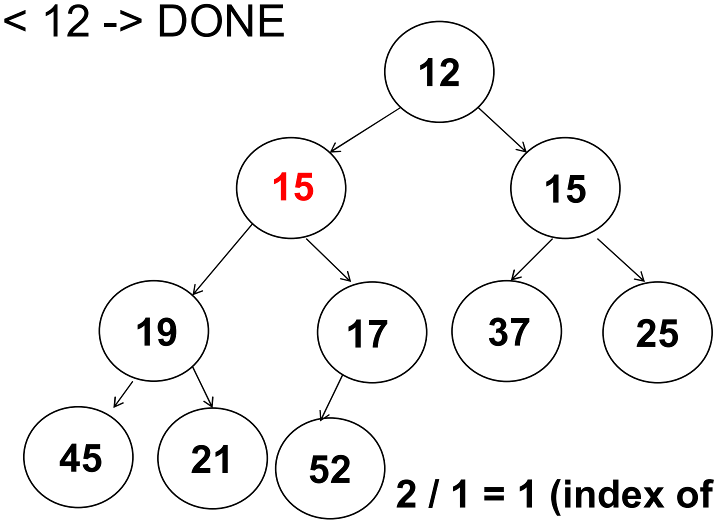


2 / 2 = 1 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**15**	**15**	**19**	**17**	**37**	**25**	**45**	**21**	**52**					

Enqueue Example With Array Shown

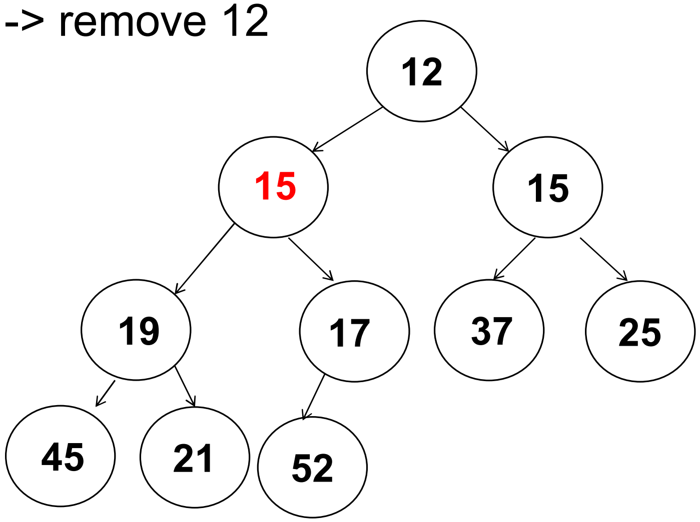
- 15 !< 12 -> DONE



2 / 1 = 1 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**15**	**16**	**19**	**17**	**37**	**25**	**45**	**21**	**52**					

- Remove -> remove 12

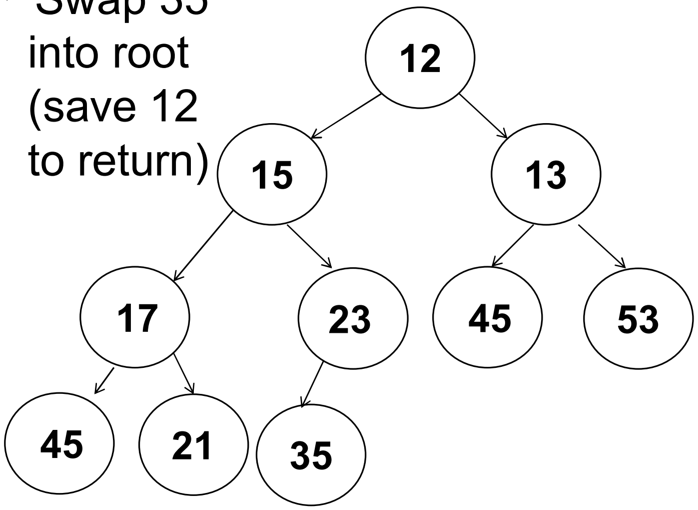


Remove / Dequeue

- min value / front of queue is in root of tree
- swap value from last node to root and move down swapping with smaller child unless values is smaller than both children

Dequeue Example

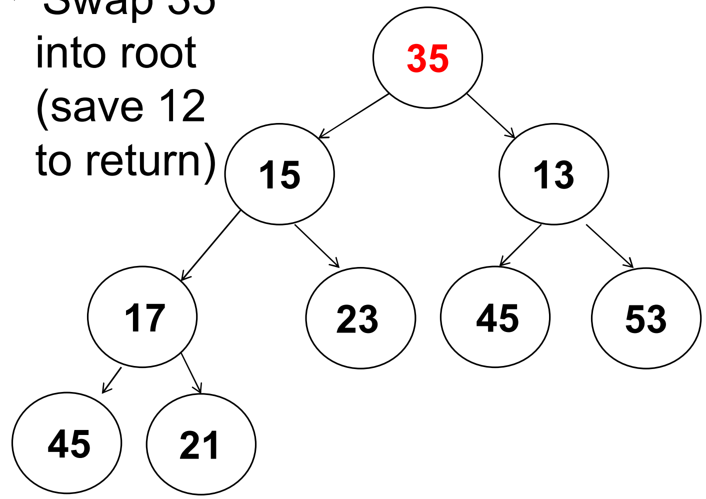
- Swap 35 into root (save 12 to return)



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**15**	**13**	**17**	**23**	**45**	**53**	**45**	**21**	**35**					

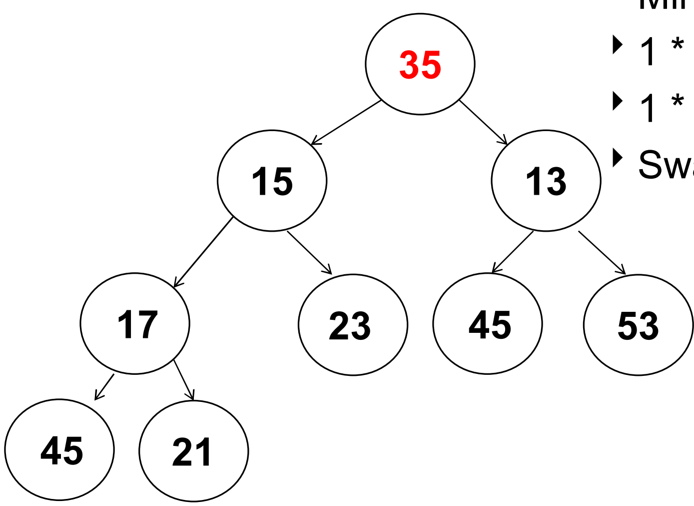
Dequeue Example

- Swap 35 into root (save 12 to return)



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	35	**15**	**13**	**17**	**23**	**45**	**53**	**45**	**21**						

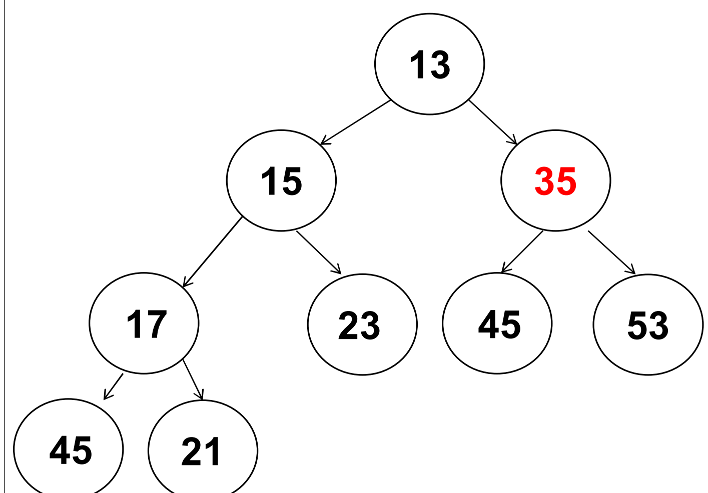
Dequeue Example



- Min child?
- 1 * 2 = 2 -> 15
- 1 * 2 + 1 = 3 -> 13
- Swap with 13

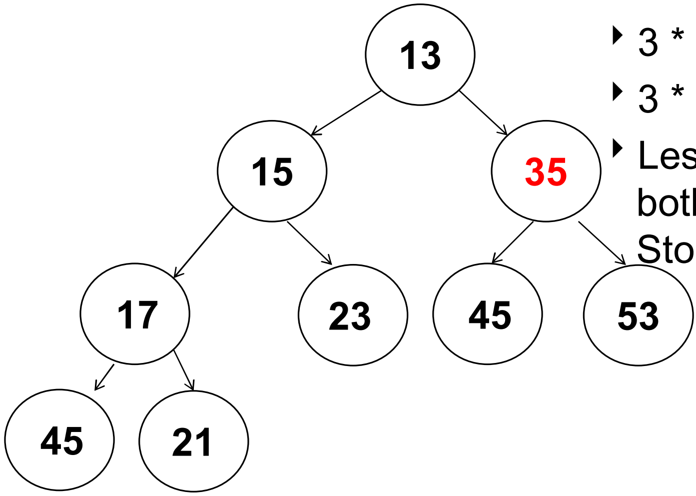
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	35	**15**	**13**	**17**	**23**	**45**	**53**	**45**	**21**						

Dequeue Example



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	13	**15**	**35**	**17**	**23**	**45**	**53**	**45**	**21**						

Dequeue Example



- Min child?
- 3 * 2 = 6 -> 45
- 3 * 2 + 1 = 7 -> 53
- Less than or equal to both of my children! Stop!

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	13	**15**	**35**	**17**	**23**	**45**	**53**	**45**	**21**						

Dequeue Code

```
public E dequeue( ) {
    E top = con[1];
    int hole = 1;
    boolean done = false;
    while ( hole * 2 < size && ! done ) {
        int child = hole * 2;
        // see which child is smaller
        if ( con[child].compareTo( con[child + 1] ) > 0 )
            child++;    // child now points to smaller

        // is replacement value bigger than child?
        if (con[size].compareTo( con[child] ) > 0 ) {
            con[hole] = con[child];
            hole = child;
        }
        else
            done = true;
    }
    con[hole] = con[size];
    size--;
    return top;
}
```


Clicker 3 - PriorityQueue Comparison

- Run a Stress test of PQ implemented with Heap and PQ implemented with BinarySearchTree
- What will result be?

A. Heap takes half the time or less of BST

B. Heap faster, but not twice as fast

C. About the same

D. BST faster, but not twice as fast

E. BST takes half the time or less of Heap