# Introduction to ML

Vitaly Shmatikov

# Reading Assignment

- Mitchell, Chapter 5.3-4

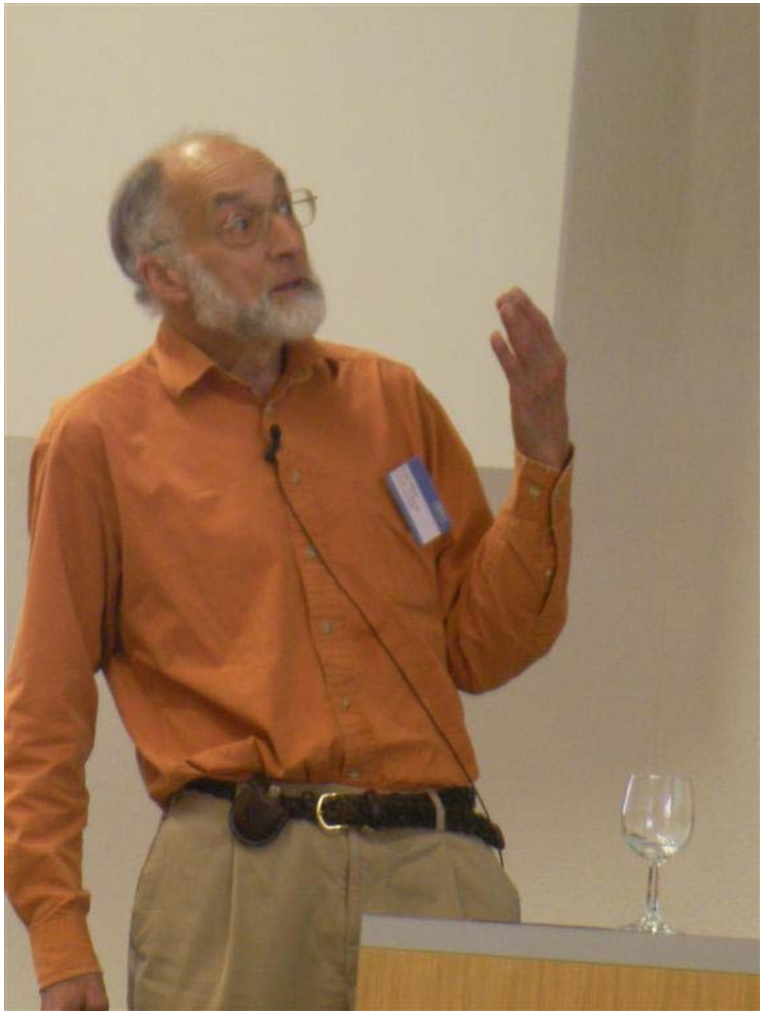
# ML

- General-purpose, non-C-like, non-OO language
  - Related languages: Haskell, Ocaml, F#, ...
- Combination of Lisp and Algol-like features
  - Expression-oriented
  - Higher-order functions
  - Garbage collection
  - Abstract data types
  - Module system
  - Exceptions
- Originally intended for interactive use

# Why Study ML ?

- Types and type checking
  - General issues in static/dynamic typing
  - Polymorphic type inference
- Memory management
  - Static scope and block structure, activation records
  - Higher-order functions
- Control
  - Type-safe exceptions
  - Tail recursion and continuations

# History of ML



- Robin Milner
  - Stanford, U. of Edinburgh, Cambridge
  - 1991 Turing Award
- Logic for Computable Functions (LCF)
  - One of the first automated theorem provers
- Meta-Language of the LCF system

# Logic for Computable Functions

- Dana Scott (1969)
  - Formulated a logic for proving properties of typed functional programs
- Robin Milner (1972)
  - Project to automate logic
  - Notation for programs
  - Notation for assertions and proofs
  - Need to write programs that find proofs
    - Too much work to construct full formal proof by hand
  - Make sure proofs are correct

# LCF Proof Search

- Tactic: function that tries to find proof

tactic(formula) = 
- succeed and return proof
- search forever
- fail

- Express tactics in the Meta-Language (ML)
- Use type system to facilitate correctness

# Tactics in ML Type System

- Tactic has a functional type

  tactic : formula $\rightarrow$ proof

- Type system must allow "failure"

$$\text{tactic(formula)} = \begin{cases} \text{succeed and return proof} \\ \text{search forever} \\ \text{fail and } \underline{\text{raise exception}} \end{cases}$$

# Function Types in ML

$f : A \to B$ means

for every $x \in A$,

$$f(x) = \begin{cases} \text{some element } y=f(x) \in B \\ \text{run forever} \\ \text{terminate by raising an exception} \end{cases}$$

In words, "if f(x) terminates normally, then $f(x) \in B$."

Addition never occurs in f(x)+3 if f(x) raises exception.

This form of function type arises directly from motivating application for ML. Integration of type system and exception mechanism mentioned in Milner's 1991 Turing Award lecture.

# Higher-Order Functions

- Tactic is a function
- Method for combining tactics is a function on functions
- Example:

$f(tactic_1, tactic_2) =$

$\lambda$ formula. try $tactic_1$(formula)

else $tactic_2$ (formula)

We haven't seen $\lambda$-expressions yet
(think of them as functions for now)

# Basic Overview of ML

- Interactive compiler: read-eval-print
  - Compiler infers type before compiling or executing
  - Type system does not allow casts or other loopholes
- Examples

```
- (5+3)-2;
> val it = 6 : int
- if 5>3 then "Bob" else "Fido";
> val it = "Bob" : string
- 5=4;
> val it = false : bool
```

# Basic Types

- Booleans
  - true, false : bool
  - if ... then ... else ... (types must match)
- Integers
  - 0, 1, 2, ... : int
  - +, * , ... : int * int → int and so on ...
- Strings
  - “Austin Powers”
- Reals
  - 1.0, 2.2, 3.14159, ... decimal point used to disambiguate

# Compound Types

- Tuples
  - (4, 5, “noxious”) : int * int * string
    type
- Lists
  - nil
  - 1 :: [2, 3, 4]
- Records
  - {name = “Fido”, hungry=true}
    : {name : string, hungry : bool}
    type

# Patterns and Declarations

- Patterns can be used in place of variables

  <pat> ::= <var> | <tuple> | <cons> | <record> ...

- Value declarations
  - General form: val <pat> = <exp>

    ```
    val myTuple = ("Conrad", "Lorenz");
    val (x,y) = myTuple;
    val myList = [1, 2, 3, 4];
    val x::rest = myList;
    ```

  - Local declarations

    ```
    let val x = 2+3 in x*4 end;
    ```

# Functions and Pattern Matching

- Anonymous function
  - fn x => x+1;   like function (...) in JavaScript
- Declaration form

  fun <name> <pat$_1$> = <exp$_1$>

  | <name> <pat$_2$> = <exp$_2$> ...

  | <name> <pat$_n$> = <exp$_n$> ...
- Examples
  - fun f (x,y) = x+y;   actual argument must match pattern (x,y)
  - fun length nil = 0

    | length (x::s) = 1 + length(s);

# Functions on Lists

- Apply function to every element of list

```
fun map (f, nil) = nil
|     map (f, x::xs) = f(x) :: map (f,xs);
```

Example: map (fn x => x+1, [1,2,3]); ⇒ [2,3,4]

- Reverse a list

```
fun reverse nil = nil
|     reverse (x::xs) = append ((reverse xs), [x]);
```

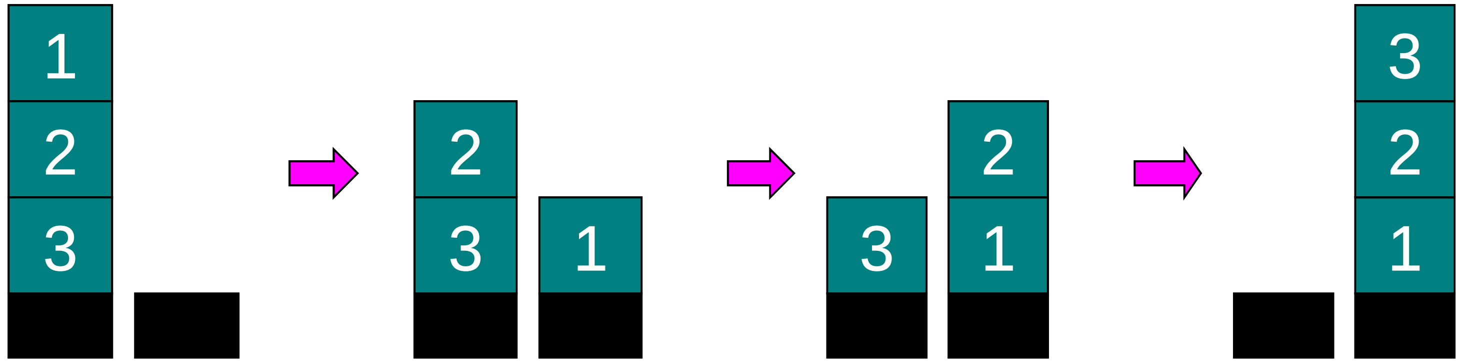
How efficient is this?
Can you do it with only one pass through the list?

- Append lists

```
fun append  (nil, ys) = ys
|     append (x::xs, ys) = x :: append(xs, ys);
```

# More Efficient Reverse Function

```
fun reverse xs =
    let fun rev(nil, z) = z
    |       rev(y::ys, z) = rev(ys, y::z)
    in rev( xs, nil )
end;
```



# Datatype Declarations

- General form

  datatype <name> = <clause> | … | <clause>

  <clause> ::= <constructor> |<constructor> of <type>

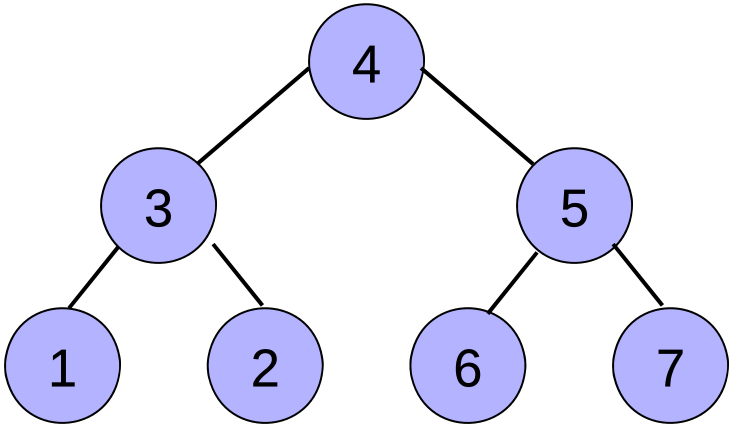
- Examples
  - datatype color = red | yellow | blue
    - Elements are red, yellow, blue
  - datatype atom = atm of string | nmbr of int
    - Elements are atm("A"), atm("B"), …, nmbr(0), nmbr(1), ...
  - datatype list = nil | cons of atom*list
    - Elements are nil, cons(atm("A"), nil), …
      cons(nmbr(2), cons(atm("ugh"), nil)), ...

# Datatypes and Pattern Matching

- Recursively defined data structure

  datatype tree = leaf of int | node of int*tree*tree

```
node(4, node(3,leaf(1), leaf(2)),
        node(5,leaf(6), leaf(7))
     )
```




- Recursive function

```
fun sum (leaf n) = n
|   sum (node(n,t1,t2)) = n + sum(t1) + sum(t2)
```

# Example: Evaluating Expressions

- Define datatype of expressions

  datatype exp = Var of int | Const of int | Plus of exp*exp;

  Write (x+3)+y as Plus(Plus(Var(1),Const(3)), Var(2))

- Evaluation function

```
fun ev(Var(n)) = Var(n)
|   ev(Const(n)) = Const(n)
|   ev(Plus(e1,e2)) = ...
```

ev(Plus(Const(3),Const(2))) ⇒ Const(5)

ev(Plus(Var(1),Plus(Const(2),Const(3)))) ⇒

ev(Plus(Var(1), Const(5))

# Case Expression

- Datatype

  datatype exp = Var of int | Const of int | Plus of exp*exp;

- Case expression

```
case e of
    Var(n) =>  ...   |
    Const(n) => .... |
    Plus(e1,e2) => ...
```

# Evaluation by Cases

```
datatype exp =  Var of int | Const of int | Plus of exp*exp;
fun ev(Var(n)) = Var(n)
|    ev(Const(n)) = Const(n)
|    ev(Plus(e1,e2)) =  (case ev(e1) of
          Var(n) => Plus(Var(n),ev(e2))      |
        Const(n) => (case ev(e2) of
                Var(m) => Plus(Const(n),Var(m))      |
              Const(m) => Const(n+m)                 |
            Plus(e3,e4) => Plus(Const(n),Plus(e3,e4)) )  |
      Plus(e3,e4) => Plus(Plus(e3,e4),ev(e2)) );
```

# ML Imperative Features

- Remember l-values and r-values?
  - Assignment y := x+3

    Refers to location (l-value) — y; Refers to contents (r-value) — x+3

- ML reference cells and assignment
  - Different types for location and contents

| | |
|---|---|
| x : int | non-assignable integer value |
| y : int ref | location whose contents must be integer |
| !y | the contents of cell y |
| ref x | expression creating new cell initialized to x |

  - ML form of assignment

| | |
|---|---|
| y := x + 3 | place value of x+3 in location (cell) y |
| y := !y + 3 | add 3 to contents of y and store in location y |

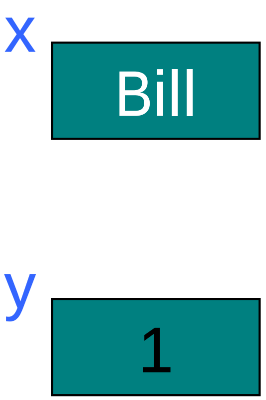
# Reference Cells in ML

- Variables in most languages
  - Variable names a storage location
  - Contents of location can be read, can be changed
- ML reference cells
  - A mutable cell is another type of value
  - Explicit operations to read contents or change contents
  - Separates naming (declaration of identifiers) from “variables”

# Imperative Examples in ML

- Create cell and change contents

```
val x = ref “Bob”;
x := “Bill”;
```




- Create cell and increment

```
val y = ref 0;
y := !y + 1;
```

- “while” loop

```
val i = ref 0;
while !i < 10 do i := !i +1;
!i;
```

# Core ML

- Basic Types
  - Unit
  - Booleans
  - Integers
  - Strings
  - Reals
  - Tuples
  - Lists
  - Records
- Patterns
- Declarations
- Functions
- Polymorphism
- Overloading
- Type declarations
- Exceptions
- Reference cells

# Related Languages

- ML family
  - Standard ML – Edinburgh, Bell Labs, Princeton, ...
  - CAML, OCAML – INRIA (France)
    - Some syntactic differences from Standard ML (SML)
    - Object system
- Haskell
  - Lazy evaluation, extended type system, monads
- F#
  - ML-like language for Microsoft .NET platform
    - *"Combining the efficiency, scripting, strong typing and productivity of ML with the stability, libraries, cross-language working and tools of .NET. "*
  - Compiler produces .NET intermediate language