CS345H: Programming Languages

Lecture 14: Introduction to Imperative Languages

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- And even perform type inference

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- Example:
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- Here, evaluating the expression `x+5` cannot change the value of any other expression

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- Answer: They may still trigger a run-time error

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- Question: What can we do about this?
- Solution: Type systems
- Any sound type system will guarantee no run-time errors
- Conclusion: We can only fully take advantage of functional features if we use a sound type system

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  - Programs are sequences of statements instead of one expression
- Imperative programming is the dominant model
- This style is much closer to the way hardware executes

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- Observe that features such as pointers, recursion and arrays are optional
- For example, FORTRAN originally only had integers and floats, loops, conditionals and goto statements

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- Question: Which style do you prefer?

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```
int i = 0;
int sum;
again:
  i++;
  int z = get_input();
  if(z < 0) goto error:
  n+=z;
  if(i < 5) goto again:
return n;
error:
  return -1;
```

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- However, as software became larger, GOTO statements started becoming problematic
- Central Problem of GOTO: “Spagetti Code”
- This means that thread of execution is very hard to follow in program text
- Jumps to a label could come from almost anyplace (in extreme cases even from other functions!)

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- In much early (and also more recent) code, GOTO not only implemented loops but was also used for code reuse
- Real Comment from numerical analyst: “Why bother writing a function if I can just jump to the label?”
- In 1968, Dijkstra wrote a very influential essay called “GOTO Statement Considered Harmful” in which he argued that GOTO statements facilitate unreadable code and should be removed from programming languages

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- Imperative programming without GOTOs is known as structural programming
- But not everyone was on board...

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- This was marketed as allowing polymorphism

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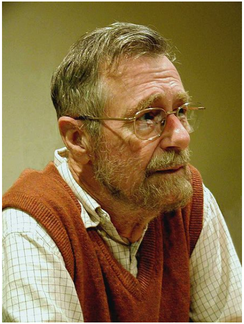

- Dijkstra's comment: "The use of COBOL cripples the mind; its teaching should, therefore, be regarded as a criminal offense."

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e & \rightarrow & id \mid e_1 + e_2 \mid e_1 - e_2 \mid int \\
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- What are some example programs in IMP1?

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- Changing the environment is the technical way of having side effects in the language

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- We then use this new environment to evaluate S_2 and return E_2

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- Observe that it is possible that id already had a value in E

Basic Statements

- Here is the assignment statement

$$\frac{\begin{array}{l} E \vdash e : v \\ E' = E[id \leftarrow v] \end{array}}{E \vdash id = e : E'}$$

- Observe that it is possible that id already had a value in E
- In this case, this rule overrides the value of id with the current value

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- Expressions and conditionals return values, while statements return environments

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$$\frac{E \vdash C : false}{E \vdash \text{while}(C) \text{ do } S \text{ od } : E}$$

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- Is it possible that this rule does not terminate?

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- Question: How does this rule make progress?
- Answer: It uses the new environment E' when reevaluating the loop body
- Is it possible that this rule does not terminate? Yes, if the loop is non-terminating

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- Also observe that for imperative languages, all expressions always evaluate to concrete values