

CS345H: Programming Languages

Lecture 16: Imperative Languages II

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Overview

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- ▶ Specifically, we wrote operational semantics for the IMP1 language
- ▶ **Today:** How to give semantics to more feature-rich imperative languages

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- ▶ A pointer is a reference to a memory location
- ▶ Pointers are naturally supported by hardware through load and store instructions
- ▶ In fact, pretty much all code turns into pointer manipulation at the assembly level

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 - ▶ Clever and efficient data structures
 - ▶ **Avoid copying of data if it can be shared**
- ▶ It is not uncommon for pointers to be 100x faster than copying data!
- ▶ For this reason, pointers are **essential** for most performance-critical task.

A Simple Pointer Language

- ▶ Let us consider the following simple language with pointers we will call IMP2:

$$P \rightarrow \varepsilon \mid P_1; P_1 \mid S$$
$$S \rightarrow \text{if}(C) \text{ then } S_1 \text{ else } s_2 \text{ fi} \mid id = e \mid *id = e \\ \mid \text{while}(C) \text{ do } S \text{ od} \mid id = \text{alloc}$$
$$e \rightarrow id \mid e_1 + e_2 \mid e_1 - e_2 \mid int \mid *id$$
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- ▶ **Addition:** Alloc allocates fresh memory

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- ▶ **Recall:** So far, we only had an environment.
- ▶ The environment mapped variables to values
- ▶ But how can we **look up** the value of a pointer?

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 - ▶ A **store** S mapping **addresses** to **values stored at this address**
- ▶ The store is emulating memory when executing a program!

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- ▶ **Statements now both change the environment and the store!**

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$$\frac{\begin{array}{l} l_f \text{ fresh} \\ S' = S[l_f \leftarrow 0] \\ S'' = S'[E(v) \leftarrow l_f] \end{array}}{E, S \vdash id = \text{alloc} : E, S''}$$

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- ▶ **Question:** How can we solve this problem?

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- ▶ This is the solution preferred by most imperative languages

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- ▶ In fact, many the cases where pointers are really useful involve some kind of aliasing
- ▶ However, in another sense, aliasing is **awful**
- ▶ Because of aliasing, storing a value into any location **can potentially change every other location's value!**
- ▶ This is very bad news for any kind of expressive type system

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- ▶ This is another typical “side effect” of adding pointers to a language

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- ▶ Observe that load and store are replaced with **array load and store** \Rightarrow pointer arrays are a generalization of pointers
- ▶ Also, assume that `alloc` allocates arrays of infinite size

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- ▶ **Question:** How can we emulate pointer load and store in IMP3?
- ▶ **Answer:** Pointer load $id = *e$ is the same as $id = e[0]$
- ▶ Pointer store $*id = e$ is the same as $id[0] = e$

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- ▶ Therefore, we only need to write two new rules
- ▶ First order of business: Array load

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 - ▶ Look up the value of l_1 in S as v_1
 - ▶ Add the index v_i to v_1 as v_2

Load in IMP3

- ▶ What do we have to do to load a value in an array?
- ▶ Specifically, how do we process $id[e]$?
 - ▶ Evaluate e to v_i
 - ▶ Look up the address l_1 of the variable id in E
 - ▶ Look up the value of l_1 in S as v_1
 - ▶ Add the index v_i to v_1 as v_2
 - ▶ Look up the value of l_2 in S

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$$\frac{\begin{array}{l} E, S \vdash e : v_i \\ l_1 = E(id) \\ v_1 = S(l_1) \\ v_2 = v_1 + v_i \\ v_3 = S(v_2) \end{array}}{E, S \vdash id[e] : v_3}$$

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- ▶ Observe how this is a **generalization** of the earlier rule for pointer load

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 - ▶ Change the value of $l_2 + v_i$ in S to e_1 's value

Store in IMP3 Cont.

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- ▶ Here is the rule for store:

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Store in IMP3 Cont.

- ▶ Here is the rule for store:

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- ▶ Again, this is a **direct** generalization of the store rule in IMP3

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- ▶ You will write semantics for an array language without pointers on the homework

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 - ▶ ...
- ▶ **But we covered all the important basics!**
- ▶ If you think a little, you can now write semantics for any missing feature

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- ▶ If you want to add a new feature, first think if you need more information!
- ▶ Sometimes, you need another mapping (like environment E and store S)
- ▶ In general, there are many correct ways to add features to operational semantics
- ▶ But your goal is to add them cleanly!