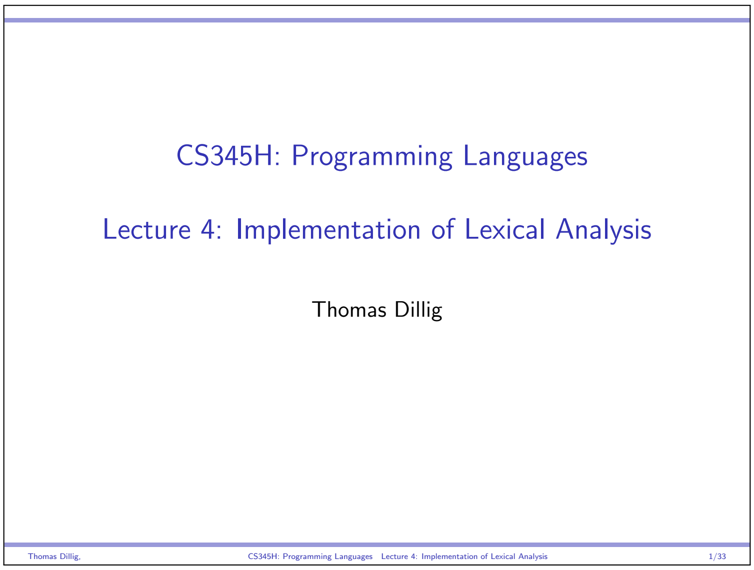
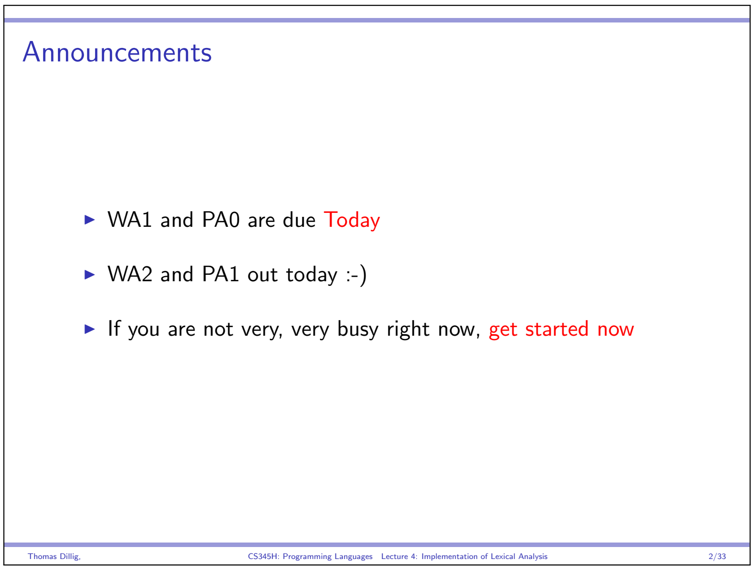


CS345H: Programming Languages

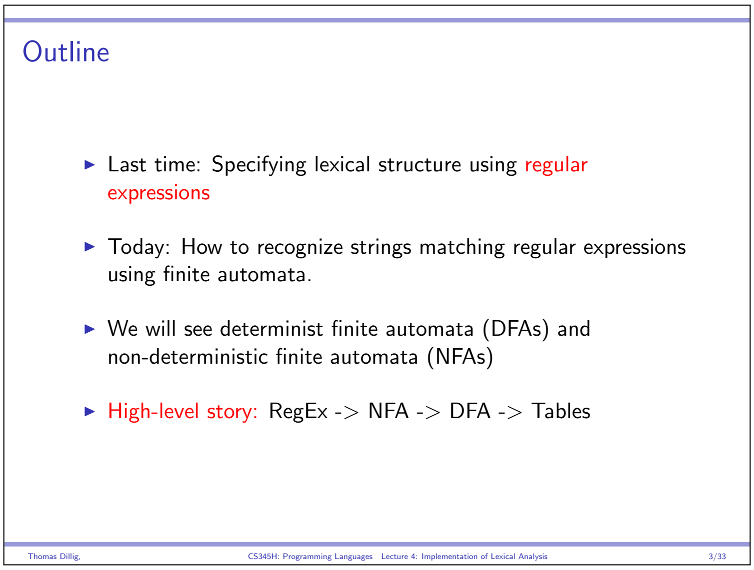
Lecture 4: Implementation of Lexical Analysis

Thomas Dillig



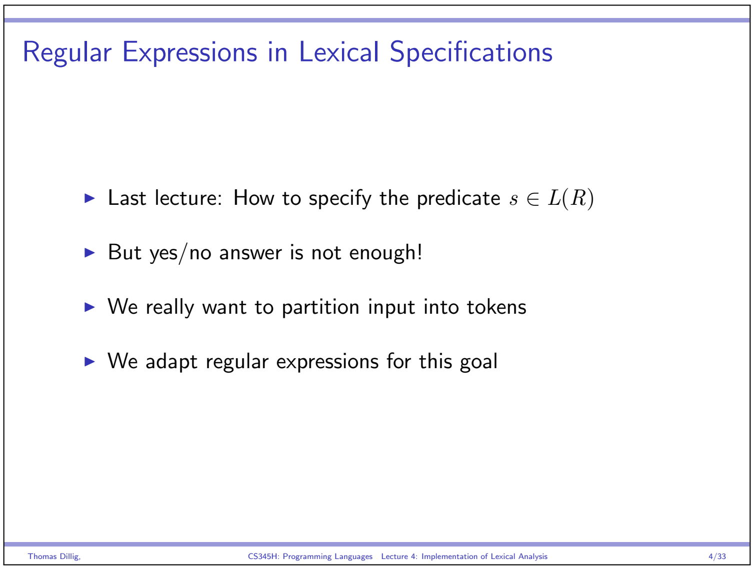
Announcements

- WA1 and PA0 are due Today
- WA2 and PA1 out today :-)
- If you are not very, very busy right now, get started now



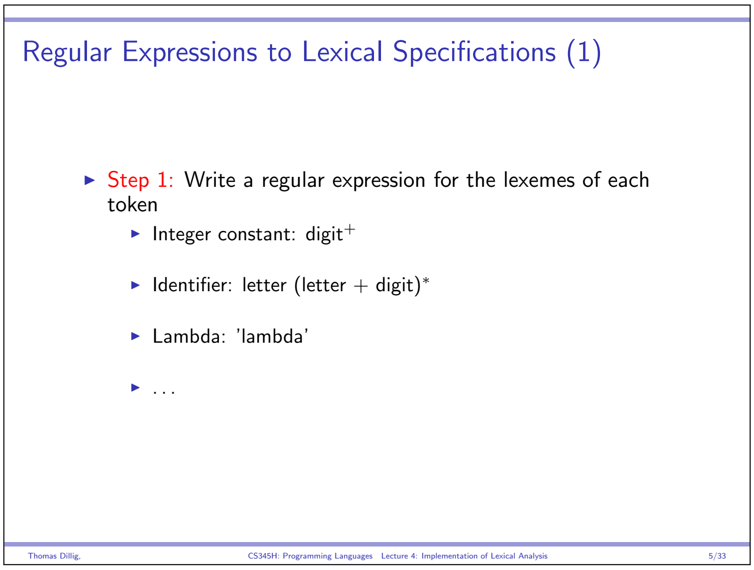
Outline

- Last time: Specifying lexical structure using regular expressions
- Today: How to recognize strings matching regular expressions using finite automata.
- We will see determinist finite automata (DFAs) and non-deterministic finite automata (NFAs)
- High-level story: RegEx -> NFA -> DFA -> Tables



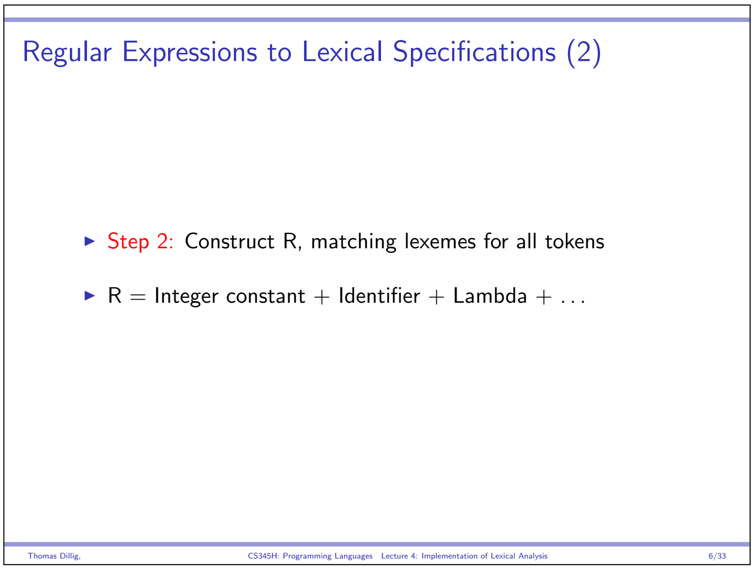
Regular Expressions in Lexical Specifications

- Last lecture: How to specify the predicate $s \in L(R)$
- But yes/no answer is not enough!
- We really want to partition input into tokens
- We adapt regular expressions for this goal



Regular Expressions to Lexical Specifications (1)

- Step 1: Write a regular expression for the lexemes of each token
  - Integer constant: digit$^+$
  - Identifier: letter (letter + digit)*
  - Lambda: 'lambda'
  - ...



Regular Expressions to Lexical Specifications (2)

- Step 2: Construct R, matching lexemes for all tokens
- R = Integer constant + Identifier + Lambda + ...

Regular Expressions to Lexical Specifications (3)

- Let the input be characters $x_1 ... x_n$
- Step 3: For each $1 \leq i \leq n$ check $x_1 ... x_j \in L(R)$ for some j
- Then, remove $x_1 ... x_j$ from input and repeat

Ambiguities I

- There are ambiguities in this algorithm. Where?
- How much input is used? What if $x_1 ... x_i \in L(R)$ and $x_1 ... x_j \in L(R)$?
- Example: identifier = letter (letter + digit)*, if = 'i' 'f'
- Rule: Pick longest possible string in $L(R)$
- This is known as “maximal munch”

Ambiguities II

- What if two rules match with the same number of characters?
- $x_1 ... x_i \in L(R_1)$ and $x_1 ... x_i \in L(R_2)$?
- Example: `"if"`
- Rule: Use rule listed first
- This is how `"if"` is matched as a keyword, not identifier

Error Handling

- What if no rule matches a prefix of the input?
- Solution 1: Get stuck ⇒ Unacceptable
- Better Solution: Write a rule matching all “bad” strings
- Question: What kind of rule and where to place it?

Where are we?

- We now know how we can partition input string into tokens assuming we can decide if a string is in the language described by a regular expression.
- Next: How to decide if $s \in L(R)$

Finite Automata

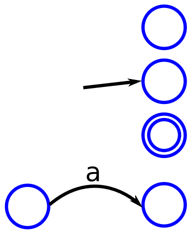
- Regular Expressions ⇔ Specification
- Finite Automata ⇔ Implementation
- A finite automata formally consists of:
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions state $\rightarrow^{\text{input}}$ state

Finite Automata

- Transition $S_1 \rightarrow^{\alpha} S_2$
- This means: In state S_1 and input character α, go to state S_2
- If end of input and in accepting state $\Rightarrow$ accept
- Otherwise $\Rightarrow$ reject

Finite Automata as State Graphs

- It is much easier to imagine finite automata visually:



A state:

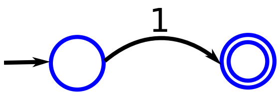
The start state:

An accepting state:

A transition:

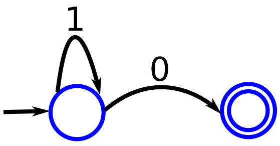
A Simple Example

- Here is an automaton that only accepts the string "1":



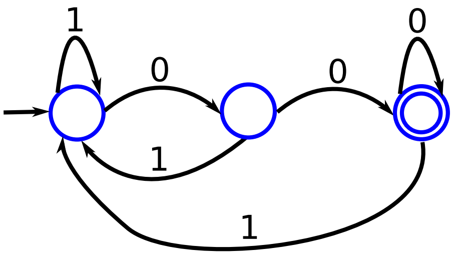
Another Simple Example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: $\{0, 1\}$



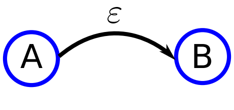
And Another Example

- Alphabet: $\{0, 1\}$
- What language does this automata recognize?



Epsilon Moves

- Another kind of transition: ε-moves



- Machine can move from state A to B without reading any input

Deterministic and Nondeterministic Automata

- Deterministic Finite Automata (DFA)
  - At most one transition per input on any state
  - No ε moves
- Nondeterministic Finite Automate (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves

Execution of Finite Automata

- A DFA can only take one path through the state graph that is completely determined by the input
- NFAs can choose:
  - Whether to make ε moves
  - Which one of multiple transitions for a single input to take

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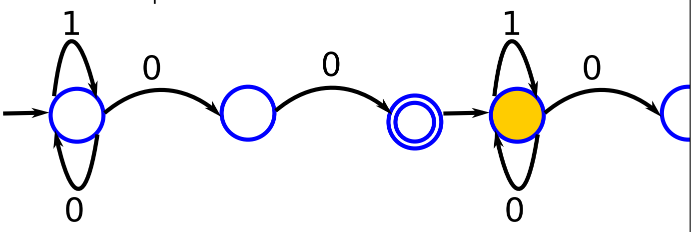
Acceptance of NFAs

- This means: A NFA can get into multiple states at the same time
- Consider again the alphabet $\Sigma = \{0, 1\}$ and the language of all strings ending in at least two 0s.
- Consider input 1 0 0



- Rule: NFA accepts if it can get to a final state

NFAs vs. DFAs

- Fundamental Result: NFAs and DFAs recognize the same set of languages (regular languages)
- DFAs are faster to execute, since there are no choices to consider
- But NFAs can be much simpler for the same language
- Result: DFAs can be exponentially larger than NFA recognizing same language

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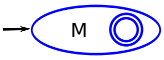
Regular Expressions to Finite Automata

- High-Level Sketch:
  - Lexical Specification
  - Regular Expressions
  - NFA
  - DFA
  - Implementation of DFA

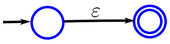
⇒ Lexer

Regular Expressions to NFA (1)

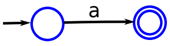
- For each kind of regular expression, define an NFA and combine
- Will use the following notation: NFA for regular expression M:



- Base cases:
  - For ε :

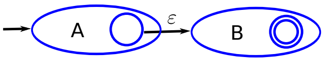


  - For input a:

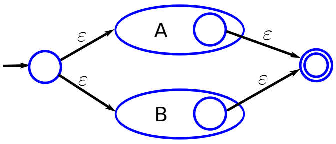


Regular Expressions to NFA (2)

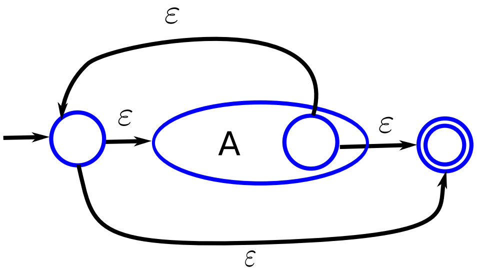
- For AB:



- For $A+B$:



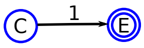
Regular Expressions to NFA (3)



- For A^*:

Example of Regular Expression to NFA conversion

- Consider the regular expression $(1+0)^*1$

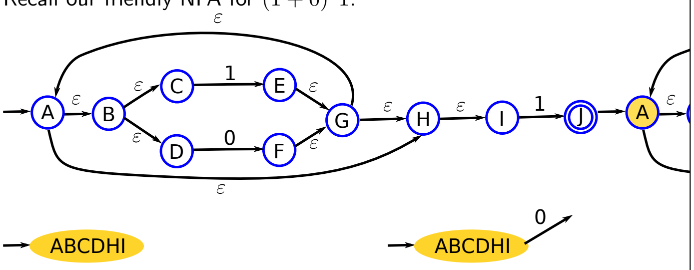


NFA to DFA: The Trick

- Insight: Simulate the NFA
- At any given time, the NFA is in a set of states
- States in the DFA ⇒ all (reachable) subsets of states in the NFA
- Start State: the set of states reachable through ε moves from the NFA start state
- Add transition $A \rightarrow^a B$ to DFA iff:
  - B is in the set of states reachable from any state in A after seeing input a, considering ε moves as well

NFA to DFA: Example

Recall our friendly NFA for $(1+0)^*1$:



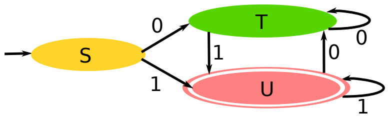
NFA to DFA: How many states?

- We need a state in the DFA for each set of states the NFA can be in
- How many different states?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets of N states? 2^N

Implementation

- A DFA can be implemented by a 2D table T
    - One dimension is “states”
    - Other dimension is “input symbols”
    - For every transition $A \rightarrow^c B$, define `T[A,c]=B`
- DFA “execution”: If in state A and input c, read `T[A,c] = B` and skip to state B
- Very efficient

Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation cont.

- Writing regular expressions as NFAs and converting them to DFAs is exactly what `flex` does
- In fact, if you open the auto-generated `flex` file `lex.yy.c`, you will see these tables emitted
- But, these DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations