Symbolic Heap Abstraction with Demand-Driven Axiomatization of Memory Invariants

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Stanford University

Relational vs. Non-Relational Heap analysis

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  - A relational analysis tracks correlations between points-to targets of two memory locations
  - A non-relational heap analysis does not.
- Relational heap analyses are more precise, but also more expensive.

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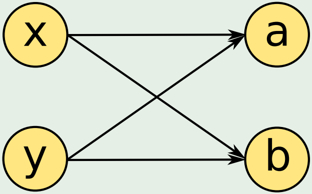

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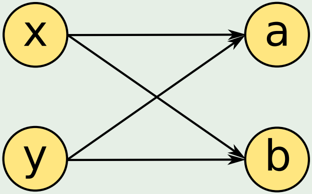

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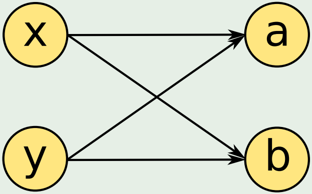

- Does not encode x and y must point to same location
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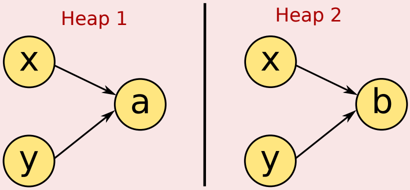

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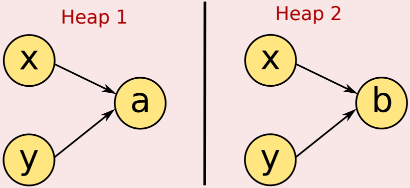

- Perform case split on possible heaps.
- Can prove assertion because in both heaps `x` and `y` point to same location.

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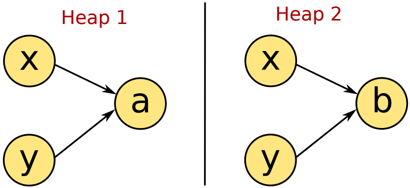

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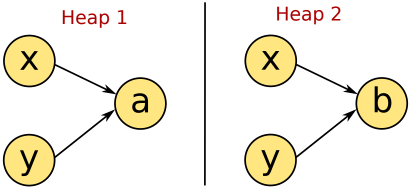

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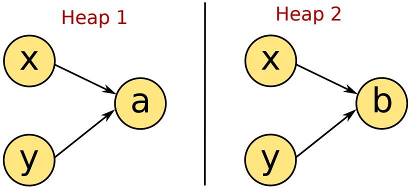

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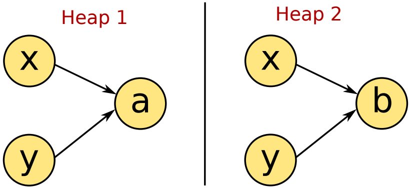

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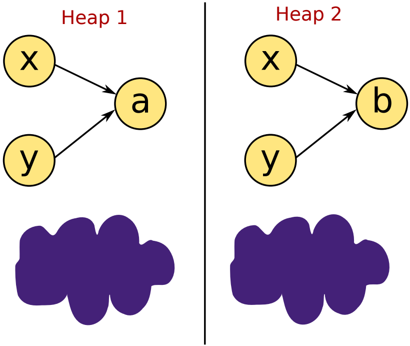

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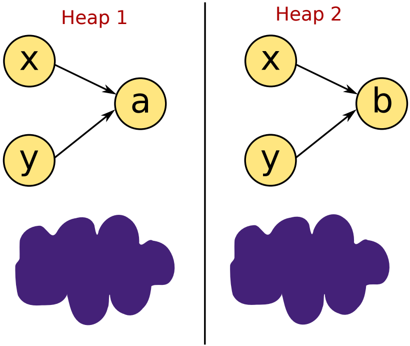

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This talk:

Scalable and precise relational heap analysis *without* performing explicit case splits on the heap

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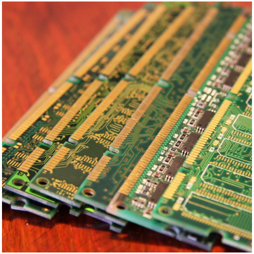

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We can achieve relational reasoning by enforcing two important memory invariants that real computer memories satisfy:

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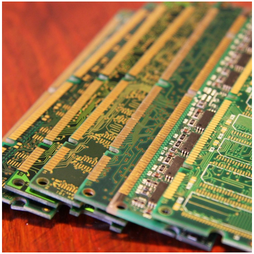

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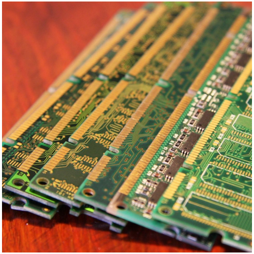

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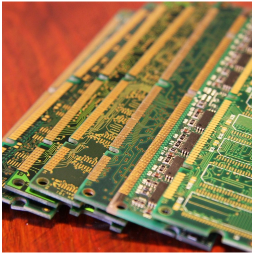

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- Existence: Every memory location has at least one value
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⇒ Heap splitting is one way of enforcing these invariants.

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- Still advantageous because:
  - Solver can often prove a constraint SAT or UNSAT without considering all cases: eager vs. lazy
  - Don't duplicate shared portions of the heap
  - No heuristics for merging "similar" heaps

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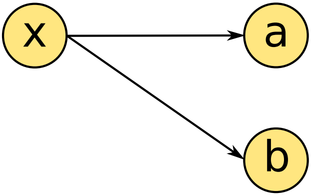

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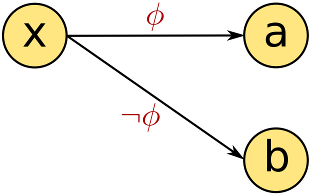

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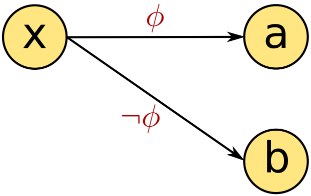

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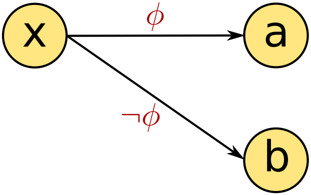

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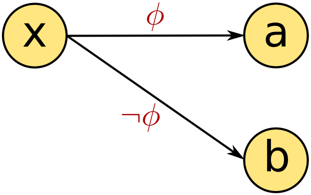

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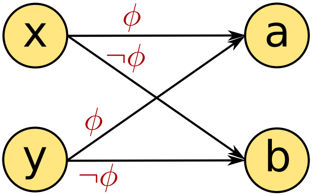

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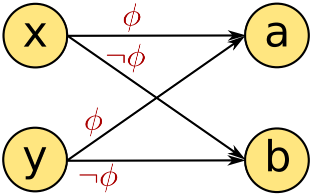

Correlation between x and y preserved

- x and y point to different locations under $\phi \wedge \neg\phi$

  $\Rightarrow$ Can prove the assertion!

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- But what about abstract locations that represent multiple concrete locations?

Memory Invariants on Summary Locations

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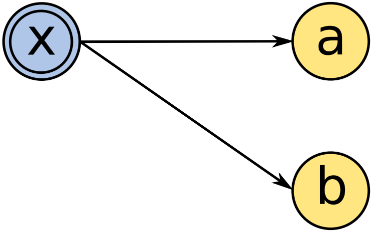

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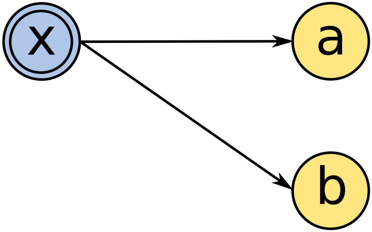

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- Graph encodes that any element in x may point to either a or b.

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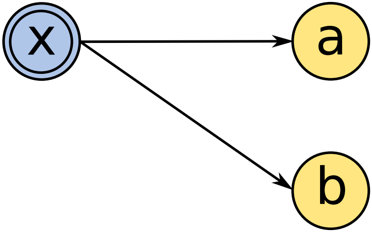

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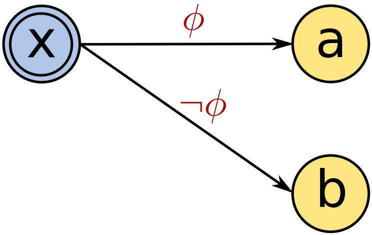

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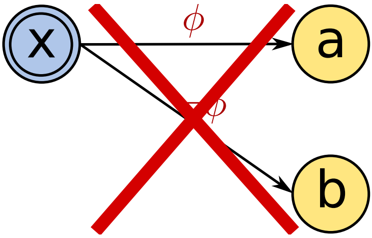

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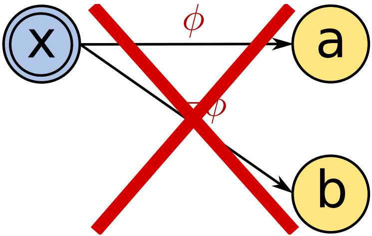

Conclusion

- To enforce memory invariants symbolically, we need a way to refer to individual elements in summary locations.

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- Use the symbolic heap from our previous work that allows distinguishing individual elements in a summary location.
  - This basic symbolic heap does not enforce memory invariants
- Describe new technique to enforce memory invariants on the symbolic heap without explicit case splits

Symbolic Heap

- Abstract locations that represent more than one concrete location are qualified by index variables.

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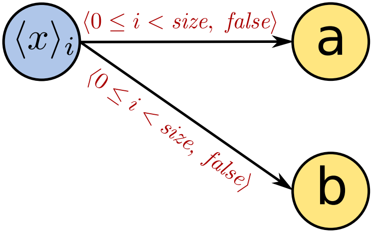

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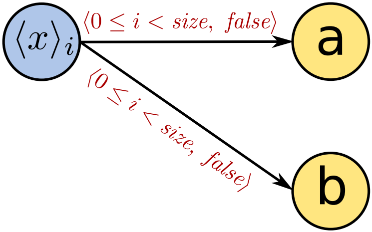

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  - Index variables allow us to refer to individual elements inside the abstract location

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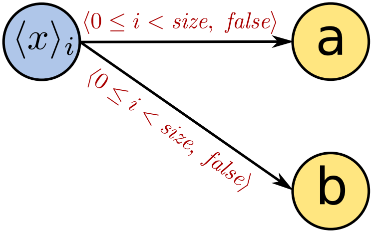

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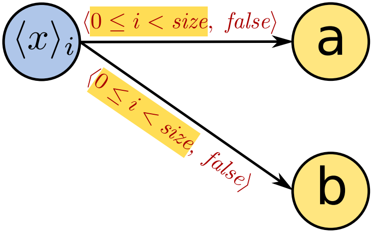

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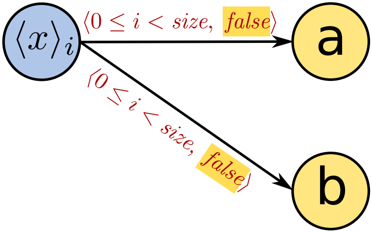

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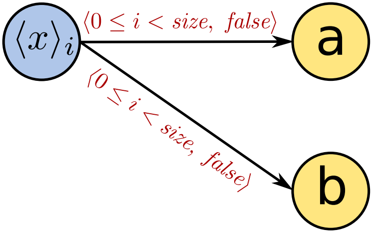

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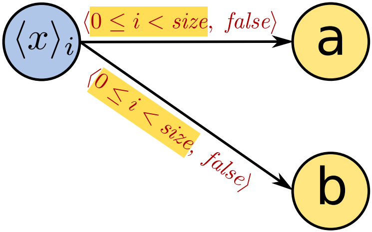

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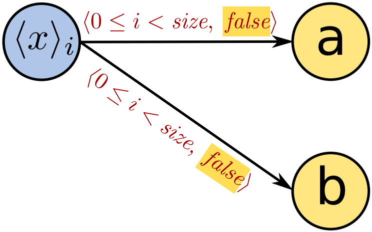

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- Existence violated because disjunction of must conditions is not valid.

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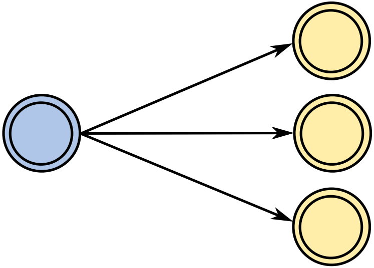

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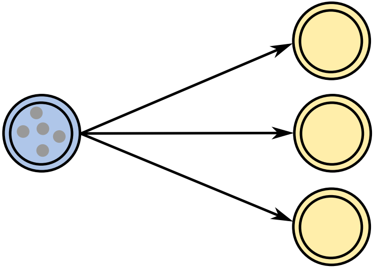

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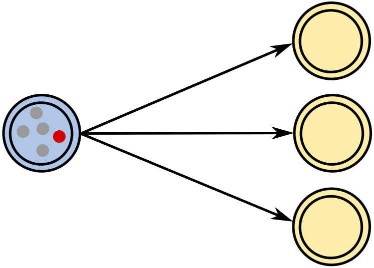

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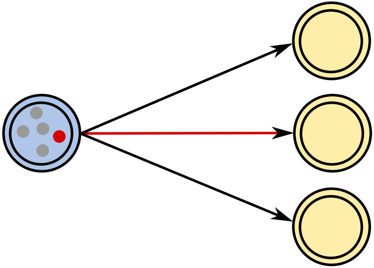

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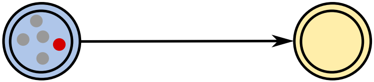

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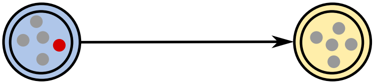

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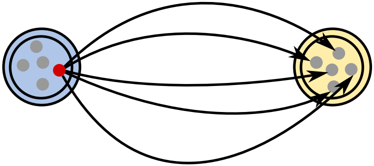

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- Θ: In this abstract target, select one concrete element.

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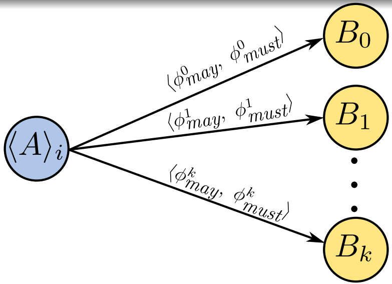

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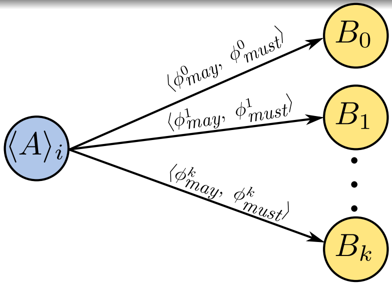

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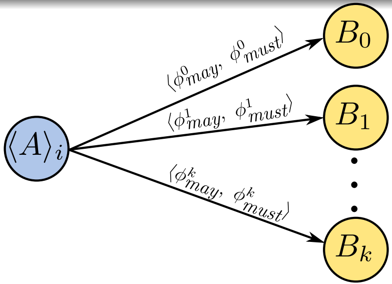

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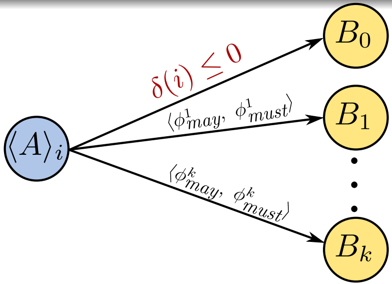

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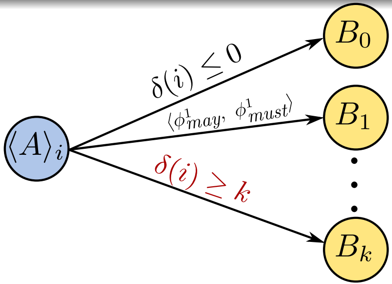

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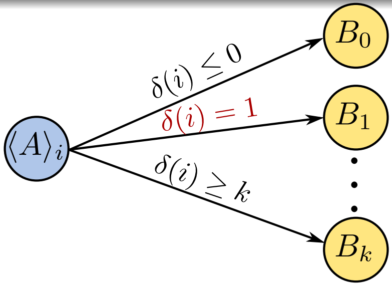

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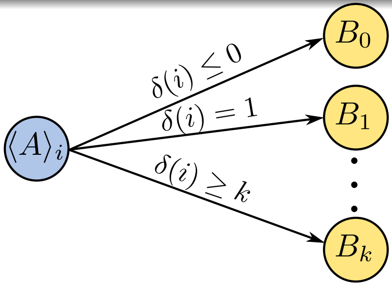

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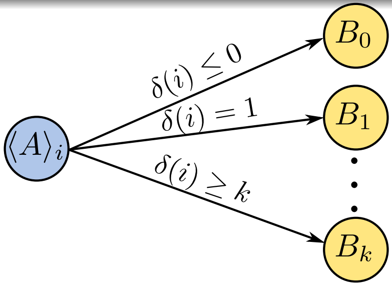

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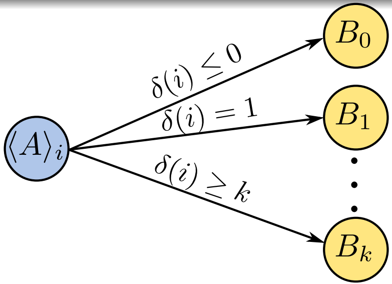

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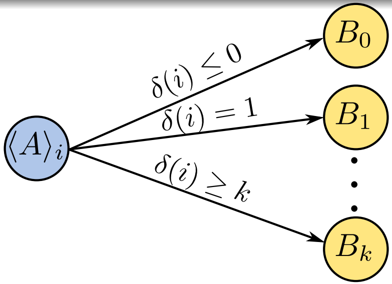

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- $\Rightarrow$ Each concrete element in A has exactly one abstract target.
- Correctly allows different indices to point to same target.

Example

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for(int i=0; i<size; i++)
{
  if(*) x[i] = a;
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}

y = x;
// 0 <= k < size
assert(x[k] == y[k]);
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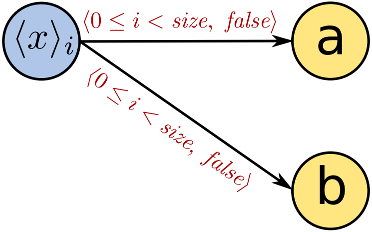

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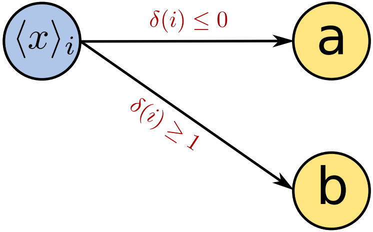

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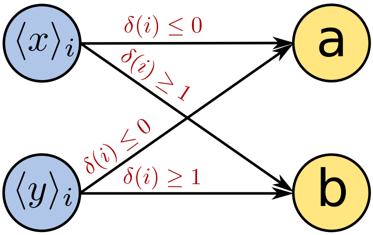

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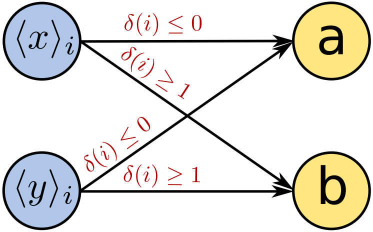

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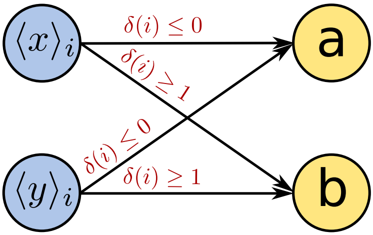

- We can now prove the assertion!
  - Because x[k] and y[k] point to different locations under $\delta(k) \leq 0 \wedge \delta(k) \geq 1 \Rightarrow$ UNSAT

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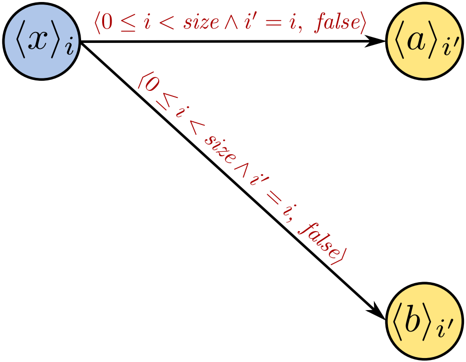

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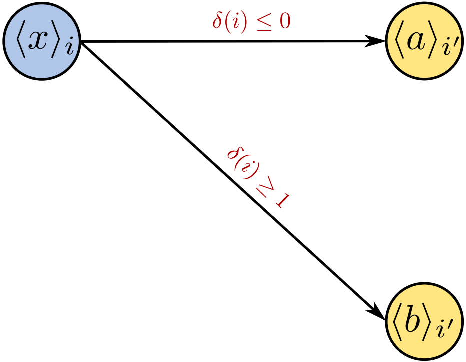

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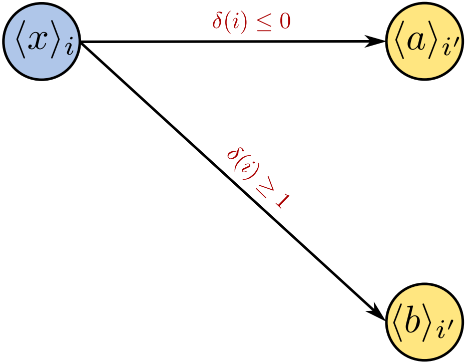

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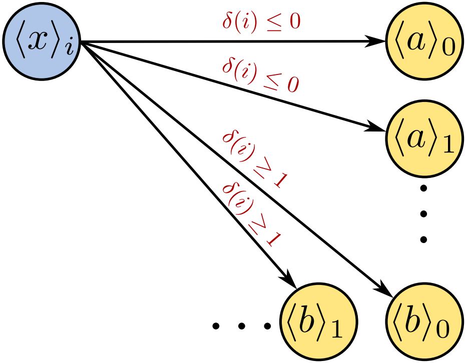

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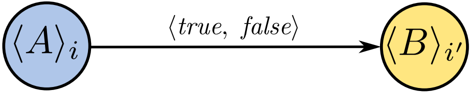

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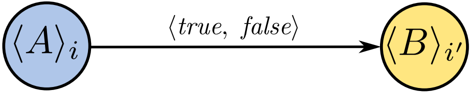

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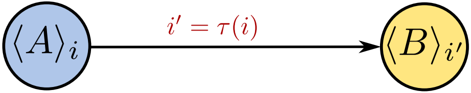

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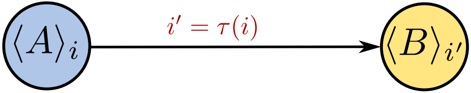

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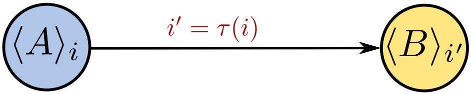

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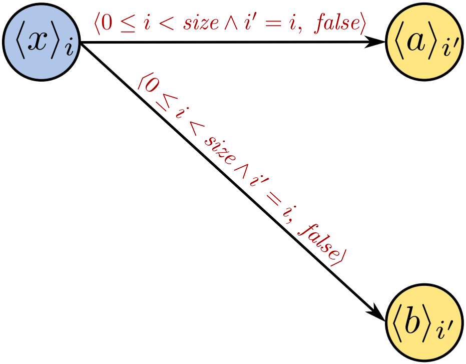

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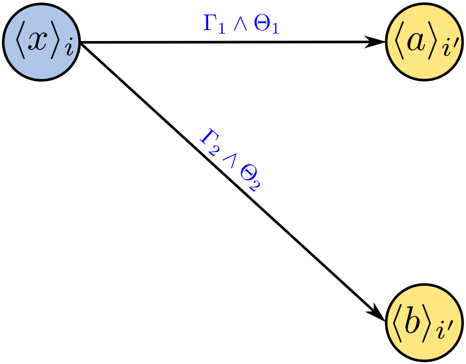

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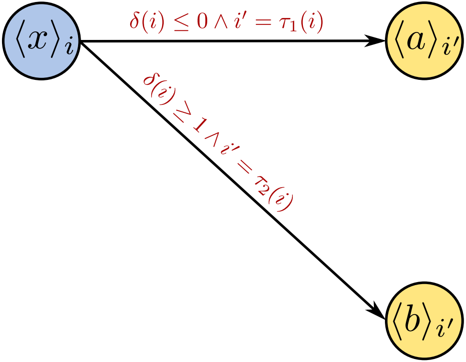

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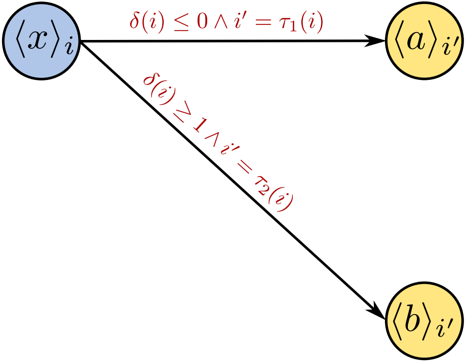

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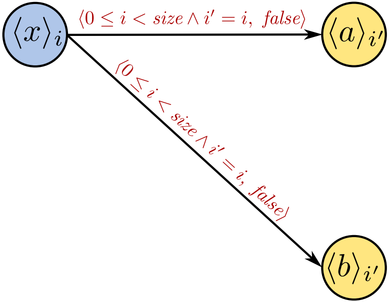

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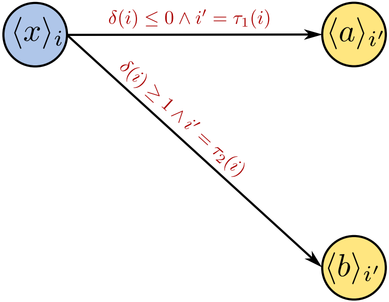

- Using original heap, can prove `x[2]` cannot point to `a[4]`.
- But using the modified heap, we can no longer prove this.

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If edge in original heap is qualified by $\langle \phi_{may}, \phi_{must} \rangle$, then introduce axioms of the form:

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- We implemented this technique as part of our Compass program analysis system

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- Verified memory safety properties (absence of buffer overruns, null dereferences, and casting errors) in a number of Unix Coreutils applications and on OpenSSH.

Results on OpenSSH

	Relational	Non-relational
Time (s)	261	788
Max memory used (MB)	208	763
# reported buffer errors	2	77
# reported null errors	3	53
# reported cast errors	0	28
Total # of errors	5	158
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- Relational technique is very precise.
- Technique without memory invariants reports many false positives.
- Surprisingly, more precise is also more efficient.
    - Memory invariant alone is sufficient to discharge many facts.

Thank You!

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