# Synthesis of Circular Compositional Program Proofs via Abduction

Boyang Li, Işıl Dillig, Tom Dillig (College of William & Mary)
Ken McMillan ( Microsoft Research)
Mooly Sagiv (Tel Aviv University)

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**Difficult, if not impossible, to design one approach that is good at everything**

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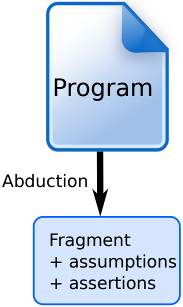

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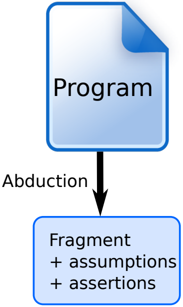

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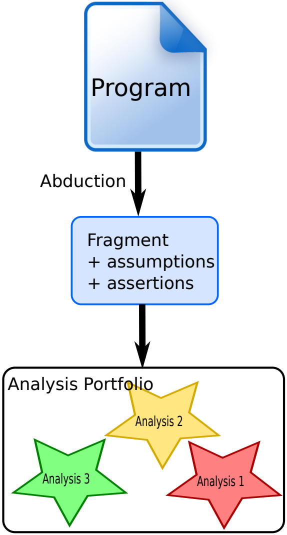
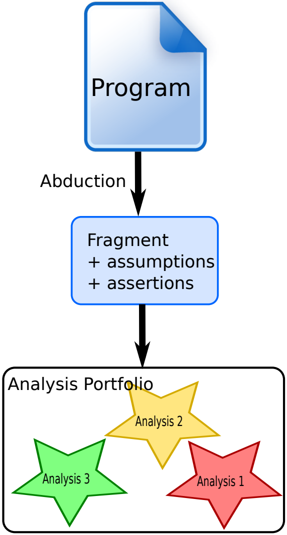


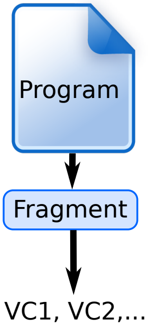
# This Talk

**New technique for circular compositional program verification**

- Decompose the program proofs into small lemmas using logical abduction
- Represent lemmas as code fragments annotated with assertions and assumptions
- Use portfolio of verification techniques to discharge fragments
- Use circular compositional reasoning to turn some assertions into assumptions



# Proof Decomposition

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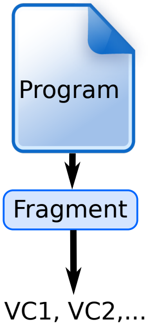


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- For any VC of the form $\phi_1 \Rightarrow \phi_2$ that is not valid, find $\psi$ such that $(\psi \wedge \phi_1) \Rightarrow \phi_2$ is valid using abduction.

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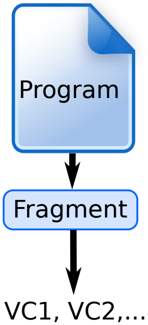


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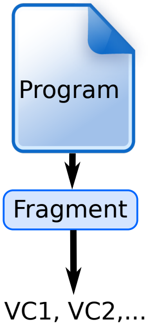


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- Now, introduce $\psi$ as new assertion in program
- And eliminate old assertion by proving it assuming $\psi$ and converting it to an assumption

# Example

- Consider the following code snippet

```
int i=1, j=0;
while(*) {j++; i+=3;}
int z = i-j;
int x=0, y=0, w=0;
while(*) {
  assert(x==y);

  z+=x+y+w;
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# Example

- Consider the following code snippet
- Code contains assertion in second loop

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# Example

- Consider the following code snippet
- Code contains assertion in second loop
- Goal: Discharge assertion using portfolio of analyses on fragments of this code

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while(*) {j++; i+=3;}
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- But not possible since precondition "$z$ is odd" is missing

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**Want to solve for missing assumptions required to prove $x = y$**

# Parametric VC Generation

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- Use $\phi_1$ and $\phi_2$ to represent unknown assumptions that make the assertion valid

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$$(\phi_2 \wedge x = y) \Rightarrow wp(\sigma, x = y)$$

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**Insight: This is an instance of logical abduction**

# Abductive Inference


abduce

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$$F \wedge E \models O \quad \text{and} \quad \mathrm{SAT}(F \wedge E)$$

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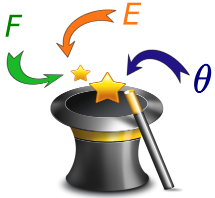


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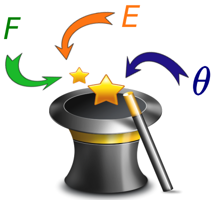


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- Use abduction to generate simple assumptions that make verification condition valid
- Known facts $F$ is verification condition, desired outcome is $true$
- Abductive solution becomes lemma in proof and can now be established separately

# Abductive Inference in Example

- Here, for

  $$(\phi_2 \wedge x = y) \Rightarrow$$
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  we compute the solution
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- Circular compositional reasoning at work!

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- Again generate parametric VCs
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  $$(\phi_1 \wedge z - i - j \wedge x = 0 \\ \wedge y = 0 \wedge w = 0 \wedge x = y) \Rightarrow \\ (w + z\%2 = 1)$$

- Solution computed via abduction $\phi_1 : z\%2 = 1$

# Invoking Client Analyses

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- Again, circular compositional reasoning at work

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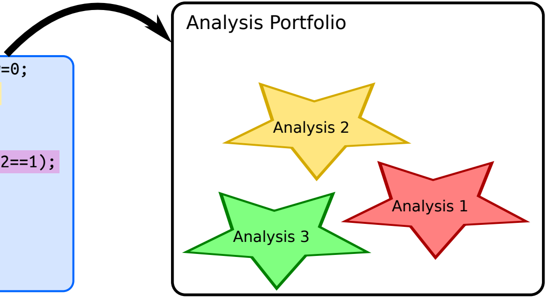


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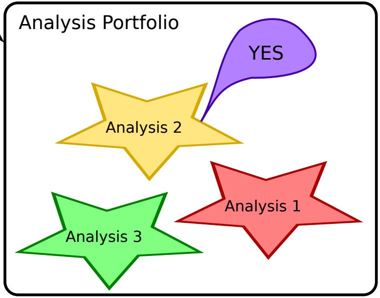


- Give fragment with assumptions and assertions to clients
- Fragment can be locally verified by divisibility analysis

## Remaining Code

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int z = i-j;
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while(*) {
  assume(x==y);
  assume((w+z)%2==1);
  z+=x+y+w;
  y++;
  x+=z%2;
  w+=2;
}
```

- Now, only one assertion left in our program

# Remaining Code

```
int i=1, j=0;
while(*) {j++; i+=3;}
int z = i-j;
int x=0, y=0, w=0;
assert(z%2==1);
while(*) {
  assume(x==y);
  assume((w+z)%2==1);
  z+=x+y+w;
  y++;
  x+=z%2;
  w+=2;
}
```

- Now, only one assertion left in our program
- Extract next fragment for this assertion and give to client analyses

# Invoking Client Analyses

```
int i=1, j=0;
while(*) {j++; i+=3;}
int z = i-j;
int x=0, y=0, w=0;
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```

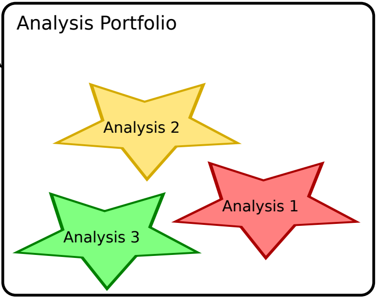


- Invoke clients on current fragment

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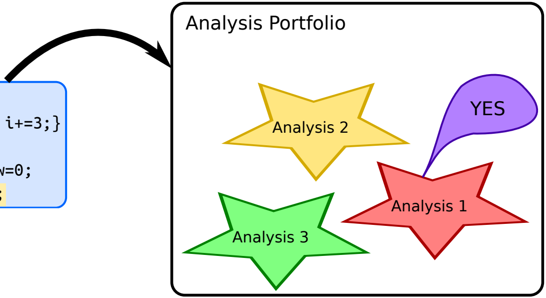


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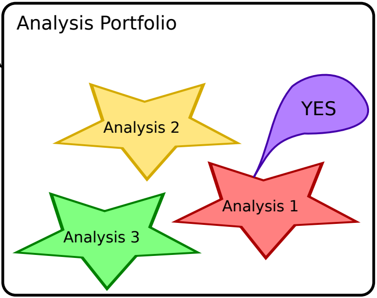


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**We have now proven the original assertion**

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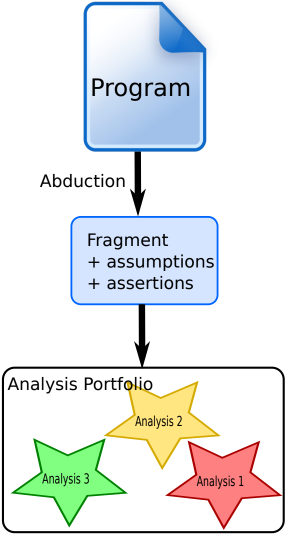
- If a subgoal cannot be shown by any client analysis, we backtrack and generate new subgoals using abductive inference

# Advantages of this Approach




- Clients only analyze (typically) small fragments

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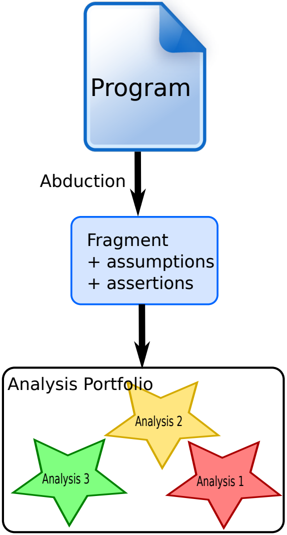


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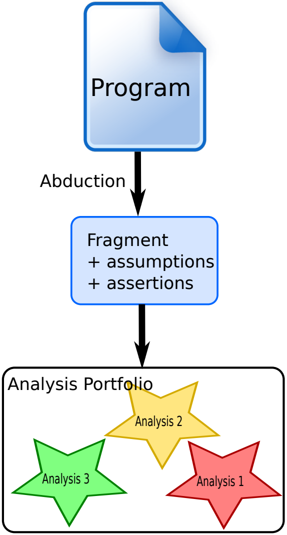


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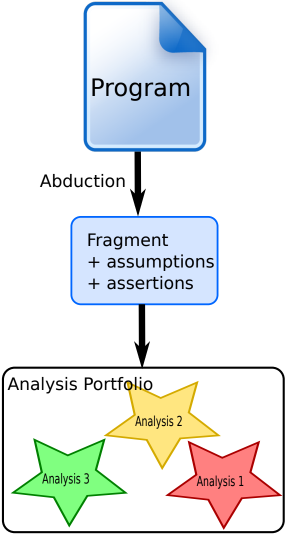


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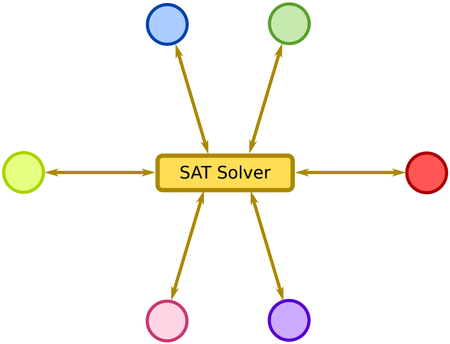
- Analyses with complementary strengths can help each other
- Can prove properties no client analysis or eager combination can prove alone

# Analogy to SMT Solver

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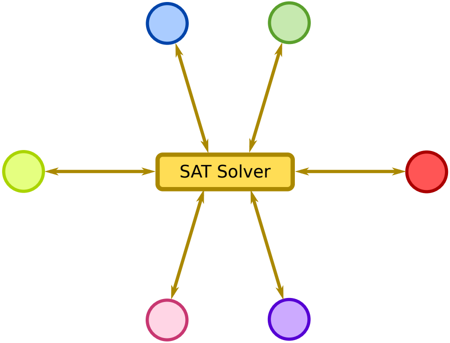


- Can view client analyses as theory solvers
- Proving assertion on program invokes clients on fragments and speculated subgoals, backtracks when needed

# Experiments

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**These tools have very different strengths and weaknesses**

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| B1 | 45 | 0.6 | 2 | ✘ | ✘ | ✔ | ✘ |
| B2 | 37 | 0.2 | 2 | ✘ | ✔ | ✘ | ✘ |
| B3 | 51 | 1.0 | 2 | ✔ | ✘ | ✔ | ✘ |
| B4 | 59 | 0.4 | 3 | ✔ | ✘ | ✔ | ✘ |
| B5 | 89 | 0.6 | 3 | ✔ | ✘ | ✔ | ✘ |
| B6 | 60 | 0.5 | 5 | ✘ | ✔ | ✘ | ✔ |
| B7 | 56 | 0.6 | 2 | ✘ | ✘ | ✔ | ✔ |
| B8 | 45 | 0.2 | 2 | ✔ | ✘ | ✔ | ✘ |
| B9 | 59 | 0.5 | 1 | ✘ | ✘ | ✔ | ✘ |
| B10 | 47 | 0.2 | 2 | ✔ | ✘ | ✔ | ✔ |

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**But all benchmarks can be proven when analyses are combined using our technique**

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| OpenSSH clientloop | 1987 | 2.8 | 3 | 2.3 | 5 |
| Coreutils su | 1057 | 3.0 | 5 | 1.7 | 6 |
| GSL Histogram | 526 | 0.6 | 4 | 3.6 | 15 |
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**Fragments extracted for queries small in practice**

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