Subdivision Surfaces

Quiz Question

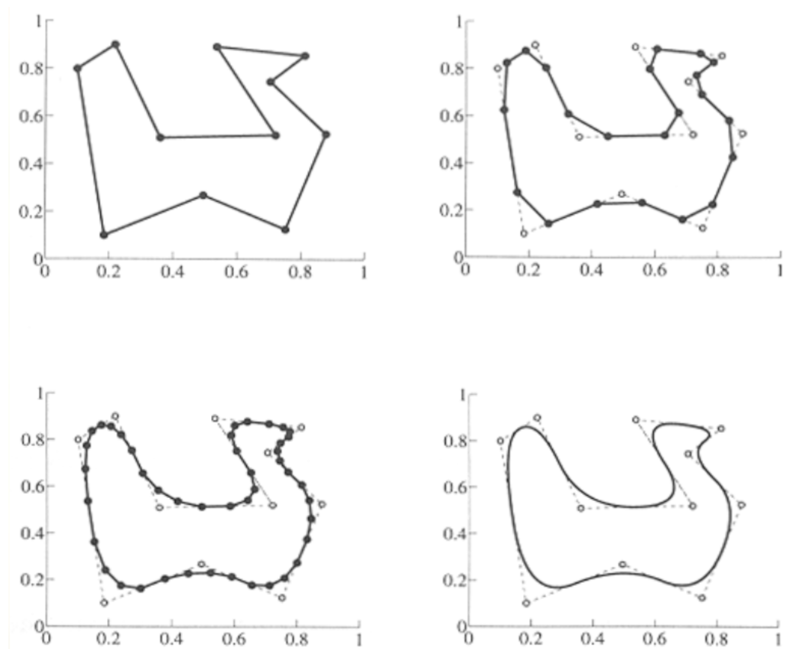
How would you create a higher resolution version from the original low resolution model?

Subdivision Curves

Idea: Repeatedly refine control polygon:

$$P^1 \rightarrow P^2 \rightarrow P^3$$

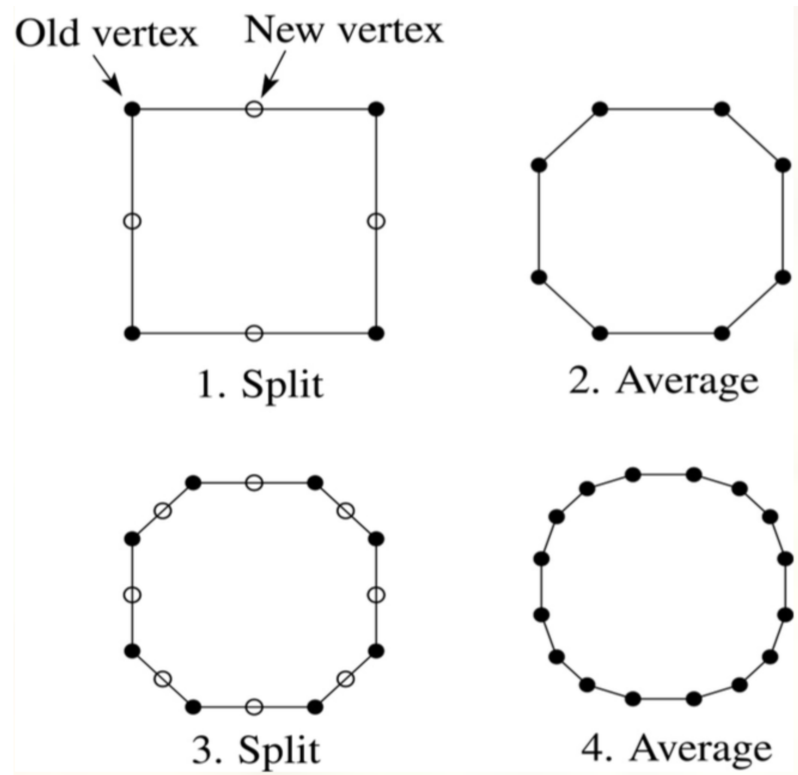
Curve will be limit of infinite process



Chaikin's Algorithm

"Corner-cutting" scheme

1. Start with piecewise linear curve
2. Insert new vertices at midpoint (splitting)
3. Average vertex with "next" neighbor (averaging)
4. Repeat splitting and averaging



Averaging Mask

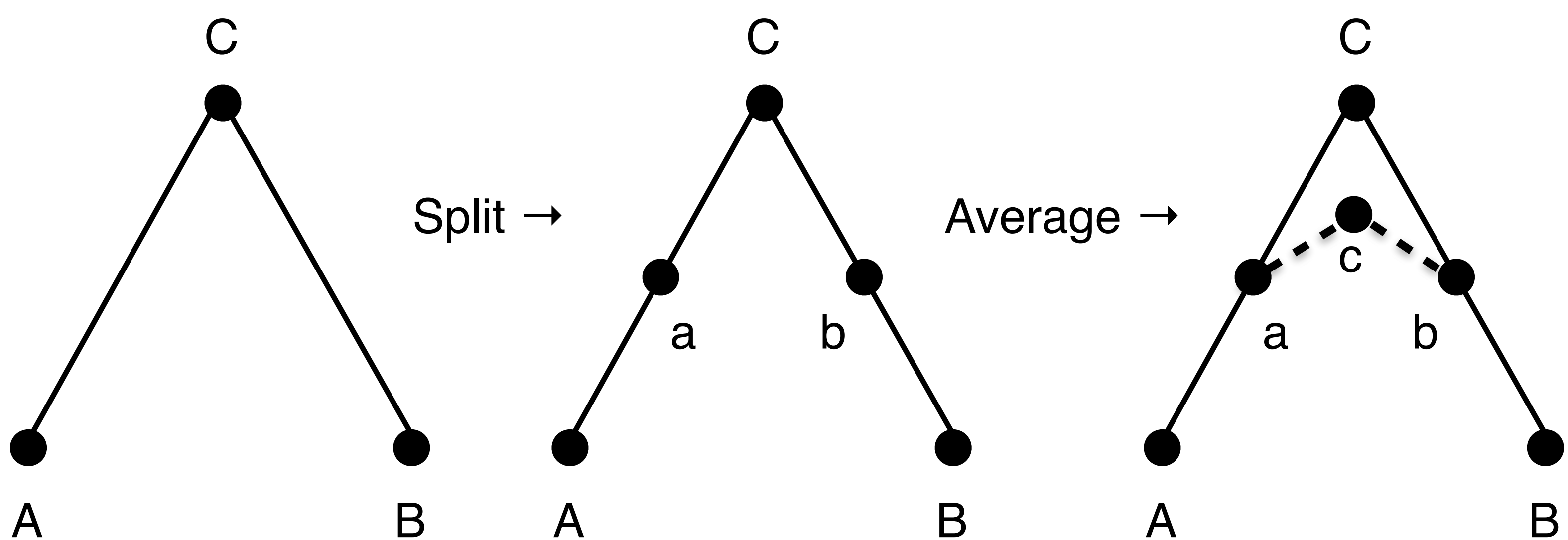
Rather than average with nearest neighbor, apply weighted averaging mask during averaging step:

$r = (\ldots, r_{-1}, r_0, r_1, \ldots)$

Chaikin's algorithm:

$r = (1/2, 1/2)$

Averaging Example

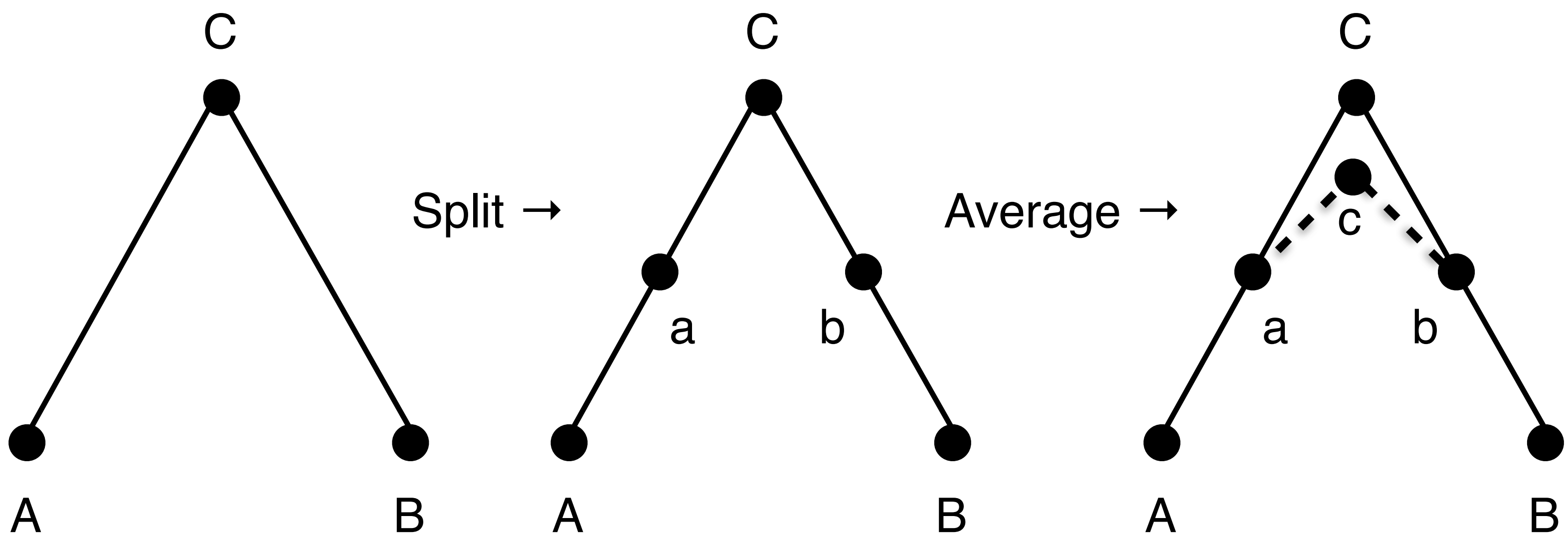


Averaging Example

Cubic B-spline subdivision mask: 1/4(1 2 1)

Split: a = 1/2(A+B), b = 1/2(B+C)

Average: c = 1/4(a + 2C + b)



Extending to Surfaces

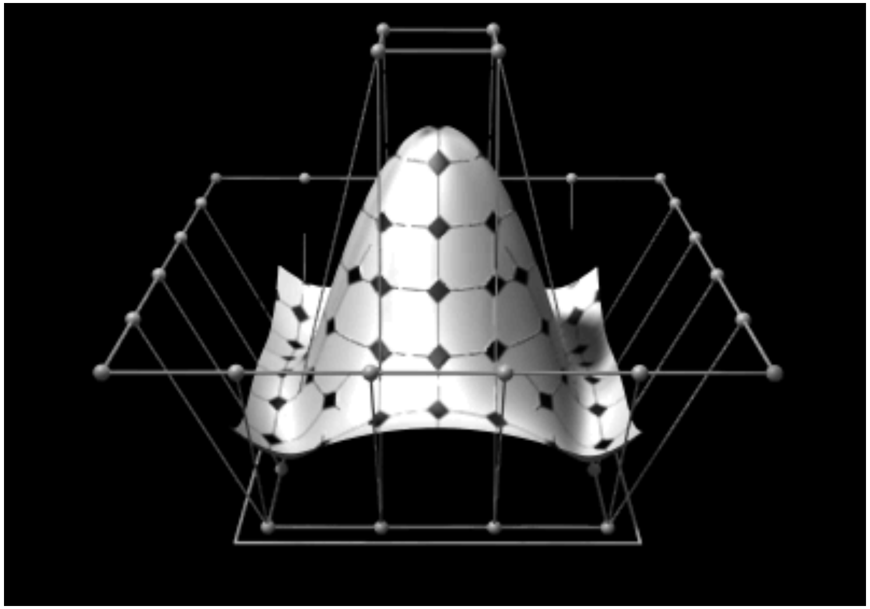
Subdivision curves extend to surfaces

Used in all major 3D modeling programs

Preserves lower polymeshs while allowing for high-quality models

NURBS

- Non-uniform rational basis splines
- Patches generated from curves
- Model curves and surfaces
- Intuitive control points
- Efficient evaluation

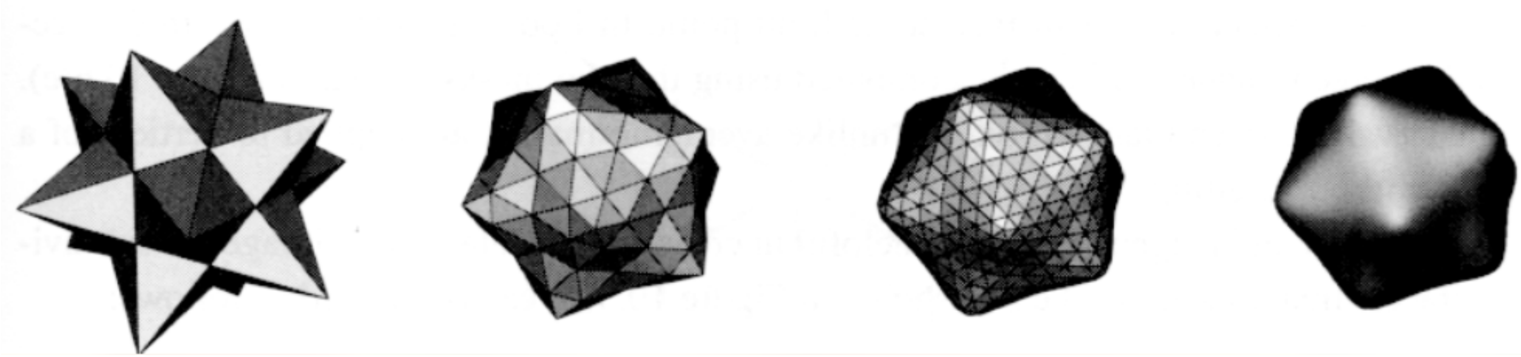


https://www.youtube.com/watch?v=m9U_XmnHQMU

Subdivision Surfaces

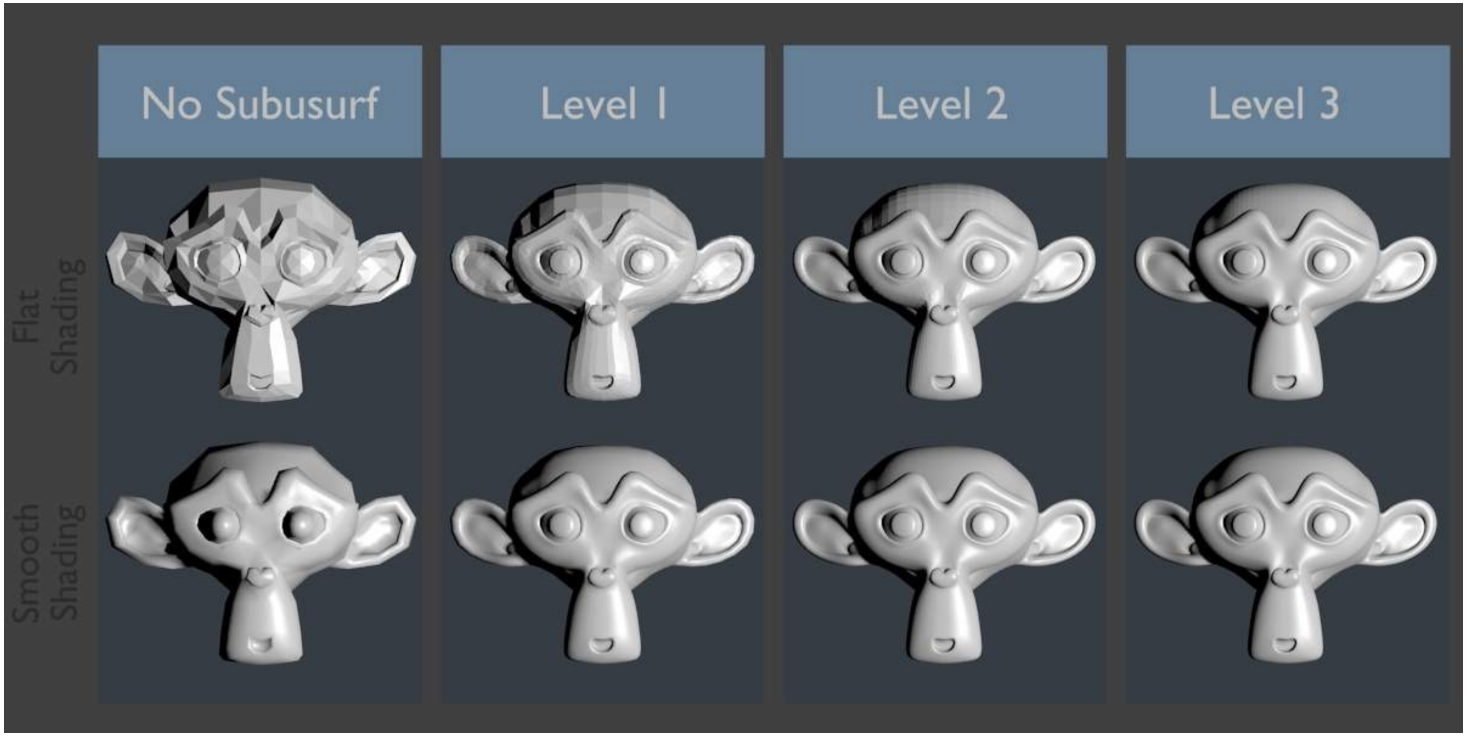
Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface using splitting and averaging steps

Allow for more regional control (good for artists)



Subdivisions for Modeling

https://www.youtube.com/watch?v=cUcif7nH4FM



Approximating Schemes

Limit surfaces approximate initial meshes

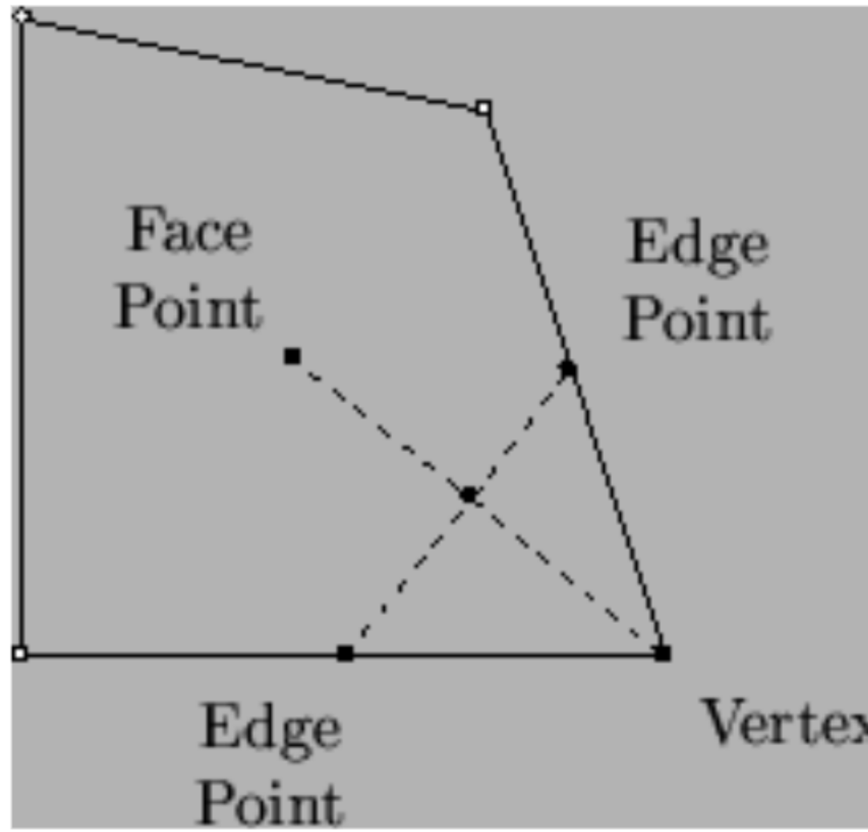
Generated control points not on surface

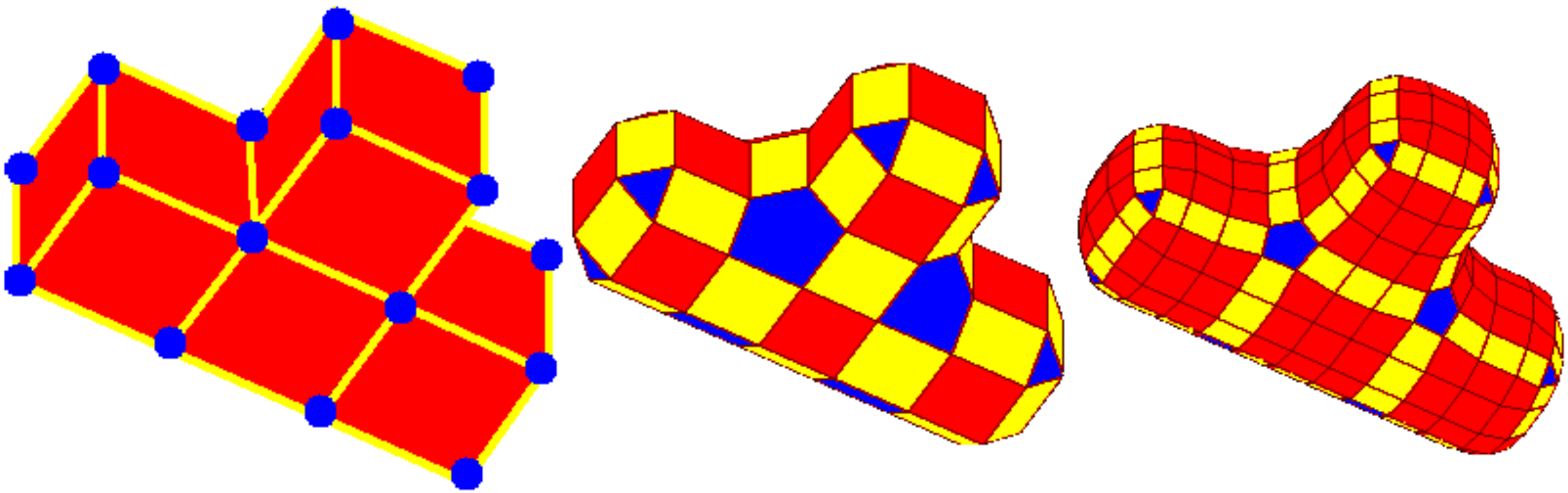
Examples:

- Doo-Sabin
- Catmull-Clark
- Loop

Doo-Sabin Scheme

- Edge points formed from midpoint of each edge
- Face point formed as centroid of polygon
- New vertex averages vertex, face point, and two edge points





Blue vertices and yellow edges show topological relationship to subdivision

Vertex Schemes

Vertices create more vertices:

- A vertex surrounded by n faces is split into n sub-vertices (one per face)

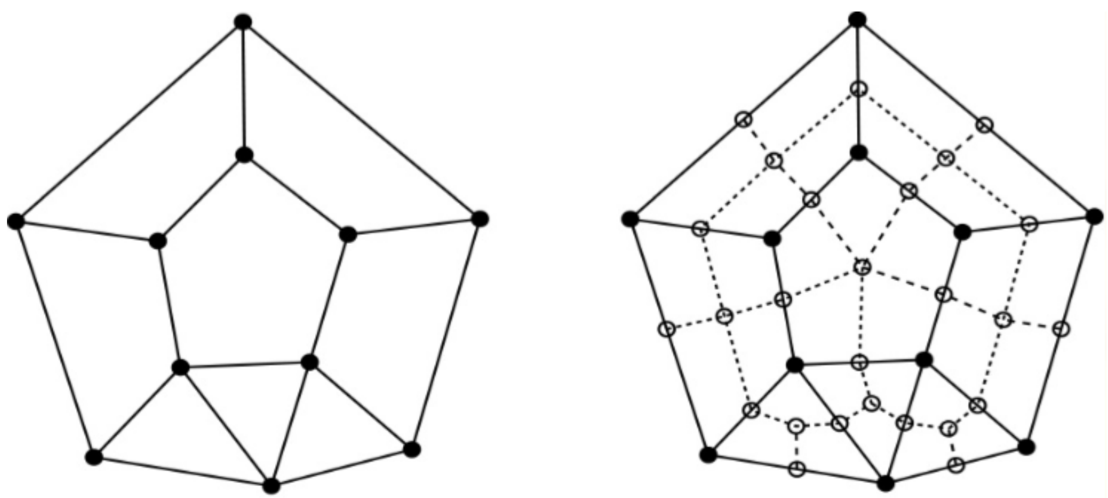
Note: **valence** is number of edges incident to a vertex

- **extraordinary vertices** do not have standard valence of topology (generally unavoidable)

Face Schemes

Faces create more faces

Can also insert vertices along edges and at face centroids



Original   After splitting

Catmull-Clark Scheme

For each face, create face point averaging original vertices

For each edge, create edge point averaging original end points and neighboring face points

For each face point, connect the face point to each edge point of the face

Catmull-Clark Scheme

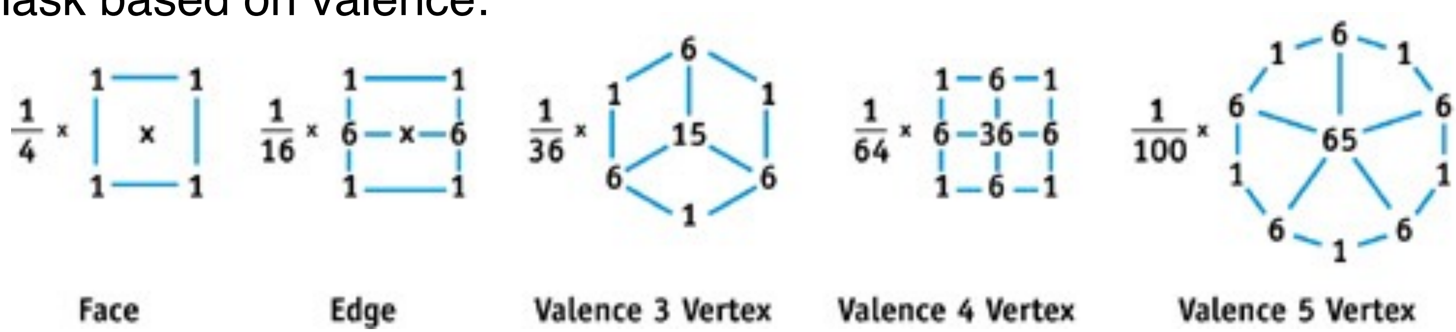
Move the original vertex (O) based on the valence (n) based on faces and edges

Average of all created face points: F

Average of all edge midpoints: E

$$newPosition = \frac{O(n-3) + F + 2E}{n}$$

Weight mask based on valence:

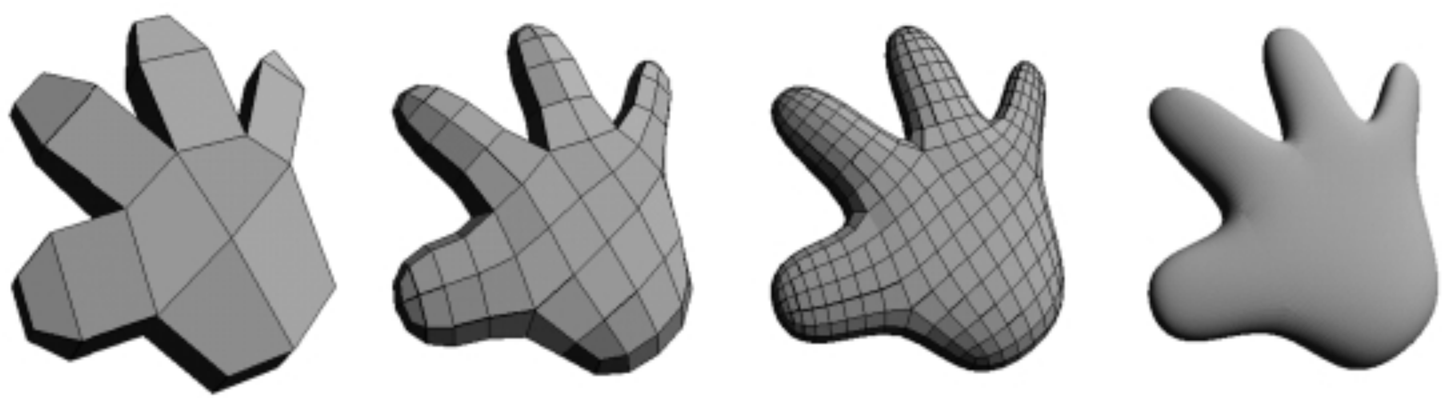


Catmull-Clark in Practice

Works best on quads (4:1 subdivision)

Turns all polys into quads

Common subdivision method in modern commercial tools



Finding the Limit

Possible to evaluate limit of Catmull-Clark surfaces without explicit subdivision

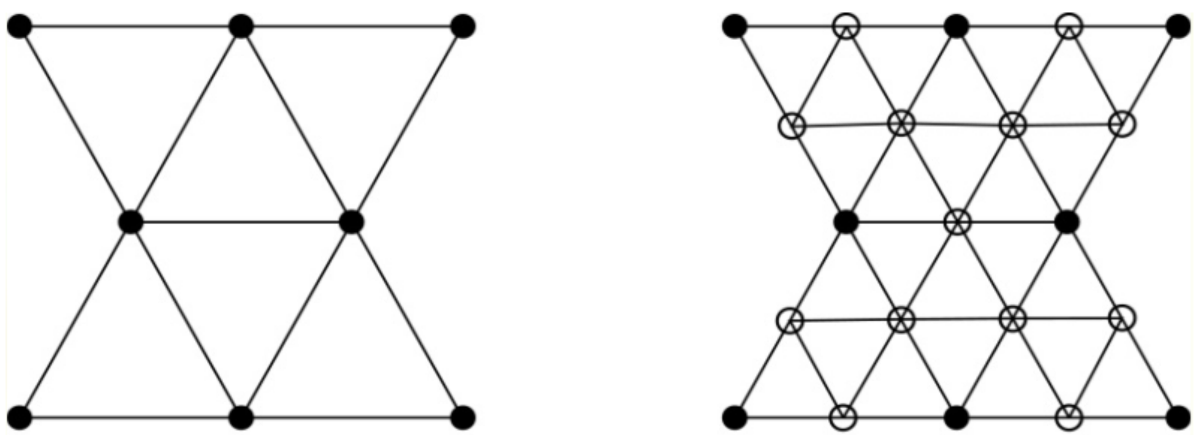
- Patches have same limit surface regardless of valence after subdivision
- Can be evaluated analytically as an eigenbasis

http://www.dgp.toronto.edu/people/stam/reality/Research/pdf/sig98.pdf

Loop Scheme

Subdivides triangles into smaller triangles (4:1 subdivision)

Each face is split into four subfaces based on weight mask



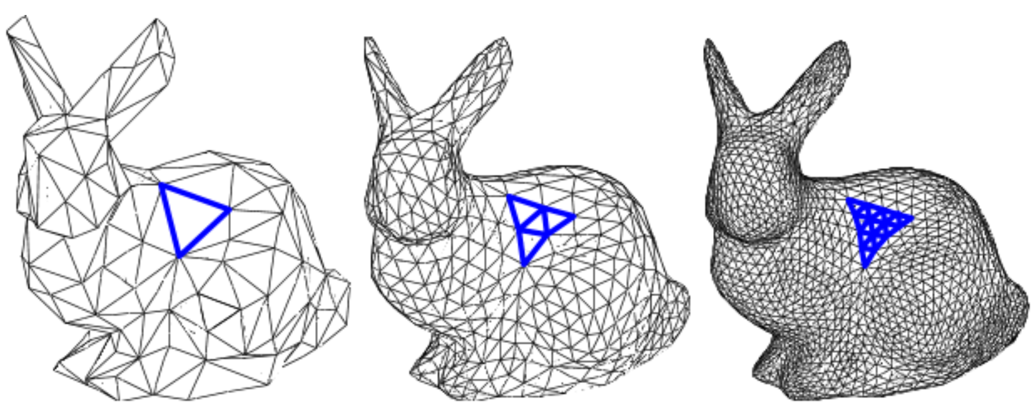
Original After splitting

Loop Scheme in Practice

Defined for triangle meshes (not a general algorithm)

Always has extraordinary vertices (valence not 6)

C^1 at extraordinary points, C^2 elsewhere



280 faces 1120 faces 4480 faces

Interpolation Schemes

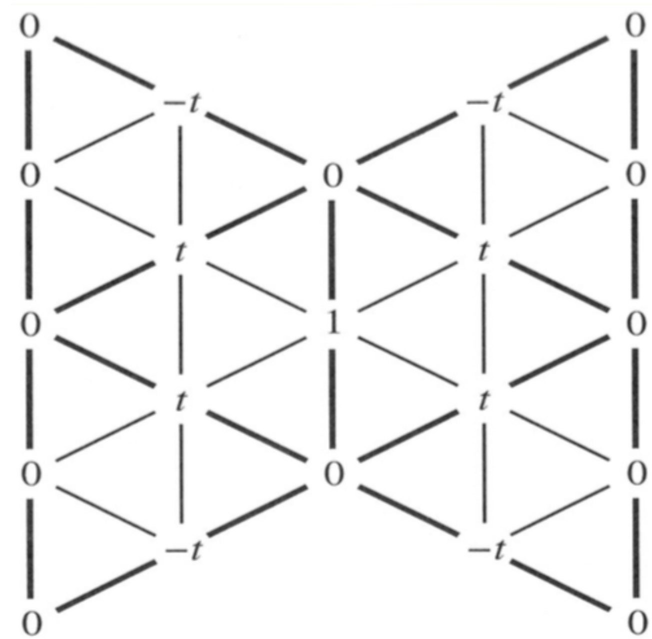
Original mesh’s control points and generated control points interpolated along limit’s surface

Examples:

- Butterfly scheme

Butterfly Subdivision

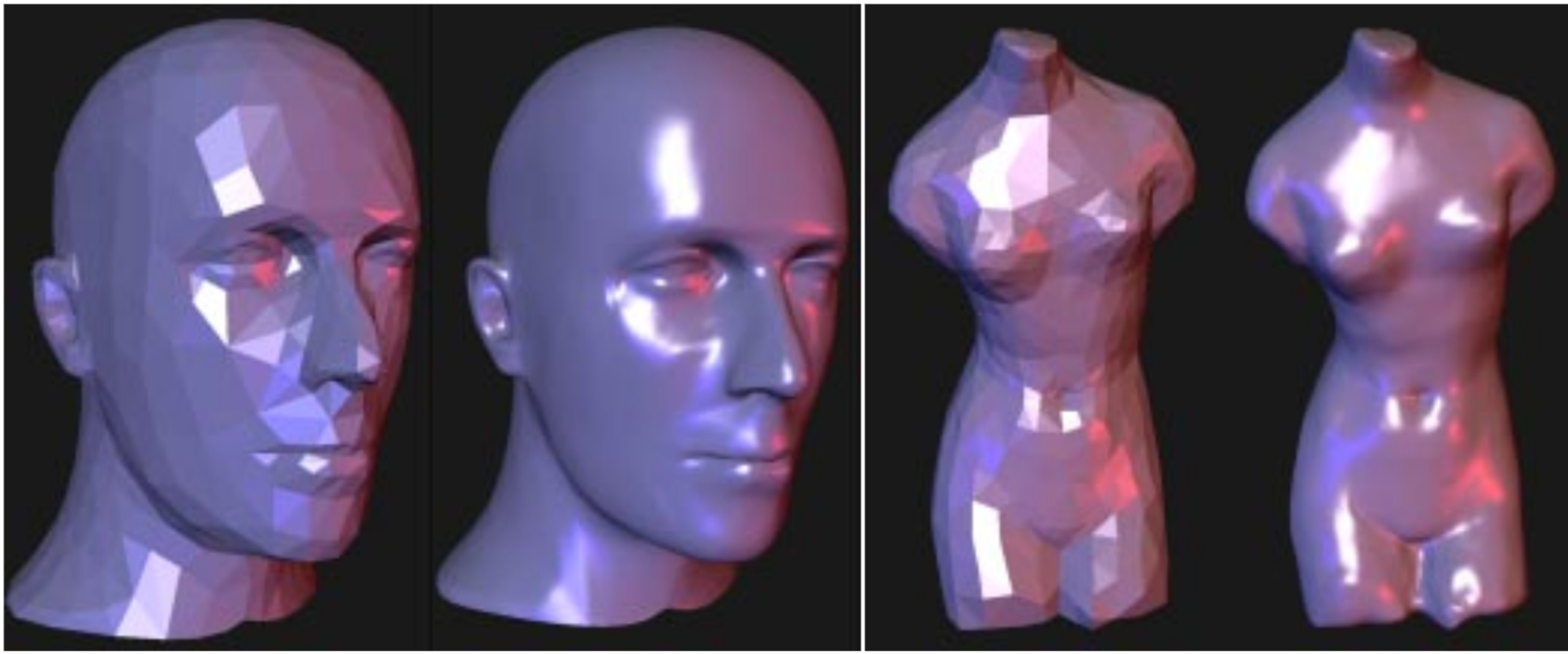
Averaging mask:



t = 0 gives original polyhedron

Smaller values of t smooths the surface

t = 1/8 has provable G^1 continuity



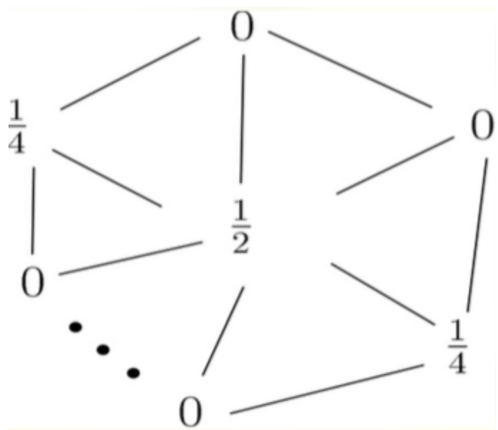
Preserving Creases

Sometimes we want features like creases to be preserved:

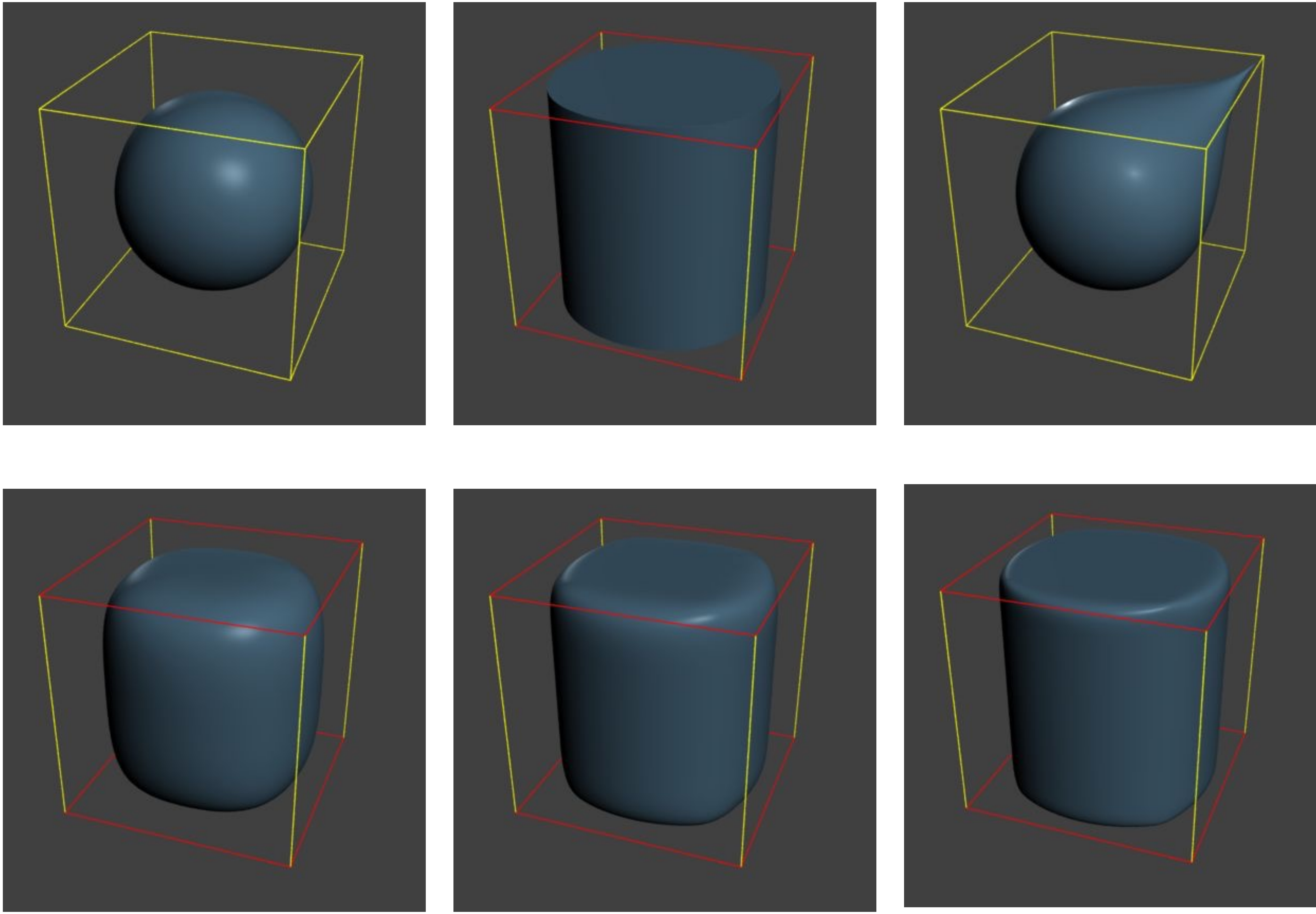
How to do this?

Trim Curves

Modify subdivision mask:



Results in G^0 continuity (no tangent plane continuity) https://www.youtube.com/watch?v=zDIn3ESrPEY



ZBrush Example

Allows for both subdivision and “dynamic” meshes

DynaMesh provides clay analogy in 3D

- Retopologizes mesh to match volume, resolution, and polygon distribution

In practice, both are used at different stages of art pipeline

https://pixologic.com/zclassroom/lesson/subdivision-vs-dynamesh

Additional Reading

http://http.developer.nvidia.com/GPUGems2/gpugems2_chapter07.html

http://graphics.pixar.com/library/Geri/paper.pdf