

CS344M

Autonomous Multiagent Systems

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Good Afternoon, Colleagues

Are there any questions?

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Logistics

- Progress reports due in 2 weeks

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- Readings for next week

Game Theory Premises

- Simultaneous actions
- No communication
- Outcome depends on **combination** of actions
- Utility (payoff) encapsulates **everything** about preferences over outcomes

Solution Concepts

- Dominant strategy
- Nash equilibrium
- Pareto optimality
- Maximum social welfare
- Maximin strategy

Prisoner's Dilemma

		Column	
		C(1)	D(2)
Row	C(1)	3, 3	0, 5
	D(2)	5, 0	1, 1

Chicken

		Column	
		C(1)	D(2)
Row	C(1)	3,3	1,5
	D(2)	5,1	0,0

Bach/Stravinsky

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- Propose a payoff matrix

Bach/Stravinsky

			Wife	
		S		B
Me	S	2,1		0,0
	B	0,0		1,2

Nash Equilibrium

- Does every game have a pure strategy Nash equilibrium?

Matching Pennies

- We each put a penny down covered
- If they match, I win, if they don't, you win

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		Player 2	
		H	T
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	T	-1, 1	1, -1

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- Not known if complexity of finding one is NP-complete or in P

Some theory

- Prove that if each player plays a dominant strategy, the result is a Nash equilibrium

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- Is the outcome of a Nash equilibrium necessarily Pareto optimal?
- Is a Pareto optimal outcome necessarily the result of Nash equilibrium strategies?
- Is the maximum social welfare outcome necessarily Pareto optimal?
- If both players play maximin, is it necessarily a Nash equilibrium?

Activity

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

Activity

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Player 1	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

Mixed strategy equilibrium

		Player 2	
		Action 1	Action 2
Player 1	Action 1	4,8	2,0
	Action 2	6,2	0,8

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Do actual numbers matter?

Rock/Paper/Scissors

- Nash equilibrium?

Rock/Paper/Scissors

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- Why is anything else **not** an equilibrium?

Rock/Paper/Scissors

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- Rock Paper Scissors tournament

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- Poker

Discussion

- What is an example game within robot soccer?

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			Goalie	
			Block	
		Right		Left
	Left	1, -1		-1, 1
Kicker				
	Right	-1, 1		1, -1

Discussion

- What is an example game within robot soccer?

			Goalie	
			Block	
		Right		Left
	Left	1, -1		-1, 1
Kicker				
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- Can we use game theory to devise better strategies?

Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky

		Wife	
		S	B
Me	S	2, 1	0, 0
	B	0, 0	1, 2

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Want only S,S or B,B - 50% each