Fages' Theorem

For any logic program, the conjunction of its completion formulas is entailed by the conjunction of its stability formulas. We would like to know under what conditions the opposite entailment holds also, so that the two conjunctions are equivalent.

About predicate constants P and Q we say that P *depends on* Q in a logic program Π if Π contains a rule such that

- its head has the form $P(\mathbf{t})$, and
- its body has an occurrence of Q that is not in the scope of any negation.

The *predicate dependency graph* of Π is the directed graph that

- has the predicate constants occurring in Π as its vertices, and
- has an edge from P to Q whenever P depends on Q in Π.

We say that Π is *tight* if its predicate dependency graph is acyclic.

Fages' Theorem [Fages, 1994]. *If a logic program is tight then the conjunction of its stability formulas is equivalent to the conjunction of its completion formulas.*

Problem 27. Find the stable models of each of the programs

$$\begin{array}{l} P(a), \\ P(b), \\ P(c), \\ Q(x) \leftarrow P(x) \wedge Q(x) \end{array}$$

and

$$\begin{array}{l} P(a), \\ P(b), \\ P(c), \\ Q(x) \leftarrow P(x) \wedge \neg\neg Q(x). \end{array}$$

References

[Fages, 1994] François Fages. Consistency of Clark's completion and existence of stable models. *Journal of Methods of Logic in Computer Science*, 1:51–60, 1994.