First-Order Formulas

In the following two problems, show that each of the given sets of sentences is satisfiable.

Problem 5.

  (i) $a \neq b$, $b = c$.

 (ii) $P(a)$, $Q(b)$, $\forall xy(x = y)$.

(iii) $\exists x P(x)$, $\exists x Q(x)$, $\neg \exists x(P(x) \wedge Q(x))$.

Problem 6.

  (i) $P(a, b)$, $\neg P(b, a)$, $\exists xy(P(x, y) \wedge P(y, x))$.

 (ii) $\forall x \exists y P(x, y)$, $\neg \exists y \forall x P(x, y)$.

The following two problems refer to a signature consisting of the object constant *Me*, the unary predicate constant *Male*, and the binary predicate constant *Parent*.

Problem 7. Consider the group of people consisting of your maternal grandmother and all her descendants. Describe it by an interpretation in the sense of first-order logic.

Problem 8. Express each of the given English sentences in logical notation.

  (i) I have no daughters.

 (ii) I have a granddaughter.

(iii) I have a brother.