Need for an Infinite Universe

Problem 10. (a) Find an interpretation satisfying the sentences

$$\forall x \exists y P(x, y),$$
$$\forall x y z (P(x, y) \land P(y, z) \rightarrow P(x, z)),$$
$$\neg \exists x P(x, x).$$

(b) Prove that any such interpretation has an infinite universe.