General Definition of a Stable Model

Let Π be a logic program, let $P_1, \dots, P_n$ be the list of all predicate constants in its signature, and let $p_1, \dots, p_n$ be distinct predicate variables. By $\Pi^\diamond(p_1, \dots, p_n)$ we denote the formula obtained by forming the conjnction of the universal closures of the rules of Π and then replacing the occurrences of $P_1, \dots, P_n$ that are not in the scope of any negation by the corresponding variables $p_1, \dots, p_n$.

As in the case of positive programs, the *stability formula* for P_i relative to Π is the sentence

$$\forall \mathbf{x}(P_i(\mathbf{x}) \leftrightarrow \forall p_1 \cdots p_n(\Pi^\diamond(p_1, \dots, p_n) \rightarrow p_i(\mathbf{x}))),$$

where $\mathbf{x}$ is a tuple of distinct object variables. A *stable model* of Π is an Herbrand interpretation that satisfies all stability formulas.

For instance, let Π be the program

$$\begin{array}{l} Person(A), \\ Person(B), \\ Male(A), \\ Female(x) \leftarrow Person(x) \wedge \neg Male(x). \end{array}$$

Then $\Pi^\diamond(p, m, f)$ is

$$p(A) \wedge p(B) \wedge m(A) \wedge \forall x(p(x) \wedge \neg Male(x) \rightarrow f(x)), \tag{1}$$

and the stability formulas are

$$\begin{array}{l} \forall y(Person(y) \leftrightarrow \forall pmf(\Pi^\diamond(p, m, f) \rightarrow p(y)), \\ \forall y(Male(y) \leftrightarrow \forall pmf(\Pi^\diamond(p, m, f) \rightarrow m(y)), \\ \forall y(Female(y) \leftrightarrow \forall pmf(\Pi^\diamond(p, m, f) \rightarrow f(y)). \end{array}$$

The conjunction of the stability formulas with the unique name assumption $A \neq B$ is equivalent to the completion of Π, that is, to the set consisting of the formulas

$$\begin{array}{l} \forall x(Person(x) \leftrightarrow x = A \vee x = B), \\ \forall x(Male(x) \leftrightarrow x = A), \\ \forall x(Female(x) \leftrightarrow Person(x) \wedge \neg Male(x)), \\ A \neq B. \end{array}$$

It follows that Π has one stable model:

$$\{Person(A),\ Person(B),\ Male(A),\ Female(B)\}.$$

Problem 25. If we replace $\neg Male(x)$ in (1) with $\neg m(x)$, how will this affect the result of simplifying the stability formulas?

Problem 26. Simplify the stability formulas of the program

$$\begin{aligned} P(x) &\leftarrow \neg Q(x), \\ Q(x) &\leftarrow \neg P(x). \end{aligned}$$