# Transitive Closure

The *Ancestor* predicate can be defined by adding the rules

$$\begin{array}{l} Ancestor(x,y) \leftarrow Parent(x,y), \\ Ancestor(x,y) \leftarrow Parent(x,z) \wedge Ancestor(z,y) \end{array} \tag{1}$$

to a group of facts defining *Parent*, such as

$$Parent(S,W),\ Parent(S,A),\ Parent(W,M). \tag{2}$$

The completion of the program consisting of rules (1) and (2) is complete on the level of ground atoms.

**Problem 18$^e$.** Use Prolog to verify this claim.

This is an application of a general idea: the transitive closure of a binary predicate $P$ can be defined by adding the rules

$$\begin{array}{l} Q(x,y) \leftarrow P(x,y), \\ Q(x,y) \leftarrow P(x,z) \wedge Q(z,y) \end{array} \tag{3}$$

to a group of facts defining $P$. If the graph defined by these facts is acyclic then the completion of the program will be complete on the level of ground atoms. Without the acyclicity assumption, this property may not hold.

**Problem 19.** Find all ground literals entailed by the completion of the program consisting of rules (3) and the facts

$$P(a,b),\ P(b,c),\ P(c,b).$$